Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.3-Tangent/98-4.3.0-a-trg-^m-b-tan-^n

Nasser M. Abbasi

December 9, 2023 Compiled on December 9, 2023 at 3:03am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [387]. This is test number [98].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (387)	0.00 (0)
Mathematica	100.00 (387)	0.00 (0)
Maple	68.99 (267)	31.01 (120)
Fricas	62.27 (241)	37.73 (146)
Maxima	35.40 (137)	64.60 (250)
Mupad	31.52 (122)	68.48 (265)
Giac	21.19 (82)	78.81 (305)
Sympy	4.65 (18)	95.35 (369)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

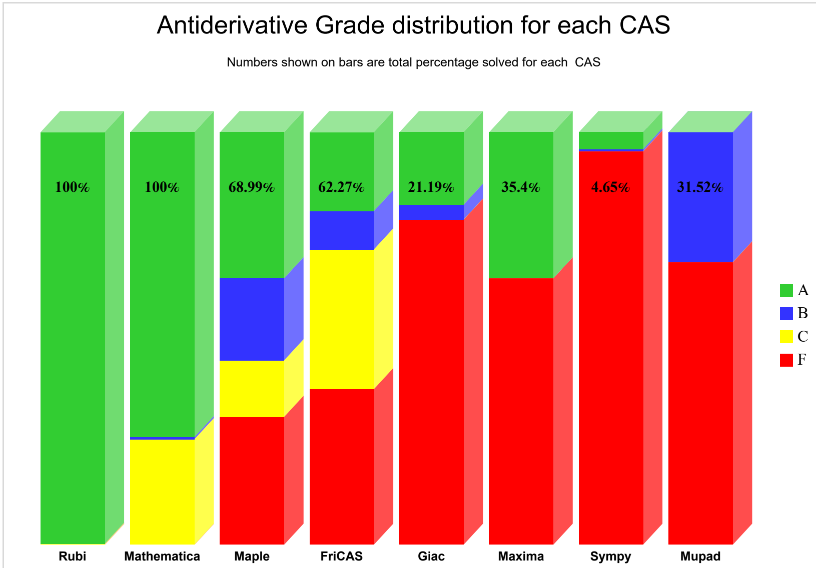
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

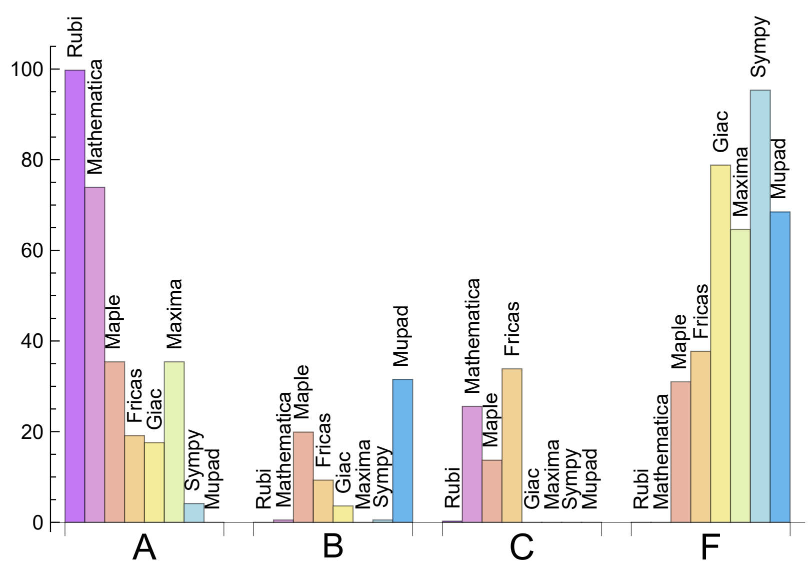
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	73.902	0.517	25.581	0.000
Maple	35.401	19.897	13.695	31.008
Maxima	35.401	0.000	0.000	64.599
Fricas	19.121	9.302	33.850	37.726
Giac	17.571	3.618	0.000	78.811
Sympy	4.134	0.517	0.000	95.349
Mupad	0.000	31.525	0.000	68.475

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	120	100.00	0.00	0.00
Fricas	146	95.89	0.00	4.11
Maxima	250	99.60	0.40	0.00
Mupad	265	0.00	100.00	0.00
Giac	305	86.23	6.23	7.54
Sympy	369	71.27	28.73	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.23
Maxima	0.35
Rubi	0.39
Giac	0.67
Mathematica	1.32
Mupad	4.55
Maple	6.11
Sympy	13.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	50.72	1.43	52.00	1.29
Mathematica	90.38	1.04	71.00	0.91
Rubi	98.91	0.97	77.00	1.00
Maxima	115.38	0.86	133.00	0.85
Mupad	146.28	2.64	77.50	1.10
Giac	206.54	2.66	176.00	1.01
Fricas	231.66	1.70	129.00	1.28
Maple	459.74	5.73	203.00	1.52

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

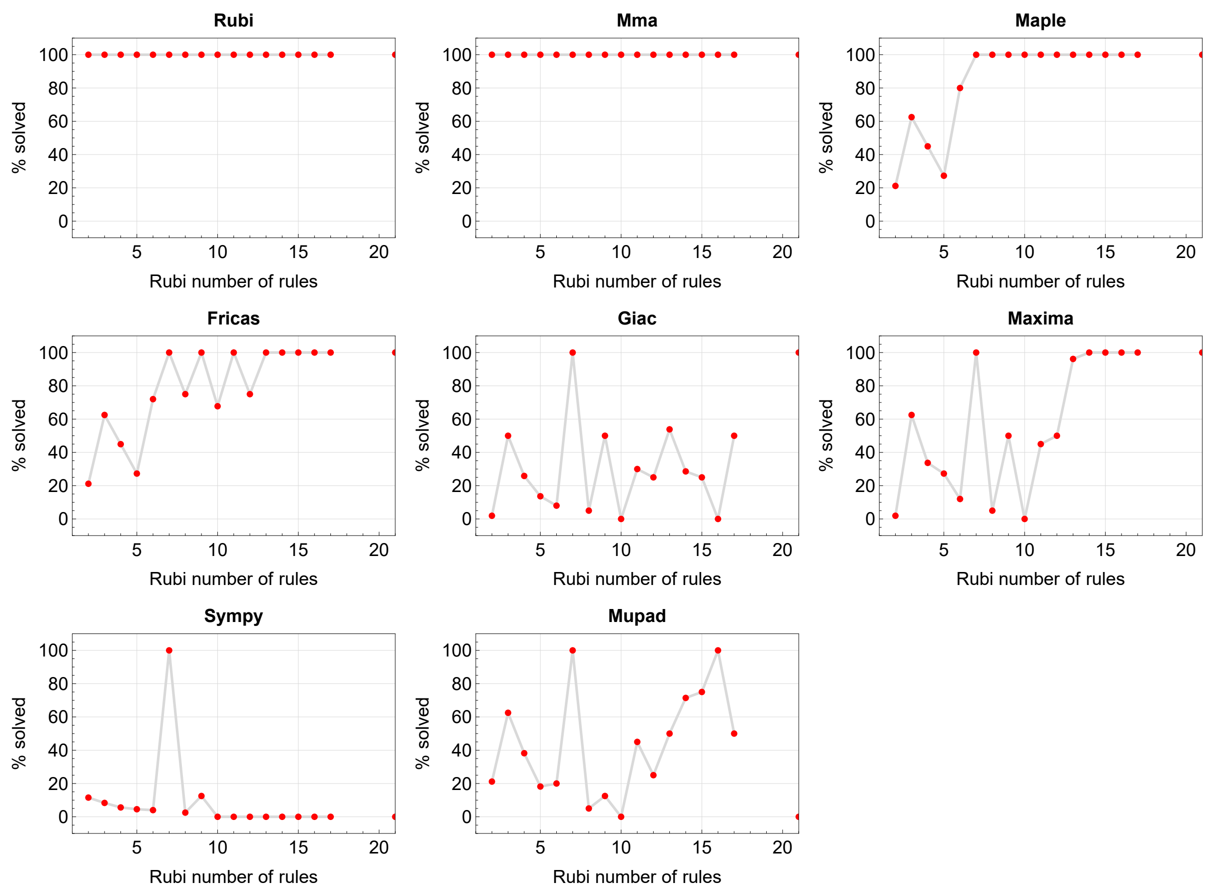


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

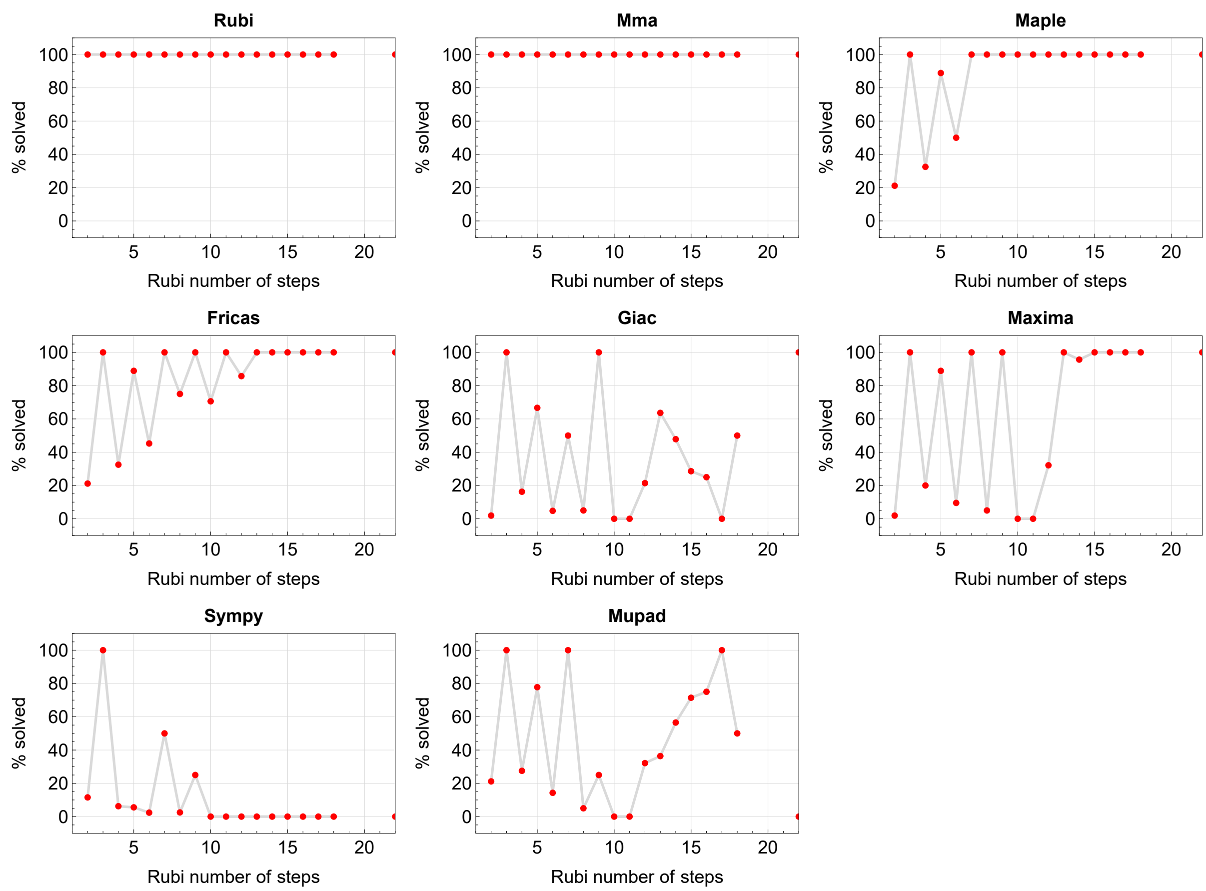


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

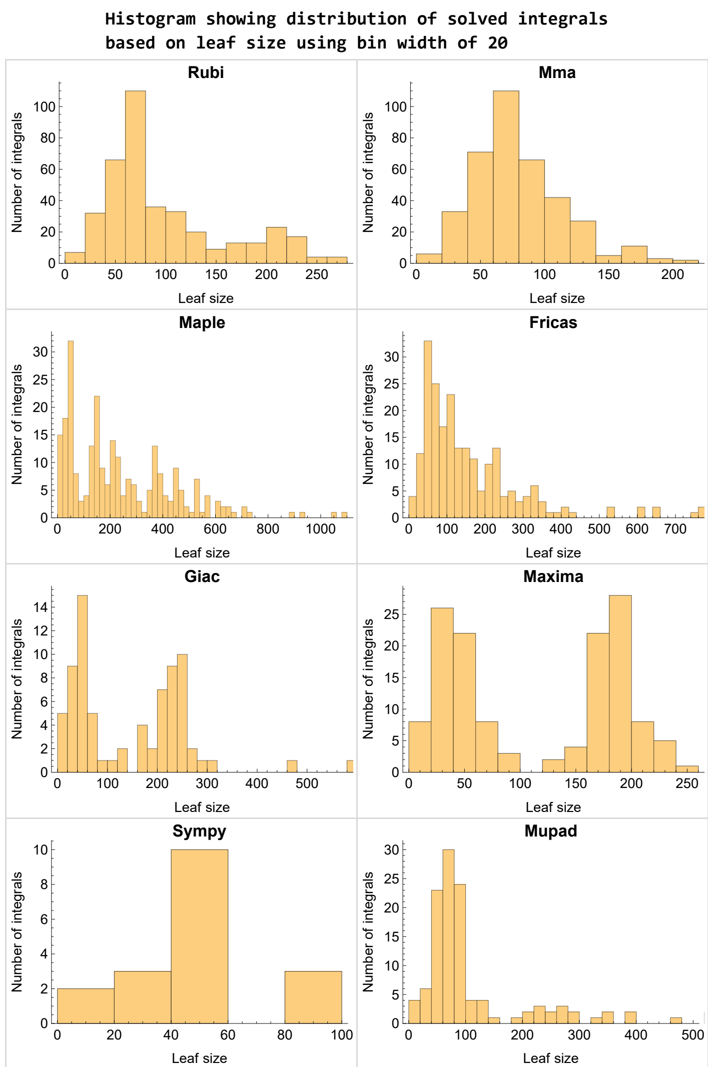


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

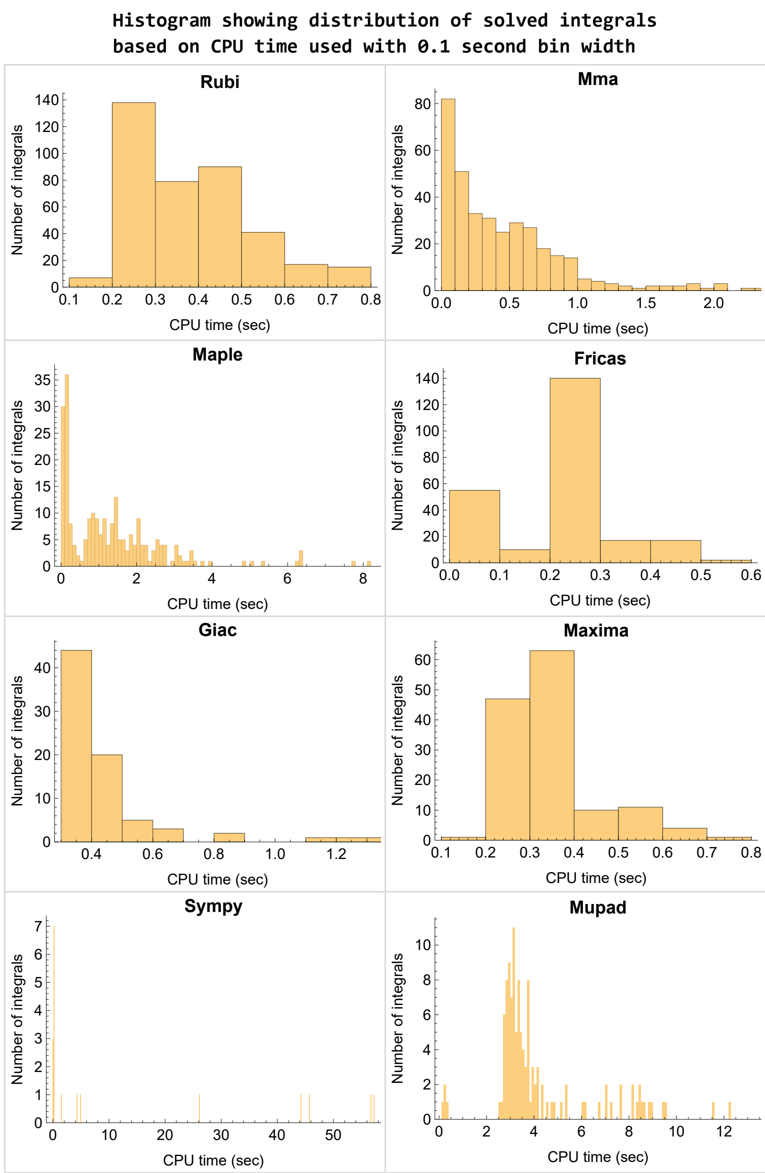


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

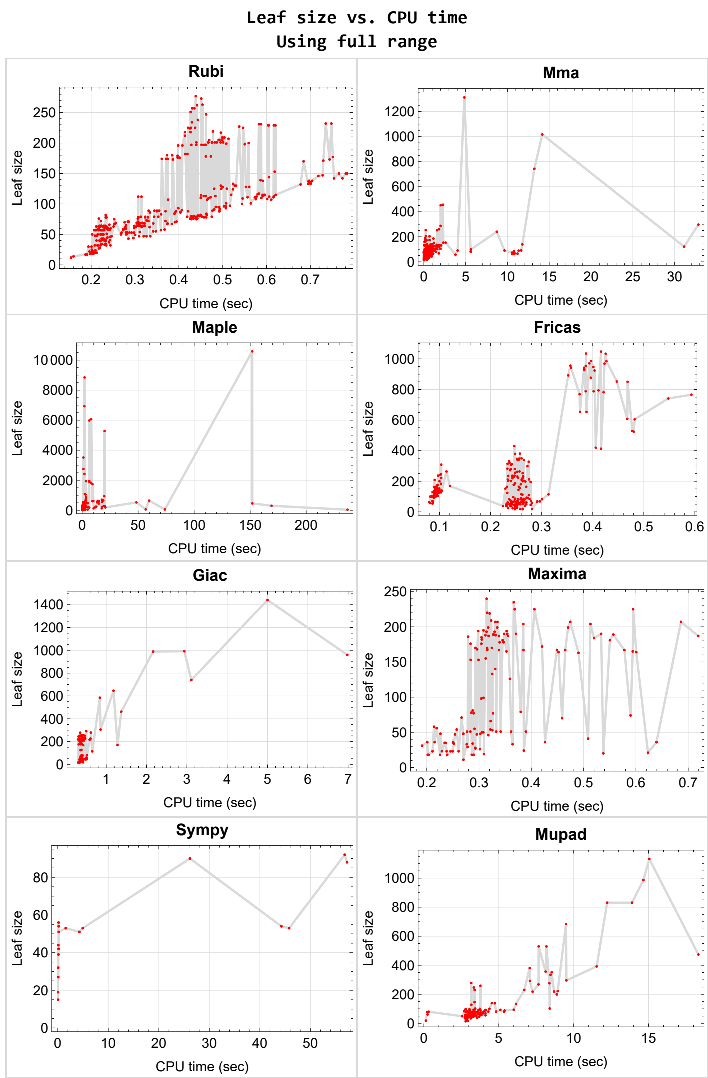


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 54, 55, 64, 65, 73, 74, 84, 85, 94, 95, 103, 104, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 229, 239, 250, 261, 291, 293, 300, 302, 307, 309, 311, 315, 317, 322, 328}

Mathematica {174, 180, 181, 183, 184, 185, 372, 373, 379, 387}

Maple {52, 54, 55, 59, 64, 65, 73, 74, 78, 84, 85, 94, 95, 99, 103, 104, 164, 165, 178, 179, 191, 192, 193, 194, 199, 200, 201, 202, 208, 214, 215, 230, 240, 245, 246, 251, 252, 257, 262, 323, 351, 352, 377, 378}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

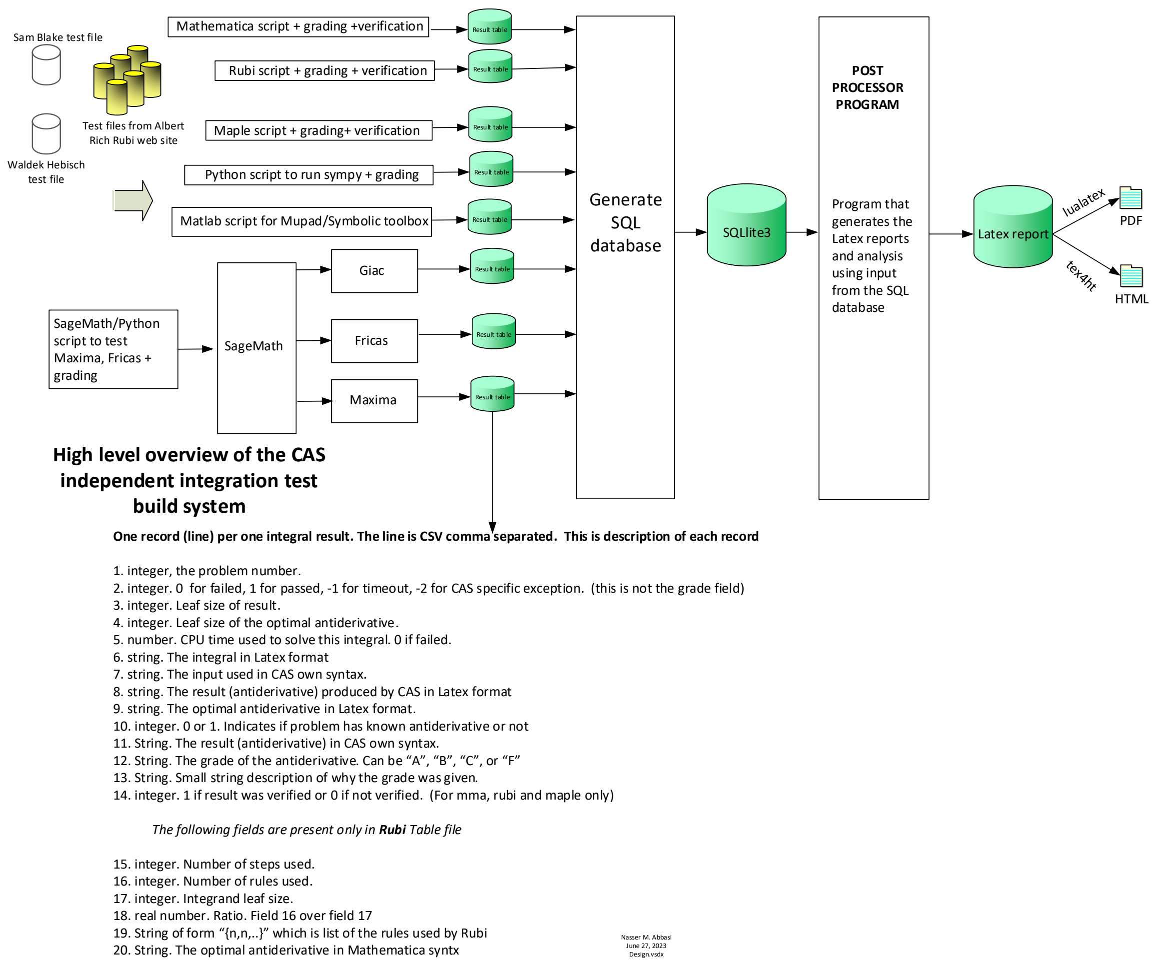
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 121, 123, 125, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 297, 300, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386 }

B grade { 379, 381 }

C grade { 17, 18, 21, 22, 39, 40, 41, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 128, 130, 132, 174, 180, 181, 183, 184, 185, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 292, 294, 296, 298, 299, 301, 302, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 372, 373, 387 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 56, 57, 58, 61, 66, 67, 68, 75, 76, 77, 79, 86, 87, 88, 96, 97, 98, 100, 101, 105, 106, 107, 118, 125, 127, 129, 131, 134, 135, 136, 137, 163, 177, 195, 196, 197, 198, 203, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 241, 247, 248, 249, 250, 253, 255, 258, 259, 260, 261, 291, 293, 295, 297, 300, 304, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 353, 363, 364, 365, 376 }

B grade { 54, 55, 60, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 102, 103, 104, 108, 109, 110, 111, 112, 113, 114, 121, 123, 133, 138, 191, 192, 193, 194, 199, 200, 201, 202, 207, 208, 209, 214, 215, 230, 231, 232, 233, 234, 235, 240, 242, 243, 244, 251, 252, 254, 256, 262, 263, 264, 265, 266, 267, 268, 269, 270, 302 }

C grade { 52, 59, 78, 99, 115, 116, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 164, 165, 178, 179, 245, 246, 257, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 351, 352, 377, 378 }

F normal fail { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 52, 57, 58, 66, 67, 68, 76, 77, 87, 88, 114, 116, 121, 123, 127, 134, 135, 163, 164, 165, 177, 178, 179, 226, 227, 236, 237, 247, 248, 258, 259, 295, 297, 306, 319, 321, 324, 326, 332, 334, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade { 22, 56, 75, 86, 96, 97, 98, 105, 106, 107, 118, 125, 129, 131, 133, 136, 137, 138, 228, 238, 249, 260, 291, 293, 300, 302, 304, 307, 309, 311, 313, 315, 317, 322, 328, 330 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 30, 31, 32, 33, 34, 35, 54, 55, 61, 62, 63, 64, 65, 71, 72, 73, 74, 80, 81, 82, 83, 84, 85, 92, 93, 94, 95, 101, 102, 103, 104, 112, 113, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 229, 230, 231, 232, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 255, 261, 262, 263, 264, 265, 269, 270, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333 }

F normal fail { 23, 42, 43, 44, 45, 53, 59, 60, 69, 70, 78, 79, 89, 90, 91, 99, 100, 108, 109, 110, 111, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 233, 234, 235, 244, 245, 246, 256, 257, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { }

F(-2) exception fail { 46, 47, 48, 49, 50, 51 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 164, 165, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade { }

C grade { }

F normal fail { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { 286 }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 12, 13, 17, 18, 19, 20, 22, 26, 30, 31, 32, 33, 34, 35, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 226, 227, 228, 229, 230, 236, 237, 238, 240, 247, 248, 249, 251, 252, 258, 259, 260, 262, 376 }

B grade { 2, 3, 4, 5, 6, 7, 8, 24, 25, 28, 29, 36, 37, 38 }

C grade { }

F normal fail { 11, 15, 21, 23, 27, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 61, 62, 63, 71, 72, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 137, 138, 142, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 235, 239, 241, 242, 243, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-1) timedout fail { 9, 10, 14, 16, 124, 134, 135, 136, 139, 140, 141, 143, 144, 149, 250, 261, 307, 308, 309 }

F(-2) exception fail { 59, 69, 70, 78, 79, 114, 115, 116, 122, 145, 146, 147, 148, 150, 151, 152, 234, 244, 245, 246, 363, 364, 365 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 56, 57, 58, 66, 67, 68, 75, 76, 77, 86, 87, 88, 96, 97, 98, 105, 106, 107, 114, 116, 121, 123, 127, 129, 134, 135, 136, 163, 164, 165, 177, 178, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 247, 248, 249, 250, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 351, 352, 353, 364, 365, 376, 377, 378 }

C grade { }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 122, 124, 125, 126, 128, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 295, 304, 319, 321, 324, 326, 330, 332 }

B grade { 353, 376 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 65, 70, 71, 84, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 111, 112, 113, 116, 117, 118, 130, 131, 137, 146, 147, 148, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 241, 242, 243, 244, 249, 250, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 301, 302, 303, 317, 318, 320, 323, 325, 331, 336, 337, 338, 340, 341, 342, 344, 345, 346, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 374, 375, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387 }

F(-1) timedout fail { 58, 59, 64, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 89, 90, 99, 107, 108, 109, 110, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 157, 158, 159, 160, 170, 179, 180, 185, 234, 235, 240, 245, 246, 247, 248, 251, 252, 257, 267, 268, 291, 297, 298, 299, 300, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 322, 327, 328, 329, 333, 334, 335, 339, 343, 347, 373, 383 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33
time (sec)	N/A	0.154	0.011	0.035	0.270	0.258	0.063	0.313	2.944

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	15	18	17	15	226	14
N.S.	1	1.00	1.64	1.07	1.29	1.21	1.07	16.14	1.00
time (sec)	N/A	0.165	0.004	0.031	0.286	0.248	0.074	0.423	2.793

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	28	31	27	32	216	30
N.S.	1	1.00	0.93	1.04	1.15	1.00	1.19	8.00	1.11
time (sec)	N/A	0.220	0.036	0.050	0.191	0.249	0.082	0.588	2.797

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	27	29	26	27	585	24
N.S.	1	1.00	1.36	0.96	1.04	0.93	0.96	20.89	0.86
time (sec)	N/A	0.232	0.005	0.046	0.278	0.252	0.104	0.842	2.881

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	38	54	39	44	462	38
N.S.	1	1.00	0.86	0.88	1.26	0.91	1.02	10.74	0.88
time (sec)	N/A	0.292	0.041	0.088	0.322	0.231	0.125	1.373	2.739

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	53	39	41	38	39	989	35
N.S.	1	1.00	1.20	0.89	0.93	0.86	0.89	22.48	0.80
time (sec)	N/A	0.300	0.017	0.063	0.509	0.225	0.140	2.163	2.907

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	49	74	51	56	740	49
N.S.	1	1.00	0.82	0.86	1.30	0.89	0.98	12.98	0.86
time (sec)	N/A	0.360	0.076	0.104	0.590	0.237	0.172	3.112	2.561

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	49	51	48	51	1441	44
N.S.	1	1.00	1.17	0.84	0.88	0.83	0.88	24.84	0.76
time (sec)	N/A	0.358	0.011	0.086	0.277	0.231	0.196	5.001	2.986

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	227	174	169	186	203	0	0	93
N.S.	1	0.98	0.75	0.73	0.80	0.88	0.00	0.00	0.40
time (sec)	N/A	0.548	0.684	0.195	0.279	0.235	0.000	0.000	3.383

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	101	154	176	202	0	0	74
N.S.	1	0.93	0.48	0.73	0.83	0.95	0.00	0.00	0.35
time (sec)	N/A	0.457	0.195	0.056	0.284	0.240	0.000	0.000	3.150

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	201	160	149	170	177	0	0	73
N.S.	1	0.96	0.76	0.71	0.81	0.84	0.00	0.00	0.35
time (sec)	N/A	0.475	0.126	0.053	0.293	0.245	0.000	0.000	3.135

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	174	71	136	153	164	0	176	49
N.S.	1	0.91	0.37	0.71	0.80	0.85	0.00	0.92	0.26
time (sec)	N/A	0.386	0.059	0.168	0.299	0.232	0.000	0.369	2.773

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	180	131	138	155	152	0	184	59
N.S.	1	0.94	0.68	0.72	0.81	0.79	0.00	0.96	0.31
time (sec)	N/A	0.380	0.081	0.099	0.313	0.229	0.000	0.385	3.121

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	82	157	167	235	0	0	76
N.S.	1	0.93	0.39	0.74	0.79	1.11	0.00	0.00	0.36
time (sec)	N/A	0.465	0.099	0.057	0.316	0.235	0.000	0.000	2.971

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	205	86	157	168	236	0	0	75
N.S.	1	0.96	0.40	0.73	0.79	1.10	0.00	0.00	0.35
time (sec)	N/A	0.479	0.145	0.056	0.296	0.247	0.000	0.000	3.355

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	225	96	171	195	256	0	0	92
N.S.	1	0.96	0.41	0.73	0.83	1.09	0.00	0.00	0.39
time (sec)	N/A	0.563	0.221	0.055	0.309	0.248	0.000	0.000	3.328

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	217	205	215	185	312	0	209	247
N.S.	1	0.89	0.84	0.88	0.76	1.28	0.00	0.86	1.02
time (sec)	N/A	0.498	0.246	0.174	0.305	0.241	0.000	0.445	3.339

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	196	185	191	168	314	0	206	259
N.S.	1	0.88	0.83	0.85	0.75	1.40	0.00	0.92	1.16
time (sec)	N/A	0.412	0.156	0.163	0.316	0.233	0.000	0.397	3.778

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	112	106	108	98	124	0	127	146
N.S.	1	0.85	0.81	0.82	0.75	0.95	0.00	0.97	1.11
time (sec)	N/A	0.319	0.228	0.106	0.304	0.233	0.000	0.385	3.387

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	112	100	108	99	299	0	125	128
N.S.	1	0.85	0.76	0.82	0.76	2.28	0.00	0.95	0.98
time (sec)	N/A	0.334	0.098	0.095	0.308	0.272	0.000	0.362	3.174

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	196	189	203	170	282	0	0	230
N.S.	1	0.88	0.84	0.91	0.76	1.26	0.00	0.00	1.03
time (sec)	N/A	0.427	0.170	0.135	0.315	0.233	0.000	0.000	3.393

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	219	254	215	182	430	0	227	278
N.S.	1	0.89	1.04	0.88	0.74	1.76	0.00	0.93	1.13
time (sec)	N/A	0.482	0.229	0.098	0.305	0.247	0.000	0.477	3.166

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	67	56	58	47	74	0	646	0
N.S.	1	0.68	0.57	0.59	0.48	0.76	0.00	6.59	0.00
time (sec)	N/A	0.399	0.280	0.161	0.296	0.239	0.000	1.181	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	49	47	48	34	52	0	226	0
N.S.	1	0.80	0.77	0.79	0.56	0.85	0.00	3.70	0.00
time (sec)	N/A	0.315	0.117	0.065	0.288	0.241	0.000	0.611	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	19	38	0	23	0
N.S.	1	1.00	1.00	1.16	0.59	1.19	0.00	0.72	0.00
time (sec)	N/A	0.245	0.029	0.066	0.309	0.241	0.000	0.361	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	33	39	45	33	50	0	0	34
N.S.	1	1.06	1.26	1.45	1.06	1.61	0.00	0.00	1.10
time (sec)	N/A	0.242	0.066	0.072	0.364	0.233	0.000	0.000	3.136

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	54	56	63	46	69	0	169	0
N.S.	1	0.82	0.85	0.95	0.70	1.05	0.00	2.56	0.00
time (sec)	N/A	0.323	0.257	0.075	0.318	0.251	0.000	0.428	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	70	66	74	66	82	0	225	0
N.S.	1	0.72	0.68	0.76	0.68	0.85	0.00	2.32	0.00
time (sec)	N/A	0.414	0.194	0.083	0.322	0.237	0.000	0.440	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	232	204	263	178	332	0	291	0
N.S.	1	0.64	0.56	0.72	0.49	0.91	0.00	0.80	0.00
time (sec)	N/A	0.785	0.678	0.173	0.339	0.236	0.000	0.504	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	198	115	236	140	279	0	253	0
N.S.	1	0.69	0.40	0.83	0.49	0.98	0.00	0.88	0.00
time (sec)	N/A	0.582	0.480	0.072	0.331	0.239	0.000	0.391	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	178	162	205	133	272	0	195	0
N.S.	1	0.70	0.64	0.80	0.52	1.07	0.00	0.76	0.00
time (sec)	N/A	0.486	0.176	0.073	0.325	0.238	0.000	0.354	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	178	87	211	126	307	0	251	0
N.S.	1	0.70	0.34	0.83	0.49	1.20	0.00	0.98	0.00
time (sec)	N/A	0.489	0.263	0.075	0.359	0.240	0.000	0.465	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	200	98	233	163	320	0	279	0
N.S.	1	0.67	0.33	0.78	0.55	1.07	0.00	0.94	0.00
time (sec)	N/A	0.573	0.212	0.075	0.490	0.237	0.000	0.618	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	232	139	272	172	348	0	305	0
N.S.	1	0.64	0.38	0.75	0.47	0.96	0.00	0.84	0.00
time (sec)	N/A	0.741	0.401	0.079	0.421	0.256	0.000	0.860	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	100	86	84	79	96	0	960	0
N.S.	1	0.55	0.47	0.46	0.43	0.53	0.00	5.27	0.00
time (sec)	N/A	0.570	0.598	0.233	0.380	0.245	0.000	6.981	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	68	66	64	53	62	0	992	0
N.S.	1	0.62	0.60	0.58	0.48	0.56	0.00	9.02	0.00
time (sec)	N/A	0.387	0.547	0.062	0.325	0.234	0.000	2.937	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	37	41	42	26	37	0	229	0
N.S.	1	0.74	0.82	0.84	0.52	0.74	0.00	4.58	0.00
time (sec)	N/A	0.247	0.075	0.063	0.302	0.231	0.000	0.458	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	38	43	40	27	39	0	45	0
N.S.	1	0.75	0.84	0.78	0.53	0.76	0.00	0.88	0.00
time (sec)	N/A	0.241	0.048	0.069	0.293	0.238	0.000	0.472	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	71	45	63	50	62	0	114	0
N.S.	1	0.60	0.38	0.53	0.42	0.52	0.00	0.96	0.00
time (sec)	N/A	0.396	0.038	0.071	0.306	0.240	0.000	0.653	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	101	45	83	70	82	0	169	0
N.S.	1	0.55	0.25	0.45	0.38	0.45	0.00	0.92	0.00
time (sec)	N/A	0.563	0.036	0.094	0.459	0.259	0.000	1.280	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.102	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	70	56	0	0	0	0	0	0
N.S.	1	1.25	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	61	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	32	32	32	5979	0	23	0	0	0
N.S.	1	1.00	1.00	186.84	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.248	0.020	6.385	0.000	0.243	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	251	122	619	225	947	0	245	0
N.S.	1	0.98	0.47	2.41	0.88	3.68	0.00	0.95	0.00
time (sec)	N/A	0.430	0.490	13.869	0.406	0.401	0.000	0.314	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	212	104	529	194	934	0	219	0
N.S.	1	0.93	0.46	2.33	0.85	4.11	0.00	0.96	0.00
time (sec)	N/A	0.424	0.324	0.920	0.353	0.383	0.000	0.309	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	37	0	16	48
N.S.	1	1.00	1.00	0.94	1.28	2.06	0.00	0.89	2.67
time (sec)	N/A	0.205	0.324	0.207	0.250	0.239	0.000	0.341	3.172

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	30	43	33	63	0	43	102
N.S.	1	0.95	0.73	1.05	0.80	1.54	0.00	1.05	2.49
time (sec)	N/A	0.238	0.322	0.800	0.276	0.243	0.000	0.336	8.404

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	58	50	52	48	82	0	58	356
N.S.	1	0.92	0.79	0.83	0.76	1.30	0.00	0.92	5.65
time (sec)	N/A	0.241	0.366	0.751	0.303	0.267	0.000	0.355	8.128

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	108	139	1740	0	0	0	0	0
N.S.	1	1.03	1.32	16.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	11.785	9.250	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	200	0	0	0	0	0
N.S.	1	1.00	0.76	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.440	3.786	0.740	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	108	0	54	0	0	0
N.S.	1	1.00	1.55	2.30	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.338	0.251	0.630	0.000	0.086	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	115	218	0	113	0	0	0
N.S.	1	1.00	1.49	2.83	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.450	0.667	0.750	0.000	0.090	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	110	124	345	0	157	0	0	0
N.S.	1	1.05	1.18	3.29	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.597	1.344	0.699	0.000	0.095	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	277	123	890	235	954	0	252	0
N.S.	1	1.00	0.44	3.21	0.85	3.44	0.00	0.91	0.00
time (sec)	N/A	0.467	0.769	3.462	0.366	0.386	0.000	0.378	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	247	238	113	1083	204	942	0	226	0
N.S.	1	0.96	0.46	4.38	0.83	3.81	0.00	0.91	0.00
time (sec)	N/A	0.454	0.464	2.329	0.385	0.383	0.000	0.330	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	24	0	16	43
N.S.	1	1.00	1.00	0.94	1.28	1.33	0.00	0.89	2.39
time (sec)	N/A	0.212	0.342	0.159	0.245	0.234	0.000	0.324	2.882

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	30	38	34	51	0	43	100
N.S.	1	0.95	0.73	0.93	0.83	1.24	0.00	1.05	2.44
time (sec)	N/A	0.242	0.287	0.638	0.252	0.249	0.000	0.359	3.741

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	58	42	47	58	71	0	64	292
N.S.	1	0.92	0.67	0.75	0.92	1.13	0.00	1.02	4.63
time (sec)	N/A	0.252	0.383	0.578	0.214	0.251	0.000	0.365	7.078

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	111	90	409	0	0	0	0	0
N.S.	1	1.01	0.82	3.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.635	4.041	0.815	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	58	395	0	0	0	0	0
N.S.	1	1.00	0.76	5.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.318	0.769	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	381	0	136	0	0	0
N.S.	1	1.00	0.80	5.01	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.457	0.339	0.625	0.000	0.098	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	138	71	371	0	176	0	0	0
N.S.	1	1.35	0.70	3.64	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.613	0.643	0.668	0.000	0.096	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	277	273	142	526	240	1048	0	278	0
N.S.	1	0.99	0.51	1.90	0.87	3.78	0.00	1.00	0.00
time (sec)	N/A	0.464	0.830	48.450	0.315	0.416	0.000	0.354	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	247	234	126	500	209	1035	0	252	0
N.S.	1	0.95	0.51	2.02	0.85	4.19	0.00	1.02	0.00
time (sec)	N/A	0.452	0.461	3.589	0.323	0.387	0.000	0.352	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	40	0	24	56
N.S.	1	1.00	1.00	0.85	1.15	2.00	0.00	1.20	2.80
time (sec)	N/A	0.226	0.400	0.296	0.211	0.248	0.000	0.362	3.127

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	32	42	36	58	0	42	64
N.S.	1	0.95	0.78	1.02	0.88	1.41	0.00	1.02	1.56
time (sec)	N/A	0.248	0.378	4.887	0.215	0.254	0.000	0.421	3.612

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	58	42	55	56	82	0	70	134
N.S.	1	0.92	0.67	0.87	0.89	1.30	0.00	1.11	2.13
time (sec)	N/A	0.250	0.504	73.866	0.219	0.248	0.000	0.394	6.155

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	142	153	1840	0	0	0	0	0
N.S.	1	1.04	1.12	13.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.770	2.609	7.734	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	109	133	233	0	0	0	0	0
N.S.	1	1.01	1.23	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.565	1.773	5.351	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	215	0	103	0	0	0
N.S.	1	1.00	0.89	2.69	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.453	0.509	1.633	0.000	0.087	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	215	0	103	0	0	0
N.S.	1	1.00	0.89	2.69	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.467	0.612	2.502	0.000	0.086	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	111	110	259	0	162	0	0	0
N.S.	1	1.01	1.00	2.35	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.614	0.697	20.387	0.000	0.093	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	146	130	456	0	209	0	0	0
N.S.	1	1.04	0.93	3.26	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.771	1.844	151.869	0.000	0.091	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	122	603	220	892	0	246	0
N.S.	1	1.00	0.47	2.35	0.86	3.47	0.00	0.96	0.00
time (sec)	N/A	0.444	0.616	9.156	0.316	0.352	0.000	0.351	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	218	109	524	188	877	0	218	0
N.S.	1	0.96	0.48	2.31	0.83	3.86	0.00	0.96	0.00
time (sec)	N/A	0.433	0.391	1.731	0.332	0.396	0.000	0.349	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	46	0	23	102
N.S.	1	1.00	1.00	0.85	1.15	2.30	0.00	1.15	5.10
time (sec)	N/A	0.213	0.290	0.158	0.238	0.246	0.000	0.322	3.431

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	40	43	35	70	0	45	530
N.S.	1	0.95	0.93	1.00	0.81	1.63	0.00	1.05	12.33
time (sec)	N/A	0.239	0.279	0.772	0.234	0.253	0.000	0.361	7.663

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	50	52	48	93	0	58	831
N.S.	1	0.92	0.77	0.80	0.74	1.43	0.00	0.89	12.78
time (sec)	N/A	0.244	0.310	0.865	0.270	0.271	0.000	0.349	12.236

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	86	390	0	0	0	0	0
N.S.	1	1.05	0.80	3.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.628	0.938	1.116	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	98	377	0	0	0	0	0
N.S.	1	1.00	1.24	4.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.463	0.766	0.988	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	60	363	0	0	0	0	0
N.S.	1	1.00	1.28	7.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	0.233	0.829	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	109	69	349	0	169	0	0	0
N.S.	1	1.51	0.96	4.85	0.00	2.35	0.00	0.00	0.00
time (sec)	N/A	0.489	0.319	0.885	0.000	0.095	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	139	104	370	0	237	0	0	0
N.S.	1	1.36	1.02	3.63	0.00	2.32	0.00	0.00	0.00
time (sec)	N/A	0.613	0.702	1.033	0.000	0.097	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	257	123	621	225	986	0	257	0
N.S.	1	1.00	0.48	2.42	0.88	3.84	0.00	1.00	0.00
time (sec)	N/A	0.459	0.589	10.056	0.368	0.397	0.000	0.428	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	218	105	529	193	971	0	228	0
N.S.	1	0.96	0.46	2.33	0.85	4.28	0.00	1.00	0.00
time (sec)	N/A	0.442	0.350	13.241	0.330	0.393	0.000	0.388	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	58	0	26	381
N.S.	1	1.00	1.00	0.85	1.15	2.90	0.00	1.30	19.05
time (sec)	N/A	0.226	0.329	0.150	0.229	0.252	0.000	0.405	7.067

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	42	48	35	84	0	45	684
N.S.	1	0.95	0.98	1.12	0.81	1.95	0.00	1.05	15.91
time (sec)	N/A	0.245	0.282	0.700	0.250	0.254	0.000	0.434	9.492

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	54	57	48	109	0	58	987
N.S.	1	0.92	0.83	0.88	0.74	1.68	0.00	0.89	15.18
time (sec)	N/A	0.252	0.314	0.769	0.270	0.275	0.000	0.500	14.657

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	112	113	102	1048	0	0	0	0	0
N.S.	1	1.01	0.91	9.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	0.498	3.034	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	113	126	151	0	0	0	0	0
N.S.	1	1.43	1.59	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	0.669	2.535	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	110	195	0	119	0	0	0
N.S.	1	1.00	1.34	2.38	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.469	0.686	0.792	0.000	0.094	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	115	136	345	0	163	0	0	0
N.S.	1	1.03	1.21	3.08	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.609	1.545	0.865	0.000	0.089	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	257	263	123	605	219	956	0	248	0
N.S.	1	1.02	0.48	2.35	0.85	3.72	0.00	0.96	0.00
time (sec)	N/A	0.459	0.868	15.087	0.320	0.356	0.000	0.379	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	224	113	527	189	943	0	220	0
N.S.	1	0.99	0.50	2.32	0.83	4.15	0.00	0.97	0.00
time (sec)	N/A	0.430	0.596	14.338	0.328	0.358	0.000	0.397	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	63	0	26	530
N.S.	1	1.00	1.00	0.85	1.15	3.15	0.00	1.30	26.50
time (sec)	N/A	0.208	0.428	0.154	0.261	0.251	0.000	0.382	8.183

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	50	48	35	91	0	45	831
N.S.	1	0.95	1.16	1.12	0.81	2.12	0.00	1.05	19.33
time (sec)	N/A	0.234	0.387	0.863	0.236	0.273	0.000	0.516	13.898

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	60	57	48	114	0	58	1132
N.S.	1	0.92	0.92	0.88	0.74	1.75	0.00	0.89	17.42
time (sec)	N/A	0.240	0.462	0.952	0.226	0.313	0.000	0.512	15.050

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	150	122	444	0	0	0	0	0
N.S.	1	1.04	0.85	3.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.752	1.857	1.460	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	117	100	431	0	0	0	0	0
N.S.	1	1.03	0.88	3.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.600	1.274	1.177	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	417	0	0	0	0	0
N.S.	1	1.00	1.15	4.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	0.767	0.945	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	366	0	0	0	0	0
N.S.	1	1.00	0.88	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.492	0.948	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	144	105	373	0	246	0	0	0
N.S.	1	1.31	0.95	3.39	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.583	1.398	0.963	0.000	0.103	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	177	116	392	0	309	0	0	0
N.S.	1	1.26	0.83	2.80	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.755	0.910	1.004	0.000	0.103	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	444	0	65	0	0	80
N.S.	1	1.00	0.75	6.53	0.00	0.96	0.00	0.00	1.18
time (sec)	N/A	0.326	0.709	1.862	0.000	0.241	0.000	0.000	5.357

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	80	156	0	102	0	0	0
N.S.	1	1.00	0.91	1.77	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.417	5.605	2.746	0.000	0.087	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	71	0	47	0	0	60
N.S.	1	1.00	1.00	2.37	0.00	1.57	0.00	0.00	2.00
time (sec)	N/A	0.205	0.458	1.624	0.000	0.238	0.000	0.000	3.300

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	79	0	64	0	0	0
N.S.	1	1.00	1.20	1.58	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.267	0.373	1.408	0.000	0.080	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	73	72	164	0	413	0	0	0
N.S.	1	0.68	0.67	1.53	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.335	0.567	1.392	0.000	0.416	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	153	0	147	0	0	0
N.S.	1	1.00	0.92	1.78	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.393	0.592	1.591	0.000	0.091	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	124	99	467	0	127	0	0	0
N.S.	1	0.98	0.79	3.71	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.534	5.629	3.939	0.000	0.095	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	436	0	57	0	0	69
N.S.	1	1.00	0.66	6.41	0.00	0.84	0.00	0.00	1.01
time (sec)	N/A	0.330	0.450	1.786	0.000	0.250	0.000	0.000	4.109

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	310	0	100	0	0	0
N.S.	1	1.00	0.99	3.69	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.395	0.508	2.641	0.000	0.085	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	268	0	45	0	0	39
N.S.	1	1.00	1.00	8.93	0.00	1.50	0.00	0.00	1.30
time (sec)	N/A	0.209	0.384	2.073	0.000	0.253	0.000	0.000	3.558

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	329	0	103	0	0	0
N.S.	1	1.00	1.02	3.66	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.413	0.549	2.130	0.000	0.093	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	110	104	221	0	524	0	0	0
N.S.	1	0.76	0.72	1.52	0.00	3.61	0.00	0.00	0.00
time (sec)	N/A	0.469	0.621	1.992	0.000	0.480	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	128	100	464	0	136	0	0	0
N.S.	1	1.04	0.81	3.77	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.565	1.170	5.078	0.000	0.093	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	48	0	71	0	0	88
N.S.	1	1.00	0.76	0.71	0.00	1.04	0.00	0.00	1.29
time (sec)	N/A	0.337	0.612	0.850	0.000	0.262	0.000	0.000	5.393

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	87	432	0	117	0	0	0
N.S.	1	1.00	0.99	4.91	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.395	0.580	2.606	0.000	0.092	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	0	53	0	0	69
N.S.	1	1.00	1.00	1.03	0.00	1.66	0.00	0.00	2.16
time (sec)	N/A	0.206	0.502	0.826	0.000	0.264	0.000	0.000	3.948

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	69	306	0	71	0	0	0
N.S.	1	1.00	1.38	6.12	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.275	0.419	1.820	0.000	0.086	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	73	80	158	0	419	0	0	0
N.S.	1	0.69	0.75	1.49	0.00	3.95	0.00	0.00	0.00
time (sec)	N/A	0.324	0.404	0.773	0.000	0.406	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	275	0	157	0	0	0
N.S.	1	1.00	1.02	3.16	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.394	0.631	1.529	0.000	0.095	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	111	112	292	0	605	0	0	0
N.S.	1	0.76	0.77	2.00	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.465	0.831	1.147	0.000	0.482	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	153	67	63	0	84	0	0	296
N.S.	1	1.05	0.46	0.43	0.00	0.58	0.00	0.00	2.03
time (sec)	N/A	0.629	1.800	0.951	0.000	0.276	0.000	0.000	9.517

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	57	53	0	71	0	0	94
N.S.	1	1.04	0.52	0.49	0.00	0.65	0.00	0.00	0.86
time (sec)	N/A	0.469	0.958	1.006	0.000	0.264	0.000	0.000	6.002

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	45	40	0	55	0	0	81
N.S.	1	1.00	1.41	1.25	0.00	1.72	0.00	0.00	2.53
time (sec)	N/A	0.215	0.595	0.845	0.000	0.254	0.000	0.000	4.821

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	107	88	207	0	529	0	0	0
N.S.	1	0.76	0.62	1.47	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.467	0.624	1.108	0.000	0.478	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	113	103	288	0	608	0	0	0
N.S.	1	0.75	0.68	1.91	0.00	4.03	0.00	0.00	0.00
time (sec)	N/A	0.472	0.597	1.098	0.000	0.468	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	173	118	178	0	140	0	0	0
N.S.	1	1.04	0.71	1.07	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.744	1.631	6.367	0.000	0.100	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	133	97	256	0	126	0	0	0
N.S.	1	1.02	0.75	1.97	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.539	0.846	3.795	0.000	0.089	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	131	0	104	0	0	0
N.S.	1	1.00	0.86	1.41	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.409	0.594	2.099	0.000	0.089	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	152	0	149	0	0	0
N.S.	1	1.00	0.92	1.77	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.403	0.506	1.794	0.000	0.093	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	128	96	171	0	200	0	0	0
N.S.	1	0.98	0.74	1.32	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.559	0.679	1.906	0.000	0.100	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	171	106	193	0	264	0	0	0
N.S.	1	1.02	0.63	1.16	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.743	0.953	2.543	0.000	0.114	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	10.770	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	10.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	10.581	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	67	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	10.709	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	10.753	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	10.645	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	10.629	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	10.829	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	11.213	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	10.573	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	10.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	121	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.338	31.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	85	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	10.766	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	72	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	10.694	0.000	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	10.864	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	70	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	10.705	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	18	17	0	17	17
N.S.	1	1.00	1.00	1.06	1.06	1.00	0.00	1.00	1.00
time (sec)	N/A	0.189	0.007	0.480	0.290	0.263	0.000	0.308	2.887

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	44	37	2751	47	57	0	0	91
N.S.	1	0.96	0.80	59.80	1.02	1.24	0.00	0.00	1.98
time (sec)	N/A	0.233	0.045	1.190	0.254	0.255	0.000	0.000	4.088

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	72	67	62	6931	71	112	0	0	219
N.S.	1	0.93	0.86	96.26	0.99	1.56	0.00	0.00	3.04
time (sec)	N/A	0.254	0.264	2.018	0.267	0.262	0.000	0.000	8.690

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.350	0.890	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	0.593	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	0.545	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	0.939	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	260	0	0	0	0	0	0
N.S.	1	1.00	3.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	1.823	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.426	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	30	28	42	0	0	53
N.S.	1	1.00	0.88	1.20	1.12	1.68	0.00	0.00	2.12
time (sec)	N/A	0.212	0.281	1.824	0.293	0.249	0.000	0.000	2.997

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	53	51	46	5281	55	86	0	0	138
N.S.	1	0.96	0.87	99.64	1.04	1.62	0.00	0.00	2.60
time (sec)	N/A	0.243	0.290	20.026	0.291	0.248	0.000	0.000	4.743

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	75	69	10580	81	144	0	0	0
N.S.	1	0.94	0.86	132.25	1.01	1.80	0.00	0.00	0.00
time (sec)	N/A	0.252	0.390	151.818	0.288	0.255	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	456	0	0	0	0	0	0
N.S.	1	1.00	5.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	2.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	252	0	0	0	0	0	0
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	1.575	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	743	0	0	0	0	0	0
N.S.	1	1.00	9.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	13.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	1017	0	0	0	0	0	0
N.S.	1	1.00	13.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	14.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	297	0	0	0	0	0	0
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	32.845	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	91	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	11.648	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	11.343	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	9.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	81	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	229	109	707	207	212	0	0	97
N.S.	1	0.99	0.47	3.05	0.89	0.91	0.00	0.00	0.42
time (sec)	N/A	0.615	0.581	17.605	0.331	0.275	0.000	0.000	2.814

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	209	91	568	190	187	0	0	83
N.S.	1	0.98	0.43	2.65	0.89	0.87	0.00	0.00	0.39
time (sec)	N/A	0.523	0.165	17.697	0.371	0.248	0.000	0.000	2.875

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	210	201	80	656	189	200	0	0	80
N.S.	1	0.96	0.38	3.12	0.90	0.95	0.00	0.00	0.38
time (sec)	N/A	0.507	0.150	18.903	0.327	0.263	0.000	0.000	3.600

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	182	132	448	167	156	0	0	61
N.S.	1	0.95	0.69	2.33	0.87	0.81	0.00	0.00	0.32
time (sec)	N/A	0.428	0.142	14.924	0.465	0.258	0.000	0.000	0.251

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	174	71	136	165	232	0	0	50
N.S.	1	0.91	0.37	0.71	0.86	1.21	0.00	0.00	0.26
time (sec)	N/A	0.380	0.070	0.151	0.594	0.264	0.000	0.000	3.028

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	205	164	149	178	253	0	0	74
N.S.	1	0.98	0.78	0.71	0.85	1.21	0.00	0.00	0.35
time (sec)	N/A	0.498	0.142	0.125	0.347	0.263	0.000	0.000	2.960

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	201	100	156	187	328	0	0	76
N.S.	1	0.94	0.47	0.73	0.87	1.53	0.00	0.00	0.36
time (sec)	N/A	0.515	0.212	0.133	0.318	0.275	0.000	0.000	3.102

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	172	171	199	339	0	0	90
N.S.	1	1.00	0.74	0.74	0.86	1.47	0.00	0.00	0.39
time (sec)	N/A	0.605	0.302	0.129	0.335	0.265	0.000	0.000	3.782

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	229	109	632	207	223	0	0	97
N.S.	1	0.98	0.47	2.70	0.88	0.95	0.00	0.00	0.41
time (sec)	N/A	0.631	0.563	3.407	0.475	0.263	0.000	0.000	2.784

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	209	91	569	190	196	0	0	83
N.S.	1	0.98	0.43	2.66	0.89	0.92	0.00	0.00	0.39
time (sec)	N/A	0.525	0.178	2.089	0.533	0.278	0.000	0.000	2.692

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	201	82	572	189	209	0	0	82
N.S.	1	0.95	0.39	2.70	0.89	0.99	0.00	0.00	0.39
time (sec)	N/A	0.521	0.149	3.089	0.557	0.261	0.000	0.000	3.108

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	192	182	134	448	167	164	0	0	61
N.S.	1	0.95	0.70	2.33	0.87	0.85	0.00	0.00	0.32
time (sec)	N/A	0.438	0.073	2.046	0.338	0.255	0.000	0.000	3.055

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	176	72	138	167	180	0	0	54
N.S.	1	0.92	0.38	0.72	0.87	0.94	0.00	0.00	0.28
time (sec)	N/A	0.413	0.059	0.187	0.385	0.259	0.000	0.000	2.864

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	201	161	149	179	262	0	0	75
N.S.	1	0.96	0.77	0.71	0.85	1.25	0.00	0.00	0.36
time (sec)	N/A	0.471	0.089	0.122	0.346	0.252	0.000	0.000	3.261

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	201	100	152	184	338	0	0	73
N.S.	1	0.95	0.47	0.72	0.87	1.60	0.00	0.00	0.35
time (sec)	N/A	0.486	0.180	0.130	0.520	0.253	0.000	0.000	3.051

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	231	172	171	199	350	0	0	91
N.S.	1	1.00	0.74	0.74	0.86	1.51	0.00	0.00	0.39
time (sec)	N/A	0.593	0.198	0.135	0.470	0.261	0.000	0.000	3.646

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	229	94	714	207	223	0	0	97
N.S.	1	0.99	0.41	3.09	0.90	0.97	0.00	0.00	0.42
time (sec)	N/A	0.620	0.242	3.461	0.686	0.254	0.000	0.000	3.044

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	209	84	484	190	198	0	0	81
N.S.	1	0.99	0.40	2.28	0.90	0.93	0.00	0.00	0.38
time (sec)	N/A	0.519	0.192	2.273	0.348	0.268	0.000	0.000	2.985

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	199	79	663	189	211	0	0	79
N.S.	1	0.95	0.38	3.17	0.90	1.01	0.00	0.00	0.38
time (sec)	N/A	0.518	0.044	3.127	0.328	0.234	0.000	0.000	0.223

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	180	131	138	165	220	0	0	57
N.S.	1	0.94	0.68	0.72	0.86	1.15	0.00	0.00	0.30
time (sec)	N/A	0.384	0.053	0.189	0.352	0.243	0.000	0.000	2.955

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	173	74	135	164	236	0	0	58
N.S.	1	0.90	0.39	0.70	0.85	1.23	0.00	0.00	0.30
time (sec)	N/A	0.391	0.004	0.187	0.452	0.246	0.000	0.000	3.097

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	205	161	151	181	264	0	0	77
N.S.	1	0.97	0.76	0.71	0.85	1.25	0.00	0.00	0.36
time (sec)	N/A	0.501	0.098	0.148	0.550	0.257	0.000	0.000	3.409

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	201	100	156	187	339	0	0	76
N.S.	1	0.94	0.47	0.73	0.87	1.58	0.00	0.00	0.36
time (sec)	N/A	0.489	0.122	0.150	0.719	0.255	0.000	0.000	3.030

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	232	229	97	734	207	231	0	0	93
N.S.	1	0.99	0.42	3.16	0.89	1.00	0.00	0.00	0.40
time (sec)	N/A	0.632	0.216	3.276	0.335	0.252	0.000	0.000	3.081

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	207	83	571	190	214	0	0	80
N.S.	1	0.98	0.39	2.71	0.90	1.01	0.00	0.00	0.38
time (sec)	N/A	0.518	0.097	2.717	0.354	0.262	0.000	0.000	0.322

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	82	157	187	379	0	0	76
N.S.	1	0.93	0.39	0.74	0.88	1.79	0.00	0.00	0.36
time (sec)	N/A	0.470	0.001	0.141	0.355	0.248	0.000	0.000	3.787

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	179	134	138	164	228	0	0	57
N.S.	1	0.93	0.70	0.72	0.85	1.19	0.00	0.00	0.30
time (sec)	N/A	0.403	0.016	0.194	0.601	0.245	0.000	0.000	2.846

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	176	74	138	167	236	0	0	58
N.S.	1	0.92	0.39	0.72	0.87	1.23	0.00	0.00	0.30
time (sec)	N/A	0.412	0.004	0.184	0.449	0.242	0.000	0.000	2.745

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	205	161	151	181	280	0	0	77
N.S.	1	0.97	0.76	0.71	0.85	1.32	0.00	0.00	0.36
time (sec)	N/A	0.496	0.115	0.158	0.345	0.247	0.000	0.000	3.339

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	201	100	156	187	347	0	0	76
N.S.	1	0.94	0.47	0.73	0.87	1.62	0.00	0.00	0.36
time (sec)	N/A	0.500	0.157	0.142	0.338	0.252	0.000	0.000	3.554

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	231	172	171	199	381	0	0	93
N.S.	1	0.99	0.74	0.73	0.85	1.63	0.00	0.00	0.40
time (sec)	N/A	0.604	0.330	0.148	0.317	0.257	0.000	0.000	3.321

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	62	52	52	51	59	0	76	334
N.S.	1	0.93	0.78	0.78	0.76	0.88	0.00	1.13	4.99
time (sec)	N/A	0.239	0.394	0.331	0.333	0.263	0.000	0.371	8.455

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	34	37	36	49	0	53	218
N.S.	1	0.96	0.76	0.82	0.80	1.09	0.00	1.18	4.84
time (sec)	N/A	0.238	0.268	0.253	0.639	0.270	0.000	0.357	7.261

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	37	0	21	53
N.S.	1	1.00	1.00	0.86	0.82	1.68	0.00	0.95	2.41
time (sec)	N/A	0.204	0.026	0.129	0.202	0.252	0.000	0.337	3.297

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	174	71	136	153	164	0	176	49
N.S.	1	0.91	0.37	0.71	0.80	0.85	0.00	0.92	0.26
time (sec)	N/A	0.378	0.057	0.166	0.284	0.270	0.000	0.317	3.406

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	227	225	102	528	193	924	0	218	0
N.S.	1	0.99	0.45	2.33	0.85	4.07	0.00	0.96	0.00
time (sec)	N/A	0.428	0.330	1.166	0.314	0.404	0.000	0.312	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	108	102	401	0	184	0	0	0
N.S.	1	1.01	0.95	3.75	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.601	0.489	1.461	0.000	0.103	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	61	367	0	131	0	0	0
N.S.	1	1.00	0.81	4.89	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.454	0.282	1.416	0.000	0.097	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	377	0	0	0	0	0
N.S.	1	1.00	1.21	8.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.179	0.946	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	94	390	0	0	0	0	0
N.S.	1	1.00	1.16	4.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	0.527	1.077	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	116	86	404	0	0	0	0	0
N.S.	1	1.05	0.77	3.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.615	0.832	1.375	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	62	52	52	51	68	0	78	392
N.S.	1	0.93	0.78	0.78	0.76	1.01	0.00	1.16	5.85
time (sec)	N/A	0.233	0.334	0.394	0.341	0.297	0.000	0.393	11.537

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	42	37	36	56	0	55	276
N.S.	1	0.96	0.93	0.82	0.80	1.24	0.00	1.22	6.13
time (sec)	N/A	0.235	0.234	0.323	0.426	0.281	0.000	0.351	8.382

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	45	0	24	100
N.S.	1	1.00	1.00	0.86	0.82	2.05	0.00	1.09	4.55
time (sec)	N/A	0.218	0.041	0.224	0.225	0.279	0.000	0.334	4.379

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	201	160	149	170	177	0	0	73
N.S.	1	0.96	0.76	0.71	0.81	0.84	0.00	0.00	0.35
time (sec)	N/A	0.461	0.179	0.175	0.325	0.246	0.000	0.000	3.142

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	225	225	110	525	188	927	0	210	0
N.S.	1	1.00	0.49	2.33	0.84	4.12	0.00	0.93	0.00
time (sec)	N/A	0.447	0.408	2.173	0.312	0.384	0.000	0.321	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	150	90	265	0	127	0	0	0
N.S.	1	1.10	0.66	1.95	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.807	0.769	1.669	0.000	0.103	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	115	80	244	0	114	0	0	0
N.S.	1	1.06	0.74	2.26	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.605	0.497	1.473	0.000	0.095	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	221	0	97	0	0	0
N.S.	1	1.00	0.86	2.76	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.440	0.331	1.386	0.000	0.088	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	219	0	0	0	0	0
N.S.	1	1.00	0.74	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.440	0.213	3.019	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	112	96	1939	0	0	0	0	0
N.S.	1	1.04	0.89	17.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.595	0.986	3.336	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	136	147	131	1958	0	0	0	0	0
N.S.	1	1.08	0.96	14.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.738	2.077	6.391	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	62	52	52	51	82	0	78	474
N.S.	1	0.93	0.78	0.78	0.76	1.22	0.00	1.16	7.07
time (sec)	N/A	0.240	0.645	0.214	0.390	0.301	0.000	0.415	18.326

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	42	37	36	69	0	55	352
N.S.	1	0.96	0.93	0.82	0.80	1.53	0.00	1.22	7.82
time (sec)	N/A	0.221	0.363	236.850	0.201	0.292	0.000	0.401	8.533

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	55	0	26	230
N.S.	1	1.00	1.00	0.86	0.82	2.50	0.00	1.18	10.45
time (sec)	N/A	0.201	0.037	1.696	0.204	0.257	0.000	0.357	6.706

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	101	154	176	202	0	0	74
N.S.	1	0.93	0.48	0.73	0.83	0.95	0.00	0.00	0.35
time (sec)	N/A	0.442	0.164	0.165	0.305	0.266	0.000	0.000	3.421

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	225	225	107	551	194	969	0	231	0
N.S.	1	1.00	0.48	2.45	0.86	4.31	0.00	1.03	0.00
time (sec)	N/A	0.440	0.400	2.482	0.298	0.423	0.000	0.425	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	253	262	125	641	225	985	0	257	0
N.S.	1	1.04	0.49	2.53	0.89	3.89	0.00	1.02	0.00
time (sec)	N/A	0.449	0.320	59.813	0.595	0.427	0.000	0.430	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	114	79	219	0	115	0	0	0
N.S.	1	1.05	0.72	2.01	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.621	0.679	1.497	0.000	0.093	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	68	200	0	97	0	0	0
N.S.	1	1.00	0.86	2.53	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.444	0.394	1.436	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	77	103	0	57	0	0	0
N.S.	1	1.00	1.64	2.19	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.323	0.185	0.967	0.000	0.087	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	126	192	0	0	0	0	0
N.S.	1	1.00	1.66	2.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	0.535	3.098	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	109	111	94	1906	0	0	0	0	0
N.S.	1	1.02	0.86	17.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.582	0.993	6.227	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	60	45	41	54	64	0	80	268
N.S.	1	0.92	0.69	0.63	0.83	0.98	0.00	1.23	4.12
time (sec)	N/A	0.248	0.534	1.559	0.257	0.291	0.000	0.522	7.656

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	32	31	36	54	0	44	64
N.S.	1	0.95	0.74	0.72	0.84	1.26	0.00	1.02	1.49
time (sec)	N/A	0.235	0.277	1.224	0.250	0.254	0.000	0.482	3.440

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	40	0	18	51
N.S.	1	1.00	1.00	0.95	0.90	2.00	0.00	0.90	2.55
time (sec)	N/A	0.208	0.045	0.252	0.239	0.267	0.000	0.425	2.922

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F(-1)	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	197	82	157	167	235	0	0	76
N.S.	1	0.93	0.39	0.74	0.79	1.11	0.00	0.00	0.36
time (sec)	N/A	0.457	0.090	0.161	0.578	0.246	0.000	0.000	2.825

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	C	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	249	247	115	937	204	1034	0	252	0
N.S.	1	0.99	0.46	3.76	0.82	4.15	0.00	1.01	0.00
time (sec)	N/A	0.471	0.495	19.772	0.513	0.425	0.000	0.415	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	142	104	380	0	222	0	0	0
N.S.	1	1.03	0.75	2.75	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.772	0.876	1.615	0.000	0.104	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	107	93	367	0	172	0	0	0
N.S.	1	1.03	0.89	3.53	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.610	0.539	1.483	0.000	0.095	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	352	0	169	0	0	0
N.S.	1	1.00	0.88	4.51	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.459	0.428	1.111	0.000	0.120	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	366	0	0	0	0	0
N.S.	1	1.00	0.85	4.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	0.408	1.106	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	115	77	380	0	0	0	0	0
N.S.	1	1.03	0.69	3.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.622	0.639	1.489	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	150	89	393	0	0	0	0	0
N.S.	1	1.06	0.63	2.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.764	0.910	1.388	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	113	299	0	119	0	0	0
N.S.	1	1.00	1.38	3.65	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.460	0.741	1.095	0.000	0.095	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	103	372	0	241	0	0	0
N.S.	1	1.04	0.94	3.38	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.594	1.989	1.281	0.000	0.102	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	54	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.941	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.755	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	0.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	58	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	59	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.291	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	130	131	195	0	788	0	0	0
N.S.	1	0.73	0.74	1.10	0.00	4.43	0.00	0.00	0.00
time (sec)	N/A	0.502	0.823	17.762	0.000	0.402	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	73	454	0	105	0	0	0
N.S.	1	1.00	0.78	4.88	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.528	10.548	2.479	0.000	0.097	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	88	101	129	0	654	0	0	0
N.S.	1	0.67	0.77	0.98	0.00	4.95	0.00	0.00	0.00
time (sec)	N/A	0.359	0.551	17.512	0.000	0.375	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	342	0	66	0	0	0
N.S.	1	1.00	1.20	6.22	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.361	0.518	2.737	0.000	0.088	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	0	50	53	0	55
N.S.	1	1.00	1.00	1.03	0.00	1.47	1.56	0.00	1.62
time (sec)	N/A	0.238	0.457	1.334	0.000	0.258	4.948	0.000	3.747

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	454	0	112	0	0	0
N.S.	1	1.00	0.74	4.78	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.500	0.917	2.902	0.000	0.091	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	53	47	0	63	0	0	69
N.S.	1	1.00	0.74	0.65	0.00	0.88	0.00	0.00	0.96
time (sec)	N/A	0.382	0.709	1.557	0.000	0.250	0.000	0.000	3.797

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	137	86	468	0	125	0	0	0
N.S.	1	1.04	0.65	3.55	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.720	1.144	2.596	0.000	0.099	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	132	83	295	0	140	0	0	0
N.S.	1	1.01	0.63	2.25	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.686	5.621	2.604	0.000	0.097	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	125	130	189	0	769	0	0	0
N.S.	1	0.74	0.77	1.12	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.526	0.900	20.763	0.000	0.374	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	272	0	91	0	0	0
N.S.	1	1.00	0.74	3.09	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.512	0.565	2.668	0.000	0.089	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	107	66	292	0	741	0	0	0
N.S.	1	0.64	0.40	1.75	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	0.389	0.703	19.504	0.000	0.548	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	67	272	0	104	0	0	0
N.S.	1	1.00	0.70	2.83	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.532	0.641	2.550	0.000	0.089	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	43	38	0	58	53	0	65
N.S.	1	1.00	1.26	1.12	0.00	1.71	1.56	0.00	1.91
time (sec)	N/A	0.238	0.949	1.456	0.000	0.250	45.793	0.000	3.706

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	138	81	292	0	116	0	0	0
N.S.	1	1.05	0.62	2.23	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.730	0.932	3.122	0.000	0.092	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	115	55	50	0	70	0	0	78
N.S.	1	1.12	0.53	0.49	0.00	0.68	0.00	0.00	0.76
time (sec)	N/A	0.536	0.994	1.403	0.000	0.279	0.000	0.000	3.955

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	170	153	300	0	852	0	0	0
N.S.	1	0.82	0.74	1.44	0.00	4.10	0.00	0.00	0.00
time (sec)	N/A	0.671	2.331	169.046	0.000	0.447	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	133	80	487	0	155	0	0	0
N.S.	1	1.02	0.61	3.72	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.721	0.737	1.891	0.000	0.093	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	125	130	233	0	788	0	0	0
N.S.	1	0.74	0.77	1.38	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.483	0.738	12.085	0.000	0.386	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	471	0	114	0	0	0
N.S.	1	1.00	0.74	5.35	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.503	0.635	2.212	0.000	0.095	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	125	140	168	0	766	0	0	0
N.S.	1	0.74	0.83	1.00	0.00	4.56	0.00	0.00	0.00
time (sec)	N/A	0.517	1.012	10.714	0.000	0.593	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	458	0	122	0	0	0
N.S.	1	1.00	0.82	4.77	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.524	0.836	1.852	0.000	0.093	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	45	40	0	68	0	0	72
N.S.	1	1.00	1.32	1.18	0.00	2.00	0.00	0.00	2.12
time (sec)	N/A	0.241	0.878	1.392	0.000	0.246	0.000	0.000	3.923

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	138	91	472	0	137	0	0	0
N.S.	1	1.05	0.69	3.60	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.740	1.266	2.188	0.000	0.104	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	130	133	203	0	782	0	0	0
N.S.	1	0.73	0.75	1.14	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.547	1.031	12.214	0.000	0.421	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	83	239	0	100	0	0	0
N.S.	1	1.00	0.90	2.60	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.522	0.657	1.873	0.000	0.082	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	88	99	133	0	653	0	0	0
N.S.	1	0.67	0.76	1.02	0.00	4.98	0.00	0.00	0.00
time (sec)	N/A	0.376	0.674	10.273	0.000	0.388	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	68	130	0	58	0	0	0
N.S.	1	1.00	1.24	2.36	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.354	0.541	2.008	0.000	0.081	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	32	0	47	51	0	52
N.S.	1	1.00	1.00	1.00	0.00	1.47	1.59	0.00	1.62
time (sec)	N/A	0.230	0.627	1.273	0.000	0.241	4.291	0.000	3.505

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	239	0	101	0	0	0
N.S.	1	1.00	0.74	2.52	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.504	0.737	1.929	0.000	0.092	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	45	0	58	88	0	64
N.S.	1	1.00	0.69	0.62	0.00	0.81	1.22	0.00	0.89
time (sec)	N/A	0.351	0.798	1.522	0.000	0.240	57.254	0.000	3.721

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	128	118	239	0	794	0	0	0
N.S.	1	0.75	0.69	1.40	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.521	1.076	12.633	0.000	0.411	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	80	365	0	130	0	0	0
N.S.	1	1.00	0.82	3.76	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.528	0.816	1.978	0.000	0.086	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	0	52	53	0	46
N.S.	1	1.00	1.00	0.91	0.00	1.62	1.66	0.00	1.44
time (sec)	N/A	0.230	0.436	1.446	0.000	0.242	1.581	0.000	3.690

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	78	365	0	129	0	0	0
N.S.	1	1.00	0.86	4.01	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.504	0.644	2.239	0.000	0.092	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	67	52	47	0	66	90	0	60
N.S.	1	0.93	0.72	0.65	0.00	0.92	1.25	0.00	0.83
time (sec)	N/A	0.386	0.604	1.474	0.000	0.243	26.170	0.000	3.861

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	133	89	452	0	141	0	0	0
N.S.	1	1.02	0.68	3.48	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.711	0.889	2.451	0.000	0.104	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	130	151	235	0	850	0	0	0
N.S.	1	0.76	0.88	1.37	0.00	4.94	0.00	0.00	0.00
time (sec)	N/A	0.556	1.225	17.886	0.000	0.468	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	81	151	0	154	0	0	0
N.S.	1	1.00	0.80	1.50	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.534	0.922	2.270	0.000	0.092	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	0	61	54	0	55
N.S.	1	1.00	1.00	1.06	0.00	1.79	1.59	0.00	1.62
time (sec)	N/A	0.231	0.609	1.267	0.000	0.270	44.265	0.000	4.114

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	80	239	0	136	0	0	0
N.S.	1	1.00	0.84	2.52	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.498	0.820	2.069	0.000	0.090	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	44	52	0	75	92	0	81
N.S.	1	1.00	0.64	0.75	0.00	1.09	1.33	0.00	1.17
time (sec)	N/A	0.336	0.789	1.356	0.000	0.266	56.796	0.000	4.306

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	135	95	259	0	153	0	0	0
N.S.	1	1.02	0.72	1.96	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.719	1.086	2.092	0.000	0.100	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	114	65	66	0	89	0	0	93
N.S.	1	1.08	0.61	0.62	0.00	0.84	0.00	0.00	0.88
time (sec)	N/A	0.463	1.125	1.390	0.000	0.260	0.000	0.000	5.107

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.171	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	66	62	6067	77	80	0	0	199
N.S.	1	0.99	0.93	90.55	1.15	1.19	0.00	0.00	2.97
time (sec)	N/A	0.244	0.329	8.101	0.327	0.261	0.000	0.000	8.878

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	45	37	2423	51	50	0	0	87
N.S.	1	1.05	0.86	56.35	1.19	1.16	0.00	0.00	2.02
time (sec)	N/A	0.247	0.151	2.058	0.361	0.258	0.000	0.000	4.058

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	20	19	42	0	19
N.S.	1	1.00	1.00	1.06	1.18	1.12	2.47	0.00	1.12
time (sec)	N/A	0.194	0.056	0.462	0.538	0.281	0.165	0.000	0.137

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	62	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.184	0.000	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	61	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	62	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.438	0.000	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	80	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	0.110	0.000	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	69	101	88	77	85	0	0	0
N.S.	1	0.93	1.36	1.19	1.04	1.15	0.00	0.00	0.00
time (sec)	N/A	0.269	1.615	0.339	0.290	0.265	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	47	78	58	51	61	0	0	139
N.S.	1	0.96	1.59	1.18	1.04	1.24	0.00	0.00	2.84
time (sec)	N/A	0.255	0.863	56.741	0.283	0.259	0.000	0.000	4.538

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	40	0	0	49
N.S.	1	1.00	1.04	1.04	1.00	1.67	0.00	0.00	2.04
time (sec)	N/A	0.217	0.015	2.755	0.386	0.251	0.000	0.000	3.278

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.078	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	0.399	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	51	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	452	0	0	0	0	0	0
N.S.	1	1.00	6.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	2.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	1313	0	0	0	0	0	0
N.S.	1	1.00	16.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	4.847	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	52	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	21	20	54	20	43
N.S.	1	1.00	1.00	1.06	1.17	1.11	3.00	1.11	2.39
time (sec)	N/A	0.200	0.022	0.281	0.623	0.263	0.179	0.354	3.254

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	43	46	36	3514	50	60	0	0	92
N.S.	1	1.07	0.84	81.72	1.16	1.40	0.00	0.00	2.14
time (sec)	N/A	0.248	0.066	1.159	0.308	0.268	0.000	0.000	4.149

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	69	67	63	8846	78	115	0	0	222
N.S.	1	0.97	0.91	128.20	1.13	1.67	0.00	0.00	3.22
time (sec)	N/A	0.260	0.245	2.139	0.285	0.260	0.000	0.000	8.935

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	240	0	0	0	0	0	0
N.S.	1	1.00	3.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	8.730	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	79	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	1.474	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	186	0	0	0	0	0	0
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	1.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	106	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	1.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.447	0.711	0.000	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.445	0.735	0.000	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	0.790	0.000	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	2.004	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [30] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	3	3	1.00	8	0.375
3	A	4	4	1.00	8	0.500
4	A	5	5	1.00	8	0.625
5	A	6	6	1.00	8	0.750
6	A	7	7	1.00	8	0.875
7	A	8	8	1.00	8	1.000
8	A	9	9	1.00	8	1.125
9	A	16	15	0.98	12	1.250
10	A	14	13	0.93	12	1.083
11	A	14	13	0.96	12	1.083
12	A	12	11	0.91	12	0.917
13	A	12	11	0.94	12	0.917
14	A	14	13	0.93	12	1.083
15	A	14	13	0.96	12	1.083
16	A	16	15	0.96	12	1.250
17	A	14	13	0.89	12	1.083
18	A	12	11	0.88	12	0.917
19	A	12	11	0.85	12	0.917
20	A	12	11	0.85	12	0.917
21	A	12	11	0.88	12	0.917
22	A	14	13	0.89	12	1.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	1.00	10	0.300
24	A	8	8	0.68	14	0.571
25	A	6	6	0.80	14	0.429
26	A	4	4	1.00	14	0.286
27	A	5	5	1.06	14	0.357
28	A	9	9	0.82	14	0.643
29	A	13	13	0.72	14	0.929
30	A	22	21	0.64	14	1.500
31	A	18	17	0.69	14	1.214
32	A	16	15	0.70	14	1.071
33	A	16	15	0.70	14	1.071
34	A	18	17	0.67	14	1.214
35	A	22	21	0.64	14	1.500
36	A	13	13	0.55	14	0.929
37	A	9	9	0.62	14	0.643
38	A	5	5	0.74	14	0.357
39	A	5	5	0.75	14	0.357
40	A	9	9	0.60	14	0.643
41	A	13	13	0.55	14	0.929
42	A	6	5	1.00	12	0.417
43	A	6	5	1.00	12	0.417
44	A	6	5	1.00	12	0.417
45	A	6	5	1.00	12	0.417
46	A	6	5	1.00	14	0.357
47	A	6	5	1.00	14	0.357
48	A	6	5	1.25	14	0.357
49	A	6	5	1.00	14	0.357
50	A	6	5	1.00	14	0.357
51	A	6	5	1.00	14	0.357
52	A	4	4	1.00	14	0.286
53	A	6	5	1.00	14	0.357
54	A	14	13	0.98	21	0.619
55	A	13	12	0.93	21	0.571
56	A	4	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	5	4	0.95	21	0.190
58	A	5	4	0.92	21	0.190
59	A	10	10	1.03	21	0.476
60	A	8	8	1.00	19	0.421
61	A	6	6	1.00	19	0.316
62	A	8	8	1.00	21	0.381
63	A	10	10	1.05	21	0.476
64	A	15	14	1.00	21	0.667
65	A	14	13	0.96	21	0.619
66	A	4	3	1.00	21	0.143
67	A	5	4	0.95	21	0.190
68	A	5	4	0.92	21	0.190
69	A	10	10	1.01	21	0.476
70	A	8	8	1.00	19	0.421
71	A	8	8	1.00	19	0.421
72	A	10	10	1.35	21	0.476
73	A	15	14	0.99	21	0.667
74	A	14	13	0.95	21	0.619
75	A	4	3	1.00	21	0.143
76	A	5	4	0.95	21	0.190
77	A	5	4	0.92	21	0.190
78	A	12	12	1.04	21	0.571
79	A	10	10	1.01	19	0.526
80	A	8	8	1.00	19	0.421
81	A	8	8	1.00	21	0.381
82	A	10	10	1.01	21	0.476
83	A	12	12	1.04	21	0.571
84	A	14	13	1.00	21	0.619
85	A	13	12	0.96	21	0.571
86	A	4	3	1.00	21	0.143
87	A	5	4	0.95	21	0.190
88	A	5	4	0.92	21	0.190
89	A	10	10	1.05	21	0.476
90	A	8	8	1.00	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	6	1.00	19	0.316
92	A	8	8	1.51	19	0.421
93	A	10	10	1.36	21	0.476
94	A	14	13	1.00	21	0.619
95	A	13	12	0.96	21	0.571
96	A	4	3	1.00	21	0.143
97	A	5	4	0.95	21	0.190
98	A	5	4	0.92	21	0.190
99	A	10	10	1.01	21	0.476
100	A	8	8	1.43	19	0.421
101	A	8	8	1.00	19	0.421
102	A	10	10	1.03	21	0.476
103	A	14	13	1.02	21	0.619
104	A	13	12	0.99	21	0.571
105	A	4	3	1.00	21	0.143
106	A	5	4	0.95	21	0.190
107	A	5	4	0.92	21	0.190
108	A	12	12	1.04	21	0.571
109	A	10	10	1.03	21	0.476
110	A	8	8	1.00	21	0.381
111	A	8	8	1.00	19	0.421
112	A	10	10	1.31	19	0.526
113	A	12	12	1.26	21	0.571
114	A	4	4	1.00	25	0.160
115	A	6	6	1.00	25	0.240
116	A	2	2	1.00	25	0.080
117	A	4	4	1.00	25	0.160
118	A	10	9	0.68	25	0.360
119	A	6	6	1.00	25	0.240
120	A	8	8	0.98	25	0.320
121	A	4	4	1.00	25	0.160
122	A	6	6	1.00	25	0.240
123	A	2	2	1.00	25	0.080
124	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	12	11	0.76	25	0.440
126	A	8	8	1.04	25	0.320
127	A	4	4	1.00	25	0.160
128	A	6	6	1.00	25	0.240
129	A	2	2	1.00	25	0.080
130	A	4	4	1.00	25	0.160
131	A	10	9	0.69	25	0.360
132	A	6	6	1.00	25	0.240
133	A	12	11	0.76	25	0.440
134	A	8	8	1.05	25	0.320
135	A	6	6	1.04	25	0.240
136	A	2	2	1.00	25	0.080
137	A	12	11	0.76	25	0.440
138	A	12	11	0.75	25	0.440
139	A	10	10	1.04	25	0.400
140	A	8	8	1.02	25	0.320
141	A	6	6	1.00	25	0.240
142	A	6	6	1.00	25	0.240
143	A	8	8	0.98	25	0.320
144	A	10	10	1.02	25	0.400
145	A	4	4	1.00	25	0.160
146	A	4	4	1.00	25	0.160
147	A	4	4	1.00	25	0.160
148	A	4	4	1.00	25	0.160
149	A	4	4	1.00	25	0.160
150	A	4	4	1.00	25	0.160
151	A	4	4	1.00	25	0.160
152	A	4	4	1.00	25	0.160
153	A	4	4	1.00	25	0.160
154	A	4	4	1.00	25	0.160
155	A	4	4	1.00	25	0.160
156	A	4	4	1.00	25	0.160
157	A	4	4	1.00	25	0.160
158	A	4	4	1.00	25	0.160

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	4	4	1.00	25	0.160
160	A	4	4	1.00	25	0.160
161	A	4	3	1.00	19	0.158
162	A	4	3	1.00	17	0.176
163	A	4	3	1.00	17	0.176
164	A	5	4	0.96	19	0.211
165	A	5	4	0.93	19	0.211
166	A	4	4	1.00	19	0.211
167	A	4	4	1.00	19	0.211
168	A	4	4	1.00	19	0.211
169	A	4	4	1.00	19	0.211
170	A	4	4	1.00	23	0.174
171	A	4	4	1.00	23	0.174
172	A	4	4	1.00	23	0.174
173	A	4	4	1.00	23	0.174
174	A	4	4	1.00	21	0.190
175	A	4	3	1.00	19	0.158
176	A	4	3	1.00	19	0.158
177	A	4	3	1.00	19	0.158
178	A	5	4	0.96	19	0.211
179	A	5	4	0.94	19	0.211
180	A	4	4	1.00	19	0.211
181	A	4	4	1.00	17	0.235
182	A	4	4	1.00	17	0.235
183	A	4	4	1.00	19	0.211
184	A	4	4	1.00	19	0.211
185	A	4	4	1.00	23	0.174
186	A	4	4	1.00	23	0.174
187	A	4	4	1.00	23	0.174
188	A	4	4	1.00	23	0.174
189	A	4	4	1.00	21	0.190
190	A	5	4	1.00	21	0.190
191	A	17	16	0.99	21	0.762
192	A	16	15	0.98	21	0.714

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	15	14	0.96	21	0.667
194	A	14	13	0.95	19	0.684
195	A	12	11	0.91	12	0.917
196	A	15	14	0.98	19	0.737
197	A	15	14	0.94	21	0.667
198	A	17	16	1.00	21	0.762
199	A	18	17	0.98	21	0.810
200	A	15	14	0.98	21	0.667
201	A	16	15	0.95	21	0.714
202	A	13	12	0.95	21	0.571
203	A	14	13	0.92	19	0.684
204	A	14	13	0.96	12	1.083
205	A	15	14	0.95	19	0.737
206	A	17	16	1.00	21	0.762
207	A	18	17	0.99	21	0.810
208	A	15	14	0.99	21	0.667
209	A	16	15	0.95	19	0.789
210	A	12	11	0.94	12	0.917
211	A	13	12	0.90	19	0.632
212	A	15	14	0.97	21	0.667
213	A	15	14	0.94	21	0.667
214	A	17	16	0.99	21	0.762
215	A	16	15	0.98	19	0.789
216	A	14	13	0.93	12	1.083
217	A	13	12	0.93	19	0.632
218	A	13	12	0.92	21	0.571
219	A	15	14	0.97	21	0.667
220	A	15	14	0.94	21	0.667
221	A	17	16	0.99	21	0.762
222	A	6	5	1.00	17	0.294
223	A	6	5	1.00	19	0.263
224	A	6	5	1.00	19	0.263
225	A	6	5	1.00	21	0.238
226	A	5	4	0.93	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	5	4	0.96	21	0.190
228	A	4	3	1.00	21	0.143
229	A	12	11	0.91	12	0.917
230	A	14	13	0.99	21	0.619
231	A	10	10	1.01	21	0.476
232	A	8	8	1.00	19	0.421
233	A	6	6	1.00	19	0.316
234	A	8	8	1.00	21	0.381
235	A	10	10	1.05	21	0.476
236	A	5	4	0.93	21	0.190
237	A	5	4	0.96	21	0.190
238	A	4	3	1.00	21	0.143
239	A	14	13	0.96	12	1.083
240	A	14	13	1.00	21	0.619
241	A	12	12	1.10	21	0.571
242	A	10	10	1.06	21	0.476
243	A	8	8	1.00	19	0.421
244	A	8	8	1.00	19	0.421
245	A	10	10	1.04	21	0.476
246	A	12	12	1.08	21	0.571
247	A	5	4	0.93	21	0.190
248	A	5	4	0.96	21	0.190
249	A	4	3	1.00	21	0.143
250	A	14	13	0.93	12	1.083
251	A	14	13	1.00	21	0.619
252	A	15	14	1.04	21	0.667
253	A	10	10	1.05	21	0.476
254	A	8	8	1.00	21	0.381
255	A	6	6	1.00	19	0.316
256	A	8	8	1.00	19	0.421
257	A	10	10	1.02	21	0.476
258	A	5	4	0.92	21	0.190
259	A	5	4	0.95	21	0.190
260	A	4	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
261	A	14	13	0.93	12	1.083
262	A	15	14	0.99	21	0.667
263	A	12	12	1.03	21	0.571
264	A	10	10	1.03	21	0.476
265	A	8	8	1.00	19	0.421
266	A	8	8	1.00	19	0.421
267	A	10	10	1.03	21	0.476
268	A	12	12	1.06	21	0.571
269	A	8	8	1.00	19	0.421
270	A	10	10	1.04	21	0.476
271	A	4	4	1.00	19	0.211
272	A	4	4	1.00	19	0.211
273	A	4	4	1.00	19	0.211
274	A	4	4	1.00	19	0.211
275	A	4	4	1.00	19	0.211
276	A	4	4	1.00	19	0.211
277	A	4	4	1.00	19	0.211
278	A	4	4	1.00	19	0.211
279	A	4	4	1.00	19	0.211
280	A	4	4	1.00	19	0.211
281	A	2	2	1.00	21	0.095
282	A	2	2	1.00	21	0.095
283	A	2	2	1.00	21	0.095
284	A	2	2	1.00	21	0.095
285	A	2	2	1.00	21	0.095
286	A	2	2	1.00	21	0.095
287	A	2	2	1.00	21	0.095
288	A	2	2	1.00	21	0.095
289	A	2	2	1.00	21	0.095
290	A	2	2	1.00	21	0.095
291	A	12	11	0.73	25	0.440
292	A	8	8	1.00	25	0.320
293	A	10	9	0.67	25	0.360
294	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	2	2	1.00	25	0.080
296	A	8	8	1.00	25	0.320
297	A	4	4	1.00	25	0.160
298	A	10	10	1.04	25	0.400
299	A	10	10	1.01	25	0.400
300	A	12	11	0.74	25	0.440
301	A	8	8	1.00	25	0.320
302	A	11	10	0.64	25	0.400
303	A	8	8	1.00	25	0.320
304	A	2	2	1.00	25	0.080
305	A	10	10	1.05	25	0.400
306	A	6	6	1.12	25	0.240
307	A	14	13	0.82	25	0.520
308	A	10	10	1.02	25	0.400
309	A	12	11	0.74	25	0.440
310	A	8	8	1.00	25	0.320
311	A	12	11	0.74	25	0.440
312	A	8	8	1.00	25	0.320
313	A	2	2	1.00	25	0.080
314	A	10	10	1.05	25	0.400
315	A	12	11	0.73	25	0.440
316	A	8	8	1.00	25	0.320
317	A	10	9	0.67	25	0.360
318	A	6	6	1.00	25	0.240
319	A	2	2	1.00	25	0.080
320	A	8	8	1.00	25	0.320
321	A	4	4	1.00	25	0.160
322	A	12	11	0.75	25	0.440
323	A	8	8	1.00	25	0.320
324	A	2	2	1.00	25	0.080
325	A	8	8	1.00	25	0.320
326	A	4	4	0.93	25	0.160
327	A	10	10	1.02	25	0.400
328	A	12	11	0.76	25	0.440

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
329	A	8	8	1.00	25	0.320
330	A	2	2	1.00	25	0.080
331	A	8	8	1.00	25	0.320
332	A	4	4	1.00	25	0.160
333	A	10	10	1.02	25	0.400
334	A	6	6	1.08	25	0.240
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	2	2	1.00	25	0.080
338	A	2	2	1.00	25	0.080
339	A	2	2	1.00	25	0.080
340	A	2	2	1.00	25	0.080
341	A	2	2	1.00	25	0.080
342	A	2	2	1.00	25	0.080
343	A	2	2	1.00	25	0.080
344	A	2	2	1.00	25	0.080
345	A	2	2	1.00	25	0.080
346	A	2	2	1.00	25	0.080
347	A	2	2	1.00	25	0.080
348	A	2	2	1.00	25	0.080
349	A	2	2	1.00	25	0.080
350	A	2	2	1.00	25	0.080
351	A	5	4	0.99	19	0.211
352	A	6	5	1.05	19	0.263
353	A	4	3	1.00	17	0.176
354	A	5	4	1.00	17	0.235
355	A	4	3	1.00	19	0.158
356	A	5	4	1.00	19	0.211
357	A	2	2	1.00	19	0.105
358	A	2	2	1.00	19	0.105
359	A	2	2	1.00	19	0.105
360	A	2	2	1.00	19	0.105
361	A	2	2	1.00	19	0.105
362	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	5	4	0.93	19	0.211
364	A	5	4	0.96	19	0.211
365	A	4	3	1.00	19	0.158
366	A	4	3	1.00	10	0.300
367	A	4	3	1.00	19	0.158
368	A	4	3	1.00	19	0.158
369	A	2	2	1.00	19	0.105
370	A	2	2	1.00	19	0.105
371	A	2	2	1.00	17	0.118
372	A	2	2	1.00	17	0.118
373	A	2	2	1.00	19	0.105
374	A	5	4	1.00	19	0.211
375	A	6	5	1.00	17	0.294
376	A	5	4	1.00	17	0.235
377	A	7	6	1.07	19	0.316
378	A	6	5	0.97	19	0.263
379	A	2	2	1.00	19	0.105
380	A	2	2	1.00	19	0.105
381	A	2	2	1.00	19	0.105
382	A	2	2	1.00	19	0.105
383	A	6	6	1.00	23	0.261
384	A	6	6	1.00	23	0.261
385	A	6	6	1.00	23	0.261
386	A	6	6	1.00	23	0.261
387	A	6	6	1.00	21	0.286

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \tan(c+dx)\,dx$

3.1.1 Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

output
```
-ln(cos(d*x+c))/d
```

3.1.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

input
```
Integrate[Tan[c + d*x],x]
```

output
```
-(Log[Cos[c + d*x]]/d)
```

3.1.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x],x]
```

output
```
-(Log[Cos[c + d*x]]/d)
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.1.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	17
default	$\frac{\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	17
norman	$\frac{\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	17
parallelrisch	$\frac{\ln\left(1+\tan^{2}\left(dx+c\right)\right)}{2d}$	17
risch	$ix+\frac{2ic}{d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	30

input
```
int(tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*ln(1+tan(d*x+c)^2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(c+dx)\,dx = -\frac{\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(d*x + c)^2 + 1))/d
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(c+dx)\,dx = \begin{cases} \frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} & \text{for } d \neq 0 \\ x\tan\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c),x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(c+dx)\,dx = \frac{\log\left(\sec\left(dx+c\right)\right)}{d}$$

input
```
integrate(tan(d*x+c),x, algorithm="maxima")
```

output
```
log(sec(d*x + c))/d
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(c+dx)\,dx = -\frac{\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

input
```
integrate(tan(d*x+c),x, algorithm="giac")
```

output
```
-log(abs(cos(d*x + c)))/d
```

3.1.9 Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(c+dx)\,dx = \frac{\ln\left({\tan(c+d\,x)}^2+1\right)}{2\,d}$$

input
```
int(tan(c + d*x),x)
```

output
```
log(tan(c + d*x)^2 + 1)/(2*d)
```

3.2 $\int \tan^2(c+dx)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \tan^2(c+dx)\,dx = -x+\frac{\tan(c+dx)}{d}$$

output
```
-x+tan(d*x+c)/d
```

3.2.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \tan^2(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}$$

input
```
Integrate[Tan[c + d*x]^2,x]
```

output
```
-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

3.2.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(c+dx)}{d}-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(c+dx)}{d}-x$$

input
```
Int[Tan[c + d*x]^2,x]
```

output
```
-x + Tan[c + d*x]/d
```

3.2.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.2.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
norman	$-x+\frac{\tan(dx+c)}{d}$	15
parallelrisch	$-\frac{dx-\tan(dx+c)}{d}$	18
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
risch	$-x+\frac{2i}{d\left(e^{2i(dx+c)}+1\right)}$	24

input
```
int(tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(d*x+c)/d
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \tan^2(c+dx)\,dx = -\frac{dx-\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(d*x - tan(d*x + c))/d
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(c+dx)\,dx = \begin{cases} -x+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\tan^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**2,x)
```

output
```
Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(c+dx)\,dx = -\frac{dx+c-\tan(dx+c)}{d}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(d*x + c - tan(d*x + c))/d
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(14) = 28.

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 16.14

$$\int \tan^2(c+dx)\,dx$$

$$= \frac{\pi - 4\,dx\tan(dx)\tan(c) - \pi\mathrm{sgn}\left(2\,\tan(dx)^2\tan(c) + 2\,\tan(dx)\tan(c)^2 - 2\,\tan(dx) - 2\,\tan(c)\right)\tan(d}{}$$

input
```
integrate(tan(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*
tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2
*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arc
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + p
i*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) -
 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))/(d*tan(d*x)*tan(
c) - d)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan^2(c + dx)\,dx = \frac{\tan(c + d\,x)}{d} - x$$

input
```
int(tan(c + d*x)^2,x)
```

output
```
tan(c + d*x)/d - x
```

3.3 $\int \tan^3(c+dx)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tan^3(c+dx)\,dx = \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d}$$

output
```
ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan^3(c+dx)\,dx = \frac{2\log(\cos(c+dx)) + \tan^2(c+dx)}{2d}$$

input
```
Integrate[Tan[c + d*x]^3,x]
```

output
```
(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2)/(2*d)
```

3.3.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)}{2d} - \int \tan(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)}{2d} - \int \tan(c+dx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\tan^2(c+dx)}{2d} + \frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^3,x]
```

output
```
Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

3.3.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.3.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-\left(\tan^2(dx+c)\right)+\ln\left(1+\tan^2(dx+c)\right)}{2d}$	28
derivativedivides	$\frac{\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	29
default	$\frac{\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	29
norman	$\frac{\tan^2(dx+c)}{2d}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2d}$	31
risch	$-ix-\frac{2ic}{d}+\frac{2\,\mathrm{e}^{2i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	56

input
```
int(tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/d
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(c+dx)\,dx=\frac{\tan\left(dx+c\right)^2+\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(tan(d*x + c)^2 + log(1/(tan(d*x + c)^2 + 1)))/d
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \tan^3(c+dx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^3\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**3,x)
```

output
```
Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d), Ne(d,
0)), (x*tan(c)**3, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tan^3(c+dx)\,dx = -\frac{\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)^2-1\right)}{2\,d}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(25) = 50$.

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 8.00

$$\int \tan^3(c+dx)\,dx$$
$$= \frac{\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan\left(dx\right)^2\tan\left(c\right)^2+\tan\left(dx\right)^2\tan\left(c\right)^2-2\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan\right.}{\tan(dx)^2\tan(c)^2+\tan(d\right.}\right.}{2\left(d\tan\left(dx\right)^2\tan\left(c\right)^2-2\,d\right.}$$

input
```
integrate(tan(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + tan(d*x)^2*tan(c)^
2 - 2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2
 + log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1)) + 1)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)
*tan(c) + d)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(c+dx)\,dx=\frac{\tan(c+d\,x)^2}{2\,d}-\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

input
```
int(tan(c + d*x)^3,x)
```

output
```
tan(c + d*x)^2/(2*d) - log(tan(c + d*x)^2 + 1)/(2*d)
```

3.4 $\int \tan^4(c+dx)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tan^4(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d}$$

input
```
Integrate[Tan[c + d*x]^4,x]
```

output
```
ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

3.4.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(c+dx)}{3d}-\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

input
```
Int[Tan[c + d*x]^4,x]
```

output
```
x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.4.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}$	27
parallelrisch	$\frac{\tan^3(dx+c)+3dx-3\tan(dx+c)}{3d}$	27
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
risch	$x-\frac{4i\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	46

input
```
int(tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(c+dx)\,dx = \frac{\tan(dx+c)^3 + 3\,dx - 3\,\tan(dx+c)}{3\,d}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(tan(d*x + c)^3 + 3*d*x - 3*tan(d*x + c))/d
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \tan^4(c+dx)\,dx = \begin{cases} x + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**4,x)
```

output
```
Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c
)**4, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(c+dx)\,dx = \frac{\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)}{3\,d}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))/d
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(26) = 52$.

Time = 0.84 (sec) , antiderivative size = 585, normalized size of antiderivative = 20.89

$$\int \tan^4(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c) + 2
*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi*tan
(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan
(d*x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(d*x)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*
pi*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c
)))*tan(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3
 + 36*d*x*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(
c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*
x)*tan(c) + 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*
tan(c) + 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan
(c) - 36*tan(d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x +
3*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)
) - 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*
x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))/(d*tan(d*x)
^3*tan(c)^3 - 3*d*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tan^4(c+dx)\,dx = x - \frac{\tan(c+d\,x) - \frac{\tan(c+d\,x)^3}{3}}{d}$$

input
```
int(tan(c + d*x)^4,x)
```

output
```
x - (tan(c + d*x) - tan(c + d*x)^3/3)/d
```

3.5 $\int \tan^5(c+dx)\,dx$

3.5.1 Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tan^5(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d} - \frac{\tan^2(c+dx)}{2d} + \frac{\tan^4(c+dx)}{4d}$$

output
```
-ln(cos(d*x+c))/d-1/2*tan(d*x+c)^2/d+1/4*tan(d*x+c)^4/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \tan^5(c+dx)\,dx = -\frac{4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx)}{4d}$$

input
```
Integrate[Tan[c + d*x]^5,x]
```

output
```
-1/4*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4)/d
```

3.5.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^4(c+dx)}{4d}-\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^5,x]
```

output
```
-(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

3.5.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.5.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\tan^4(dx+c)-2(\tan^2(dx+c))+2\ln(1+\tan^2(dx+c))}{4d}$	38
derivativedivides	$\frac{\frac{(\tan^4(dx+c))}{4}-\frac{(\tan^2(dx+c))}{2}+\frac{\ln(1+\tan^2(dx+c))}{2}}{d}$	39
default	$\frac{\frac{(\tan^4(dx+c))}{4}-\frac{(\tan^2(dx+c))}{2}+\frac{\ln(1+\tan^2(dx+c))}{2}}{d}$	39
norman	$-\frac{\tan^2(dx+c)}{2d}+\frac{\tan^4(dx+c)}{4d}+\frac{\ln(1+\tan^2(dx+c))}{2d}$	44
risch	$ix+\frac{2ic}{d}-\frac{4(e^{6i(dx+c)}+e^{4i(dx+c)}+e^{2i(dx+c)})}{d(e^{2i(dx+c)}+1)^4}-\frac{\ln(e^{2i(dx+c)}+1)}{d}$	76

input
```
int(tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)^2))/d
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \tan^5(c+dx)\,dx = \frac{\tan(dx+c)^4 - 2\,\tan(dx+c)^2 - 2\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/4*(tan(d*x + c)^4 - 2*tan(d*x + c)^2 - 2*log(1/(tan(d*x + c)^2 + 1)))/d
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(c+dx)\,dx = \begin{cases} \frac{\log(\tan^2(c+dx)+1)}{2d} + \frac{\tan^4(c+dx)}{4d} - \frac{\tan^2(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x\tan^5(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**5,x)
```

output
```
Piecewise((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c
+ d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**5, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \tan^5(c+dx)\,dx = \frac{\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 2\,\log\left(\sin(dx+c)^2-1\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/4*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*lo
g(sin(d*x + c)^2 - 1))/d
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(39) = 78$.

Time = 1.37 (sec) , antiderivative size = 462, normalized size of antiderivative = 10.74

$$\int \tan^5(c + dx)\, dx =$$

$$-\frac{2 \log\left(\frac{4\left(\tan(dx)^2\tan(c)^2 - 2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^4\tan(c)^4 + 3\tan(dx)^4\tan(c)^4 - 8\log\left(\frac{4\left(\tan(dx)^2\tan(c\right.}{\tan(dx)^2\tan(c)}\right.}{}$$

input
```
integrate(tan(d*x+c)^5,x, algorithm="giac")
```

output
```
-1/4*(2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 + 3*tan(d*x)^4*ta
n(c)^4 - 8*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^4
*tan(c)^2 - 8*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^2*tan(c)^4 + 12*log(4*(tan(
d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - tan(d*x)^4 - 8*tan(d*x)^3*tan(c) +
 4*tan(d*x)^2*tan(c)^2 - 8*tan(d*x)*tan(c)^3 - tan(c)^4 - 8*log(4*(tan(d*x
)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
tan(c)^2 + 1))*tan(d*x)*tan(c) + 2*tan(d*x)^2 - 8*tan(d*x)*tan(c) + 2*tan(
c)^2 + 2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 3)/(d*tan(d*x)^4*tan(c)^4 - 4*d*ta
n(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*tan(c) + d)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \tan^5(c + dx)\, dx = \frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2} - \frac{\tan(c+d\,x)^2}{2} + \frac{\tan(c+d\,x)^4}{4}}{d}$$

input
```
int(tan(c + d*x)^5,x)
```

output
```
(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4)/d
```

3.6 $\int \tan^6(c+dx)\,dx$

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \tan^6(c+dx)\,dx = -x+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

output
```
-x+tan(d*x+c)/d-1/3*tan(d*x+c)^3/d+1/5*tan(d*x+c)^5/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \tan^6(c+dx)\,dx = -\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

input
```
Integrate[Tan[c + d*x]^6,x]
```

output
```
-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c
+ d*x]^5/(5*d)
```

3.6.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^5(c+dx)}{5d}-\int \tan(c+dx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^2(c+dx)dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

input
```
Int[Tan[c + d*x]^6,x]
```

output
```
-x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

3.6.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.6.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$-\frac{-3\left(\tan^{5}\left(dx+c\right)\right)+5\left(\tan^{3}\left(dx+c\right)\right)+15dx-15\tan\left(dx+c\right)}{15d}$	39
derivativedivides	$\frac{\frac{\left(\tan^{5}\left(dx+c\right)\right)}{5}-\frac{\left(\tan^{3}\left(dx+c\right)\right)}{3}+\tan\left(dx+c\right)-\arctan\left(\tan\left(dx+c\right)\right)}{d}$	41
default	$\frac{\frac{\left(\tan^{5}\left(dx+c\right)\right)}{5}-\frac{\left(\tan^{3}\left(dx+c\right)\right)}{3}+\tan\left(dx+c\right)-\arctan\left(\tan\left(dx+c\right)\right)}{d}$	41
norman	$-x+\frac{\tan\left(dx+c\right)}{d}-\frac{\tan^{3}\left(dx+c\right)}{3d}+\frac{\tan^{5}\left(dx+c\right)}{5d}$	41
risch	$-x+\frac{2i\left(45\,\mathrm{e}^{8i(dx+c)}+90\,\mathrm{e}^{6i(dx+c)}+140\,\mathrm{e}^{4i(dx+c)}+70\,\mathrm{e}^{2i(dx+c)}+23\right)}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$	70

input
```
int(tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*d*x-15*tan(d*x+c))/d
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx + 15\,\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x + 15*tan(d*x + c))/d
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \tan^6(c+dx)\,dx = \begin{cases} -x + \frac{\tan^5(c+dx)}{5d} - \frac{\tan^3(c+dx)}{3d} + \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^6(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**6,x)
```

output
```
Piecewise((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*
x)/d, Ne(d, 0)), (x*tan(c)**6, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \tan^6(c+dx)\,dx = \frac{3\,\tan\left(dx+c\right)^5 - 5\,\tan\left(dx+c\right)^3 - 15\,dx - 15\,c + 15\,\tan\left(dx+c\right)}{15\,d}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c
))/d
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(40) = 80$.

Time = 2.16 (sec) , antiderivative size = 989, normalized size of antiderivative = 22.48

$$\int \tan^6(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^6,x, algorithm="giac")
```

output
```
1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*
tan(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))
*tan(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)
)*tan(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)
^4 + 75*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x
) + tan(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^
4*tan(c)^5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*ta
n(d*x)^5*tan(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(
c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) +
tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)
^3 + 300*tan(d*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2
*tan(c)^2 + 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d
*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c
) + 150*pi*tan(d*x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x
) + tan(c)))*tan(d*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(...
```

3.6.9 Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \tan^6(c+dx)\,dx = \frac{\frac{\tan(c+d\,x)^5}{5} - \frac{\tan(c+d\,x)^3}{3} + \tan(c+d\,x)}{d} - x$$

input
```
int(tan(c + d*x)^6,x)
```

output
```
(tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5)/d - x
```

3.6. $\int \tan^6(c+dx)\,dx$

3.7 $\int \tan^7(c+dx)\,dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \tan^7(c+dx)\,dx = \frac{\log(\cos(c+dx))}{d} + \frac{\tan^2(c+dx)}{2d} - \frac{\tan^4(c+dx)}{4d} + \frac{\tan^6(c+dx)}{6d}$$

output
```
ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d-1/4*tan(d*x+c)^4/d+1/6*tan(d*x+c)^6/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \tan^7(c+dx)\,dx = \frac{12\log(\cos(c+dx)) + 6\tan^2(c+dx) - 3\tan^4(c+dx) + 2\tan^6(c+dx)}{12d}$$

input
```
Integrate[Tan[c + d*x]^7,x]
```

output
```
(12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*
x]^6)/(12*d)
```

3.7.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^7dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^6(c+dx)}{6d}-\int \tan^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^6(c+dx)}{6d}-\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^3(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}$$

$$\Big\downarrow \text{3954}$$

$$-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan(c+dx)dx+\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]^7,x]
```

output
```
Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c
+ d*x]^6/(6*d)
```

3.7.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{6}(dx+c)\right)}{6}-\frac{\left(\tan^{4}(dx+c)\right)}{4}+\frac{\left(\tan^{2}(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2}}{d}$	49
default	$\frac{\frac{\left(\tan^{6}(dx+c)\right)}{6}-\frac{\left(\tan^{4}(dx+c)\right)}{4}+\frac{\left(\tan^{2}(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2}}{d}$	49
parallelrisch	$-\frac{-2(\tan^{6}(dx+c))+3(\tan^{4}(dx+c))-6(\tan^{2}(dx+c))+6\ln(1+\tan^{2}(dx+c))}{12d}$	50
norman	$\frac{\tan^{2}(dx+c)}{2d}-\frac{\tan^{4}(dx+c)}{4d}+\frac{\tan^{6}(dx+c)}{6d}-\frac{\ln(1+\tan^{2}(dx+c))}{2d}$	57
risch	$-ix-\frac{2ic}{d}+\frac{6\,e^{10i(dx+c)}+12\,e^{8i(dx+c)}+\frac{68\,e^{6i(dx+c)}}{3}+12\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^{6}}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	103

input
```
int(tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c
)^2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \tan^7(c+dx)\,dx = \frac{2\,\tan\left(dx+c\right)^6 - 3\,\tan\left(dx+c\right)^4 + 6\,\tan\left(dx+c\right)^2 + 6\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{12\,d}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/12*(2*tan(d*x + c)^6 - 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 6*log(1/(ta
n(d*x + c)^2 + 1)))/d
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \tan^7(c+dx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^6\left(c+dx\right)}{6d} - \frac{\tan^4\left(c+dx\right)}{4d} + \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^7\left(c\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*x+c)**7,x)
```

output
```
Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**6/(6*d) - tan(c
 + d*x)**4/(4*d) + tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**7, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int \tan^7(c+dx)\,dx = -\frac{\frac{18\sin(dx+c)^4-27\sin(dx+c)^2+11}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 6\log\left(\sin\left(dx+c\right)^2-1\right)}{12\,d}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
-1/12*((18*sin(d*x + c)^4 - 27*sin(d*x + c)^2 + 11)/(sin(d*x + c)^6 - 3*si
n(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 6*log(sin(d*x + c)^2 - 1))/d
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(51) = 102.

Time = 3.11 (sec) , antiderivative size = 740, normalized size of antiderivative = 12.98

$$\int \tan^7(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^7,x, algorithm="giac")
```

output
```
1/12*(6*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^6*tan(c)^6 + 11*tan(d*x)^6*t
an(c)^6 - 36*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)
^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)
^6*tan(c)^4 - 54*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)^4*tan(c)^6 + 90*log(4*(t
an(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x
)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(c)^4 - 3*tan(d*x)^6*tan(c)^2 - 36*tan(
d*x)^5*tan(c)^3 + 99*tan(d*x)^4*tan(c)^4 - 36*tan(d*x)^3*tan(c)^5 - 3*tan(
d*x)^2*tan(c)^6 - 120*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/
(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3 + 2
*tan(d*x)^6 + 18*tan(d*x)^5*tan(c) + 90*tan(d*x)^4*tan(c)^2 - 72*tan(d*x)^
3*tan(c)^3 + 90*tan(d*x)^2*tan(c)^4 + 18*tan(d*x)*tan(c)^5 + 2*tan(c)^6 +
90*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 - 3*tan(d*x)^4 - 36*ta
n(d*x)^3*tan(c) + 99*tan(d*x)^2*tan(c)^2 - 36*tan(d*x)*tan(c)^3 - 3*tan(c)
^4 - 36*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + 6*tan(d*x)^2 - 54*t
an(d*x)*tan(c) + 6*tan(c)^2 + 6*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 11)/(d*tan(
d*x)^6*tan(c)^6 - 6*d*tan(d*x)^5*tan(c)^5 + 15*d*tan(d*x)^4*tan(c)^4 - ...
```

3.7.9 Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \tan^7(c+dx)\,dx = -\frac{\frac{\ln\left(\tan(c+dx)^2+1\right)}{2} - \frac{\tan(c+dx)^2}{2} + \frac{\tan(c+dx)^4}{4} - \frac{\tan(c+dx)^6}{6}}{d}$$

input
```
int(tan(c + d*x)^7,x)
```

output
```
-(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4 - tan(c
+ d*x)^6/6)/d
```

3.8 $\int \tan^8(c+dx)\,dx$

3.8.1 Optimal result

Integrand size = 8, antiderivative size = 58

$$\int \tan^8(c+dx)\,dx = x - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}$$

output
```
x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d-1/5*tan(d*x+c)^5/d+1/7*tan(d*x+c)^7/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \tan^8(c+dx)\,dx = \frac{\arctan(\tan(c+dx))}{d} - \frac{\tan(c+dx)}{d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^7(c+dx)}{7d}$$

input
```
Integrate[Tan[c + d*x]^8,x]
```

output
```
ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d
*x]^5/(5*d) + Tan[c + d*x]^7/(7*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^8(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^8dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^7(c+dx)}{7d}-\int \tan^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^7(c+dx)}{7d}-\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan^4(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3954}$$

$$-\int \tan^2(c+dx)dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan(c+dx)^2dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^7(c+dx)}{7d} - \frac{\tan^5(c+dx)}{5d} + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} + x$$

input
```
Int[Tan[c + d*x]^8,x]
```

output
```
x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c +
 d*x]^7/(7*d)
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

3.8.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

method	result	size
parallelrisch	$\frac{15\left(\tan^7(dx+c)\right)-21\left(\tan^5(dx+c)\right)+35\left(\tan^3(dx+c)\right)+105dx-105\tan(dx+c)}{105d}$	49
derivativedivides	$\frac{\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
default	$\frac{\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}-\frac{\tan^5(dx+c)}{5d}+\frac{\tan^7(dx+c)}{7d}$	53
risch	$x-\frac{8i\left(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44\right)}{105d\left(e^{2i(dx+c)}+1\right)^7}$	90

input
```
int(tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/105*(15*tan(d*x+c)^7-21*tan(d*x+c)^5+35*tan(d*x+c)^3+105*d*x-105*tan(d*x
+c))/d
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \tan^8(c+dx)\,dx$$
$$=\frac{15\,\tan\left(dx+c\right)^{7}-21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+105\,dx-105\,\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
 - 105*tan(d*x + c))/d
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx=\begin{cases} x+\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d} & \text{for } d\neq 0\\ x\tan^8(c) & \text{otherwise}\end{cases}$$

input
```
integrate(tan(d*x+c)**8,x)
```

output
```
Piecewise((x + tan(c + d*x)**7/(7*d) - tan(c + d*x)**5/(5*d) + tan(c + d*x
)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**8, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \tan^8(c+dx)\,dx = \frac{15\,\tan\left(dx+c\right)^7 - 21\,\tan\left(dx+c\right)^5 + 35\,\tan\left(dx+c\right)^3 + 105\,dx + 105\,c - 105\,\tan\left(dx+c\right)}{105\,d}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
 + 105*c - 105*tan(d*x + c))/d
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. 2(52) = 104.

Time = 5.00 (sec) , antiderivative size = 1441, normalized size of antiderivative = 24.84

$$\int \tan^8(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^8,x, algorithm="giac")
```

output
```
1/420*(105*pi + 420*d*x*tan(d*x)^7*tan(c)^7 - 105*pi*sgn(2*tan(d*x)^2*tan(
c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^7*tan(c)^7 - 10
5*pi*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + ta
n(c)))*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(d*x)^7*tan(c)^7 - 2940*d*x*tan(d*x)^6*tan(c)^6 + 735*pi*sgn(2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)
^6*tan(c)^6 + 735*pi*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x)*tan(c) -
1)/(tan(d*x) + tan(c)))*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 + 420*tan(d*x)^7*tan(c)^6 +
 420*tan(d*x)^6*tan(c)^7 + 8820*d*x*tan(d*x)^5*tan(c)^5 - 2205*pi*sgn(2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*
tan(c)^5 - 140*tan(d*x)^7*tan(c)^4 - 2205*pi*tan(d*x)^5*tan(c)^5 + 4410*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^5*tan(c)^5 + 4410
*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 2
940*tan(d*x)^6*tan(c)^5 - 2940*tan(d*x)^5*tan(c)^6 - 140*tan(d*x)^4*tan(c)
^7 - 14700*d*x*tan(d*x)^4*tan(c)^4 + 3675*pi*sgn(2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 84*tan(d*x
)^7*tan(c)^2 + 980*tan(d*x)^6*tan(c)^3 + 3675*pi*tan(d*x)^4*tan(c)^4 - 735
0*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^4*tan(c)^4 -
7350*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c...
```

3.8.9 Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \tan^8(c+dx)\,dx = x - \frac{-\frac{\tan(c+d\,x)^7}{7} + \frac{\tan(c+d\,x)^5}{5} - \frac{\tan(c+d\,x)^3}{3} + \tan(c+d\,x)}{d}$$

input
```
int(tan(c + d*x)^8,x)
```

output
```
x - (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5 - tan(c + d*x)^7/7
)/d
```

3.9 $\int (b\tan(c+dx))^{7/2}\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 232

$$\begin{aligned}\int (b\tan(c+dx))^{7/2}\,dx =& -\frac{b^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} \\ &+\frac{b^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} \\ &-\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &+\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} \\ &-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}\end{aligned}$$

output
```
-1/2*b^(7/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/2*
b^(7/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(7/
2)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1
/4*b^(7/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2
^(1/2)-2*b^3*(b*tan(d*x+c))^(1/2)/d+2/5*b*(b*tan(d*x+c))^(5/2)/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.75

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{(b\tan(c+dx))^{7/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+\right.}{2\sqrt{2}}\right.}{d\tan^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(7/2),x]
```

output
```
((b*Tan[c + d*x])^(7/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])
 + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]^(
5/2))/5))/(d*Tan[c + d*x]^(7/2))
```

3.9.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(c+dx))^{7/2}dx$$

$$\Big\downarrow \; 3954$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-b^2\int\frac{1}{\sqrt{b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-b^2\int\frac{1}{\sqrt{b\tan(c+dx)}}dx\right)$$

$$\Big\downarrow \text{3957}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^3\int\frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}\right)$$

$$\Big\downarrow \text{266}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\int\frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int\frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+d}{2b}\right.}{d}\right.$$

$$\Big\downarrow \text{1082}$$

3.9. $\int(b\tan(c+dx))^{7/2}\,dx$

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}\cdots}{2b}\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 217

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}\right)$$

$\downarrow$ 1479

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 25

$$b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{\frac{2b(b\tan(c+dx))^{5/2}}{5d}-2b^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 27

3.9. $\int(b\tan(c+dx))^{7/2}\,dx$

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\cdots\right.}{d}\cdots\right.$$

$\downarrow$ 1103

$$\frac{2b(b\tan(c+dx))^{5/2}}{5d}-b^2\left(\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{\cdots}\right.}{\cdots}}{2b}\cdots\right.}{d}\cdots\right.$$

input

```
Int[(b*Tan[c + d*x])^(7/2),x]
```

output

```
(2*b*(b*Tan[c + d*x])^(5/2))/(5*d) - b^2*((-2*b^3*((-(ArcTan[1 - Sqrt[2]*S
qrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c
 + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log[b - Sqrt[2]*b^(3/2)*Tan[c +
d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b]) + Log[b + Sqrt[2]*b^(3/2)*Tan
[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/(2*b)))/d + (2*b*Sqrt
[b*Tan[c + d*x]])/d)
```

3.9.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.9. $\int (b\tan(c+dx))^{7/2}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.9.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(d}{(b^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan(dx+c)}+\frac{b^2(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(d}{(b^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{d}$

input
```
int((b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(1/5*(b*tan(d*x+c))^(5/2)-b^2*(b*tan(d*x+c))^(1/2)+1/8*b^2*(b^2)^(1/
4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2
)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2
)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)
/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)))
```

3.9.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{5\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^3+\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\right)+5i\left(-\frac{b^{14}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^3+\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/10*(5*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 + (-b^14/d^4)^(1/
4)*d) + 5*I*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 + I*(-b^14/d^
4)^(1/4)*d) - 5*I*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 - I*(-b
^14/d^4)^(1/4)*d) - 5*(-b^14/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^3 - (
-b^14/d^4)^(1/4)*d) + 4*(b^3*tan(d*x + c)^2 - 5*b^3)*sqrt(b*tan(d*x + c)))
/d
```

3.9.6 Sympy [F]

$$\int (b\tan(c+dx))^{7/2}\,dx = \int (b\tan(c+dx))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(7/2),x)
```

output
```
Integral((b*tan(c + d*x))**(7/2), x)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.80

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{10\sqrt{2}b^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+10\sqrt{2}b^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+5\sqrt{}}{}$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/20*(10*sqrt(2)*b^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*ta
n(d*x + c)))/sqrt(b)) + 10*sqrt(2)*b^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sq
rt(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + 5*sqrt(2)*b^(9/2)*log(b*tan(d*x
 + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 5*sqrt(2)*b^(9/2)*log(
b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) + 8*(b*tan(d*x
+ c))^(5/2)*b^2 - 40*sqrt(b*tan(d*x + c))*b^4)/(b*d)
```

3.9.8 Giac [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.9.9 Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{5/2}}{5\,d} - \frac{2\,b^3\,\sqrt{b\tan\left(c+d\,x\right)}}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d} - \frac{(-1)^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}\,1\mathrm{i}}{\sqrt{b}}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(7/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(5/2))/(5*d) - (2*b^3*(b*tan(c + d*x))^(1/2))/d - ((
-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
- ((-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2)*1i)/b^(1/2))
)/d
```

3.10 $\int (b\tan(c+dx))^{5/2}\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 212

$$\int (b\tan(c+dx))^{5/2}\,dx=\frac{b^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}-\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}+\frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

output
```
1/2*b^(5/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b
^(5/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(5/2
)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1/
4*b^(5/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^
(1/2)+2/3*b*(b*tan(d*x+c))^(3/2)/d
```

3.10.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{b(b\tan(c+dx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(\right.\right.}{3d\tan^{\frac{7}{4}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(5/2),x]
```

output
```
(b*(b*Tan[c + d*x])^(3/2)*(-3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*
x])^(1/4) + 3*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*T
an[c + d*x]^(7/4)))/(3*d*Tan[c + d*x]^(7/4))
```

3.10.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-b^2\int \sqrt{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-b^2\int \sqrt{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{b^3\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$\downarrow$ 266

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$\downarrow$ 826

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$\downarrow$ 1476

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\right.}{d}$$

$\downarrow$ 1082

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c\right.}{d}$$

$\downarrow$ 217

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$\downarrow$ 1479

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d}-\frac{2b^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{d}$$

$\downarrow$ 25

3.10. $\int (b\tan(c+dx))^{5/2}\,dx$

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{2b^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}} - \frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\right.}{\sqrt{}}\right.\right.}{d}$$

$\downarrow$ 27

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{2b^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}} - \frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\right.}{\sqrt{2}}\right.\right.}{d}$$

$\downarrow$ 1103

$$\frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{2b^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}} - \frac{\log\left(\sqrt{2}b^{3/2}\right.}{}\right.\right.}{d}$$

input
```
Int[(b*Tan[c + d*x])^(5/2),x]
```

output
```
(-2*b^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) +
ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - S
qrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Lo
g[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b
]))/2))/d + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.10. $\int (b\tan(c+dx))^{5/2}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.10.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}}{}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$

input
```
int((b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(1/3*(b*tan(d*x+c))^(3/2)-1/8*b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x
+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b
^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c
))^(1/2)+1)))
```

3.10.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{4\sqrt{b\tan(dx+c)}b^2\tan(dx+c) - 3\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b^7 + \left(-\frac{b^{10}}{d^4}\right)^{\frac{3}{4}} d^3\right) + 3i\left(\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(b*tan(d*x + c))*b^2*tan(d*x + c) - 3*(-b^10/d^4)^(1/4)*d*log(s
qrt(b*tan(d*x + c))*b^7 + (-b^10/d^4)^(3/4)*d^3) + 3*I*(-b^10/d^4)^(1/4)*d
*log(sqrt(b*tan(d*x + c))*b^7 + I*(-b^10/d^4)^(3/4)*d^3) - 3*I*(-b^10/d^4)
^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^7 - I*(-b^10/d^4)^(3/4)*d^3) + 3*(-b^1
0/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b^7 - (-b^10/d^4)^(3/4)*d^3))/d
```

3.10.6 Sympy [F]

$$\int (b\tan(c+dx))^{5/2}\,dx = \int (b\tan(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((b*tan(c + d*x))**(5/2), x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int (b\tan(c+dx))^{5/2}\,dx = -\frac{3b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}\right.}{\sqrt{b}}\right.}{12\,bd}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b)) - 8*(b*tan(d
*x + c))^(3/2)*b^2)/(b*d)
```

3.10.8 Giac [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.10.9 Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int (b\tan(c+dx))^{5/2}\,dx = \frac{2\,b\,(b\tan(c+d\,x))^{3/2}}{3\,d} - \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d} + \frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(5/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(3/2))/(3*d) - ((-1)^(1/4)*b^(5/2)*atan(((-1)^(1/4)*
(b*tan(c + d*x))^(1/2))/b^(1/2)))/d + ((-1)^(1/4)*b^(5/2)*atanh(((-1)^(1/4
)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/d
```

3.11 $\int (b\tan(c+dx))^{3/2}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{2b\sqrt{b\tan(c+dx)}}{d}$$

output
```
1/2*b^(3/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b
^(3/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/4*b^(3/2
)*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)-1/
4*b^(3/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^
(1/2)+2*b*(b*tan(d*x+c))^(1/2)/d
```

3.11.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(\cdots\right.}{2\cdots} \cdots\right.}{d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x])^(3/2),x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[c + d*x]])*(b*Tan[c + d*x])^(3/2))/(d*Tan[c + d*x]^(3/2
))
```

3.11.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(c+dx))^{3/2}dx$$

$$\Big\downarrow \; 3954$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{b^3\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\int\frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 755$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int\frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int\frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int\frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1479$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-}{2b}\right)}{d}$$

3.11. $\int (b\tan(c+dx))^{3/2}\,dx$

$$\Big\downarrow \ 25$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arctan}}{2b}\right.}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arctan}}{2b}\right.}{d}$$

$$\Big\downarrow \ 1103$$

$$\frac{2b\sqrt{b\tan(c+dx)}}{d}-\frac{2b^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}}{2b}\right.}{d}$$

input
```
Int[(b*Tan[c + d*x])^(3/2),x]
```

output
```
(-2*b^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) +
ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*
Log[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b
]) + Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]
*Sqrt[b]))/(2*b)))/d + (2*b*Sqrt[b*Tan[c + d*x]])/d
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.11.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right)\right)}{8}\right)}{d}$
default	$\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right)\right)}{8}\right)}{d}$

input
```
int((b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*((b*tan(d*x+c))^(1/2)-1/8*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)
+1)))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int (b\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b+\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\right)+i\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(\sqrt{b\tan(dx+c)}b+i\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\right)-i\left(\cdots\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b + (-b^6/d^4)^(1/4)*d)
+ I*(-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b + I*(-b^6/d^4)^(1/4)*d)
- I*(-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b - I*(-b^6/d^4)^(1/4)*d)
- (-b^6/d^4)^(1/4)*d*log(sqrt(b*tan(d*x + c))*b - (-b^6/d^4)^(1/4)*d) - 4*
sqrt(b*tan(d*x + c))*b)/d
```

3.11.6 Sympy [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \int (b\tan(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((b*tan(c + d*x))**(3/2), x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int (b\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}b^{\frac{5}{2}}\log\left(b\tan(\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b)) + 2*sqrt(2)*b^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*b^(5/2)*log(b*tan(d*x + c
) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*b^(5/2)*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 8*sqrt(b*tan(d*x +
c))*b^2)/(b*d)
```

3.11.8 Giac [F]

$$\int (b\tan(c+dx))^{3/2}\,dx = \int (b\tan(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c))^(3/2), x)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{2\,b\,\sqrt{b\tan(c+d\,x)}}{d} + \frac{(-1)^{1/4}\,b^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d} + \frac{(-1)^{1/4}\,b^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

input
```
int((b*tan(c + d*x))^(3/2),x)
```

output
```
(2*b*(b*tan(c + d*x))^(1/2))/d + ((-1)^(1/4)*b^(3/2)*atan(((-1)^(1/4)*(b*t
an(c + d*x))^(1/2))/b^(1/2))*1i)/d + ((-1)^(1/4)*b^(3/2)*atanh(((-1)^(1/4)
*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
```

3.11. $\int (b\tan(c+dx))^{3/2}\,dx$

3.12 $\int \sqrt{b\tan(c+dx)}\,dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{b\tan(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d}$$

output
```
-1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/2*
arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/4*ln(b^
(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2^(1/2)-1
/4*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2
^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$=\frac{\left(\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\right)\sqrt[4]{-\tan(c+dx)}\sqrt{b\tan(c+dx)}}{d\tan^{\frac{3}{4}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]],x]
```

output
```
((ArcTan[(-Tan[c + d*x]^2)^(1/4)] - ArcTanh[(-Tan[c + d*x]^2)^(1/4)])*(-Ta
n[c + d*x])^(1/4)*Sqrt[b*Tan[c + d*x]])/(d*Tan[c + d*x]^(3/4))
```

3.12.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{b\tan(c+dx)}dx$$
$$\Big\downarrow \text{3957}$$
$$\frac{b\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \text{266}$$
$$\frac{2b\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$
$$\Big\downarrow \text{826}$$

$$\frac{2b\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$\downarrow$ 1476

$$\frac{2b\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\right.}{d}$$

$\downarrow$ 1082

$$\frac{2b\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c\right.}{d}$$

$\downarrow$ 217

$$\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{d}$$

$\downarrow$ 1479

$$\frac{2b\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\right.}{\sqrt{}}\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\right.}{\sqrt{2}}\right.\right.}{d}$$

$\downarrow$ 27

$$\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\,\mathrm{ta}\right.}{\sqrt{2}}\right.\right.}{d}$$

$\downarrow$ 1103

$$\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3/2}\right.}{}\right.\right.}{d}$$

3.12. $\int \sqrt{b\tan(c+dx)}\,dx$

input
```
Int[Sqrt[b*Tan[c + d*x]],x]
```

output
```
(2*b*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + Arc
Tan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt
[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b
 + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))
/2))/d
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.12.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right.}{4d(b^2)^{\frac{1}{4}}}$
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right.}{4d(b^2)^{\frac{1}{4}}}$

input
```
int((b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*b/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2
^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \sqrt{b\tan(c+dx)}\,dx =& \frac{1}{2}\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(d^3\left(-\frac{b^2}{d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)\\ &-\frac{1}{2}i\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(i\,d^3\left(-\frac{b^2}{d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)\\ &+\frac{1}{2}i\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(-i\,d^3\left(-\frac{b^2}{d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)\\ &-\frac{1}{2}\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(-d^3\left(-\frac{b^2}{d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}b\right)\end{aligned}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-b^2/d^4)^(1/4)*log(d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))*b) -
1/2*I*(-b^2/d^4)^(1/4)*log(I*d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x + c))*b
) + 1/2*I*(-b^2/d^4)^(1/4)*log(-I*d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x +
c))*b) - 1/2*(-b^2/d^4)^(1/4)*log(-d^3*(-b^2/d^4)^(3/4) + sqrt(b*tan(d*x +
 c))*b)
```

3.12.6 Sympy [F]

$$\int \sqrt{b\tan(c+dx)}\,dx = \int \sqrt{b\tan\left(c+dx\right)}\,dx$$

input
```
integrate((b*tan(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)), x)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}\right.}{4\,d}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqr
t(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c
) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} + \frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} - \frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\right.}{d}}{4\,b}$$

input
```
integrate((b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d -
sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqr
t(abs(b)) + abs(b))/d + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*
sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d)/b
```

3.12.9 Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{(-1)^{1/4}\sqrt{b}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)\right)}{d}$$

input
```
int((b*tan(c + d*x))^(1/2),x)
```

output
```
((-1)^(1/4)*b^(1/2)*(atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)) - a
tanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))))/d
```

3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}d}$$

output
```
-1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)+1/2*
arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)-1/4*ln(b^
(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b^(1/2)+1
/4*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b
^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= \frac{\left(-2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\mathrm{t}\right.}{2\sqrt{2}d\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]],x]
```

output
```
((-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1
+ Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(2*Sqrt[
2]*d*Sqrt[b*Tan[c + d*x]])
```

3.13.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 755$$

$$\frac{2b\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{2b\bigg(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan\cdots}}{2b}\cdots}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{2b\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{2b\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

$$\Big\downarrow \; 1479$$

$$\frac{2b\bigg(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\cdots}{2b}\cdots}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{2b\bigg(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(\cdots\right)}{\cdots}}{2b}\cdots}{d}$$

$$\Big\downarrow \; 27$$

3.13. $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$

$$\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan(}{}}{2b}\right.}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2b\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}}{2b}\right)}{d}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]],x]
```

output
```
(2*b*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + Arc
Tan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log
[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b])
+ Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sq
rt[b]))/(2*b)))/d
```

3.13.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.13. $\int \frac{1}{\sqrt{b \tan(c+dx)}} \, dx$

3.13.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+\cdots}}{(b^2)^{\frac{1}{4}}}\cdots\right)\right)}{4db}$
default	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+\cdots}}{(b^2)^{\frac{1}{4}}}\cdots\right)\right)}{4db}$

input
```
int(1/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d/b*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2
^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-
2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx &= \frac{1}{2}\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &\quad+\frac{1}{2}i\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(i\,bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &\quad-\frac{1}{2}i\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-i\,bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\\ &\quad-\frac{1}{2}\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan(dx+c)}\right)\end{aligned}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-1/(b^2*d^4))^(1/4)*log(b*d*(-1/(b^2*d^4))^(1/4) + sqrt(b*tan(d*x + c
))) + 1/2*I*(-1/(b^2*d^4))^(1/4)*log(I*b*d*(-1/(b^2*d^4))^(1/4) + sqrt(b*t
an(d*x + c))) - 1/2*I*(-1/(b^2*d^4))^(1/4)*log(-I*b*d*(-1/(b^2*d^4))^(1/4)
 + sqrt(b*tan(d*x + c))) - 1/2*(-1/(b^2*d^4))^(1/4)*log(-b*d*(-1/(b^2*d^4)
)^(1/4) + sqrt(b*tan(d*x + c)))
```

3.13.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)), x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$
$$= \frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\tan\right.}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c)
 + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))/(b*d)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd} + \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{4\,bd} - \frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sqrt
(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*sqrt(abs(b))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(
b*d) + 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*
x + c))*sqrt(abs(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(
d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = -\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{\sqrt{b}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(1/2),x)
```

output
```
- ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(1/
2)*d) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)
/(b^(1/2)*d)
```

3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{2}{bd\sqrt{b\tan(c+dx)}}$$

output
```
1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/4*ln(b^(
1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^(1/2)+1/
4*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^
(1/2)-2/b/d/(b*tan(d*x+c))^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}}{bd\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-3/2),x]
```

output
```
(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4) + ArcTanh[(-
Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Tan[c + d*x]])
```

3.14.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\cdots\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c\cdots}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\cdots\right.\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$$\Big\downarrow \text{25}$$

3.14. $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\cdots\right.}{\sqrt{\cdots}}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$\big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\cdots\right.}{\sqrt{2}\cdots}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

$\big\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3/2}\cdots\right.}{\cdots}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x])^(-3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcT
an[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt[
2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b
+ Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/
2))/(b*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.14. $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.14.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)\right)}{8b^2(b^2)^{\frac{1}{4}}}\right)}{d}$
default	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)\right)}{8b^2(b^2)^{\frac{1}{4}}}\right)}{d}$

input
```
int(1/(b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/8/b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d
*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))
^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/b^2/(b*ta
n(d*x+c))^(1/2))
```

3.14.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx =$$
$$-\frac{b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\log\left(b^5d^3\left(-\frac{1}{b^6d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan(dx+c)}\right)\tan(dx+c)-i\,b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\log\left(i\,b^5d^3\left(-\frac{1}{b^6d^4}\right)^{\frac{3}{4}}+\sqrt{\right.}\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*(-1/(b^6*d^4))^(1/4)*log(b^5*d^3*(-1/(b^6*d^4))^(3/4) + sqrt(b
*tan(d*x + c)))*tan(d*x + c) - I*b^2*d*(-1/(b^6*d^4))^(1/4)*log(I*b^5*d^3*
(-1/(b^6*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c) + I*b^2*d*(-1/(b
^6*d^4))^(1/4)*log(-I*b^5*d^3*(-1/(b^6*d^4))^(3/4) + sqrt(b*tan(d*x + c)))
*tan(d*x + c) - b^2*d*(-1/(b^6*d^4))^(1/4)*log(-b^5*d^3*(-1/(b^6*d^4))^(3/
4) + sqrt(b*tan(d*x + c)))*tan(d*x + c) + 4*sqrt(b*tan(d*x + c)))/(b^2*d*t
an(d*x + c))
```

3.14.6 Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(3/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-3/2), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*
sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c)
 - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + 8/sqrt(b*tan(d*x +
c)))/(b*d)
```

3.14.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.14.9 Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\tan(c+d\,x)}}$$

input
```
int(1/(b*tan(c + d*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*d
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2
)*d) - 2/(b*d*(b*tan(c + d*x))^(1/2))
```

3.14. $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

3.15.1 Optimal result

Integrand size = 12, antiderivative size = 214

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

output
```
1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)+1/4*ln(b^(
1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^(1/2)-1/
4*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^
(1/2)-2/3/b/d/(b*tan(d*x+c))^(3/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\left(-\tan^2(c+dx)\right)^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(c\right.}{3bd(b\tan(c+dx))^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-5/2),x]
```

output
```
(-2 + 3*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4) + 3*ArcTan
h[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(3*b*d*(b*Tan[c + d*x]
)^(3/2))
```

3.15.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3955$$

$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}dx}{b^2}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3957$$

$$-\frac{\int \frac{1}{\sqrt{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 266

$$-\frac{2\int \frac{1}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 755

$$-\frac{2\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\,\mathrm{ta}}}{2b}\right.}{bd}$$
$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}\right)}{bd}-$$
$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}\right)}{bd}-$$
$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1479

3.15. $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

$$-\frac{2\left(-\frac{\frac{\int -\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\dots}{2b}\right)}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arctan}\dots}{2b}\right)}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}}{2b}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\text{arctan}\dots}{2b}\right)}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}}{2b}+\frac{\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}}{2b}\right)}{bd}$$

$$\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

input
```
Int[(b*Tan[c + d*x])^(-5/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcT
an[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/(Sqrt[2]*Sqrt[b]))/(2*b) + (-1/2*Log[
b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(Sqrt[2]*Sqrt[b]) +
 Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c + d*x]^2]/(2*Sqrt[2]*Sqr
t[b]))/(2*b)))/(b*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

3.15. $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.15.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\text{arct}}{8b^4}\right.}{d}$
default	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\,\text{arct}}{8b^4}\right.}{d}$

input
```
int(1/(b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(-1/3/b^2/(b*tan(d*x+c))^(3/2)-1/8/b^4*(b^2)^(1/4)*2^(1/2)*(ln((b*ta
n(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+
c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)
/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(
d*x+c))^(1/2)+1)))
```

3.15.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{3\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}+\sqrt{b\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^2+3i\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(i\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*b^3*d*(-1/(b^10*d^4))^(1/4)*log(b^3*d*(-1/(b^10*d^4))^(1/4) + sqrt
(b*tan(d*x + c)))*tan(d*x + c)^2 + 3*I*b^3*d*(-1/(b^10*d^4))^(1/4)*log(I*b
^3*d*(-1/(b^10*d^4))^(1/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^2 - 3*I*b^
3*d*(-1/(b^10*d^4))^(1/4)*log(-I*b^3*d*(-1/(b^10*d^4))^(1/4) + sqrt(b*tan(
d*x + c)))*tan(d*x + c)^2 - 3*b^3*d*(-1/(b^10*d^4))^(1/4)*log(-b^3*d*(-1/(
b^10*d^4))^(1/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^2 + 4*sqrt(b*tan(d*x
 + c)))/(b^3*d*tan(d*x + c)^2)
```

3.15.6 Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(5/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-5/2), x)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{3\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{b^{\frac{3}{2}}}-}{12\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
c)))/sqrt(b))/b^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/b^(3/2) + 3*sqrt(2)*log(b*tan(d*x + c) + s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) - 3*sqrt(2)*log(b*tan(d*x
 + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) + 8/(b*tan(d*x +
 c))^(3/2))/(b*d)
```

3.15.8 Giac [F]

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c))^(-5/2), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.35

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3\,b\,d\,(b\tan(c+d\,x))^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)\,1\mathrm{i}}{b^{5/2}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(5/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)
*d) - 2/(3*b*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*
tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*d)
```

3.15. $\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx$

3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

3.16.1 Optimal result

Integrand size = 12, antiderivative size = 234

$$\begin{aligned}\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx &= -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}\\
&+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d}+\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d}\\
&-\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d}\\
&-\frac{2}{5bd(b\tan(c+dx))^{5/2}}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}\end{aligned}$$

output
```
-1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/2*
arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/4*ln(b^
(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2^(1/2)-1
/4*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2
^(1/2)+2/b^3/d/(b*tan(d*x+c))^(1/2)-2/5/b/d/(b*tan(d*x+c))^(5/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.41

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{10-2\cot^2(c+dx)+5\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}-5\text{arctanh}}{5b^3d\sqrt{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-7/2),x]
```

output
```
(10 - 2*Cot[c + d*x]^2 + 5*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^
2)^(1/4) - 5*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4))/(5*
b^3*d*Sqrt[b*Tan[c + d*x]])
```

3.16.3 Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {3042, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}dx$$

$$\Big\downarrow \; 3955$$

$$-\frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}dx}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}dx}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \; 3955$$

$$-\frac{-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \sqrt{b\tan(c+dx)}dx}{b^2}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 3957

$$-\frac{-\frac{\int \frac{\sqrt{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 266

$$-\frac{-\frac{2\int \frac{b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 826

$$-\frac{-\frac{2\left(\frac{1}{2}\int \frac{b^2\tan^2(c+dx)+b}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 1476

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}+\frac{1}{2}\int \frac{1}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\cdots}{bd}\cdots}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 1082

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-1}d\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{\cdots}{bd\sqrt{\cdots}}}{b^2}$$

$$\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$\Big\downarrow$ 217

3.16. $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int\frac{b-b^2\tan^2(c+dx)}{b^4\tan^4(c+dx)+b^2}d\sqrt{b\tan(c+dx)}\right)}{bd}-\frac{2}{bd\sqrt{b\tan(c+dx)}}}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 1479$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)\cdots\right.}{\sqrt{2}\sqrt{b}}\cdots\right.}{bd}\cdots}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+\cdots\right.}{\sqrt{2}\sqrt{b}}\cdots\right.}{bd}\cdots}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{b}-2\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{2}\sqrt{b}}-\frac{\int\frac{\sqrt{b}+\sqrt{2}\sqrt{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{2}b^{3/2}\tan(c+dx)+b}d\sqrt{b\tan(c+dx)}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+\cdots\right.}{\sqrt{2}\sqrt{b}}\cdots\right.}{bd}\cdots}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

$$\Big\downarrow \ 1103$$

$$-\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{b}\tan(c+dx)+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{b}\tan(c+dx)\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c+dx)+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(\sqrt{2}b^{3/2}\tan(c+dx)+b^2\tan^2(c\cdots\right.}{2\sqrt{2}\sqrt{b}}\cdots\right.}{bd}\cdots}{b^2}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

input
```
Int[(b*Tan[c + d*x])^(-7/2),x]
```

3.16. $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

output
```
-2/(5*b*d*(b*Tan[c + d*x])^(5/2)) - ((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[b]*Ta
n[c + d*x]]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + Sqrt[2]*Sqrt[b]*Tan[c + d*x]]/
(Sqrt[2]*Sqrt[b]))/2 + (Log[b - Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*Tan[c +
 d*x]^2]/(2*Sqrt[2]*Sqrt[b]) - Log[b + Sqrt[2]*b^(3/2)*Tan[c + d*x] + b^2*
Tan[c + d*x]^2]/(2*Sqrt[2]*Sqrt[b]))/2))/(b*d) - 2/(b*d*Sqrt[b*Tan[c + d*x
]]))/b^2
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.16. $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

3.16.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)\right)}{8b^{4}\left(b^{2}\right)^{\frac{1}{4}}}$
default	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)\right)}{8b^{4}\left(b^{2}\right)^{\frac{1}{4}}}$

input
```
int(1/(b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*b*(1/8/b^4/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^
(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/5/b^2/(b*t
an(d*x+c))^(5/2)+1/b^4/(b*tan(d*x+c))^(1/2))
```

3.16.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.09

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{5\,b^4 d\left(-\frac{1}{b^{14}d^4}\right)^{\frac{1}{4}}\log\left(b^{11}d^3\left(-\frac{1}{b^{14}d^4}\right)^{\frac{3}{4}}+\sqrt{b\tan\left(dx+c\right)}\right)\tan\left(dx+c\right)^3-5i\,b^4 d\left(-\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/10*(5*b^4*d*(-1/(b^14*d^4))^(1/4)*log(b^11*d^3*(-1/(b^14*d^4))^(3/4) + s
qrt(b*tan(d*x + c)))*tan(d*x + c)^3 - 5*I*b^4*d*(-1/(b^14*d^4))^(1/4)*log(
I*b^11*d^3*(-1/(b^14*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^3 +
5*I*b^4*d*(-1/(b^14*d^4))^(1/4)*log(-I*b^11*d^3*(-1/(b^14*d^4))^(3/4) + sq
rt(b*tan(d*x + c)))*tan(d*x + c)^3 - 5*b^4*d*(-1/(b^14*d^4))^(1/4)*log(-b^
11*d^3*(-1/(b^14*d^4))^(3/4) + sqrt(b*tan(d*x + c)))*tan(d*x + c)^3 + 4*sq
rt(b*tan(d*x + c))*(5*tan(d*x + c)^2 - 1))/(b^4*d*tan(d*x + c)^3)
```

3.16.6 Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\tan(c+dx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(7/2),x)
```

output
```
Integral((b*tan(c + d*x))**(-7/2), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\cdots\right.}{\sqrt{b}}\cdots\right)}{b^2}\cdots}{20\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
1/20*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x
+ c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) -
 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/b^2 + 8*(5*b^2*ta
n(d*x + c)^2 - b^2)/((b*tan(d*x + c))^(5/2)*b^2))/(b*d)
```

3.16.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

3.16.9 Mupad [B] (verification not implemented)

Time = 3.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.39

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = \frac{(-1)^{1/4}\,\text{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}-\frac{2\tan(c+d\,x)^2}{b}}{d\,(b\tan(c+d\,x))^{5/2}} - \frac{(-1)^{1/4}\,\text{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(7/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
 - (2/(5*b) - (2*tan(c + d*x)^2)/b)/(d*(b*tan(c + d*x))^(5/2)) - ((-1)^(1/
4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
```

3.17 $\int (b\tan(c+dx))^{4/3}\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 243

$$\begin{aligned}\int (b\tan(c+dx))^{4/3}\,dx = &-\frac{b^{4/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}\\ &+\frac{b^{4/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}-\frac{b^{4/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}\\ &+\frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}\\ &-\frac{\sqrt{3}b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}\\ &+\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}\end{aligned}$$

output
```
-b^(4/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(-3^(1/2
)+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(3^(1/2)+2*(b*tan(d*
x+c))^(1/3)/b^(1/3))/d+1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c
))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*3^
(1/2)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d+3*b*(b*tan(d*x+
c))^(1/3)/d
```

3.17.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.84

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{b\sqrt[3]{b\tan(c+dx)}\left(-i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)+i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)-(-1)^{5/6}\log\ldots\right.}{\ldots}$$

input
```
Integrate[(b*Tan[c + d*x])^(4/3),x]
```

output
```
(b*(b*Tan[c + d*x])^(1/3)*((-I)*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] + I*Log[
1 + I*(Tan[c + d*x]^2)^(1/6)] - (-1)^(5/6)*Log[1 - (-1)^(1/6)*(Tan[c + d*x
]^2)^(1/6)] + (-1)^(5/6)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-1)
^(1/6)*Log[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(1/6)*Log[1 + (-1
)^(5/6)*(Tan[c + d*x]^2)^(1/6)] + 6*(Tan[c + d*x]^2)^(1/6)))/(2*d*(Tan[c +
 d*x]^2)^(1/6))
```

3.17.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3954, 3042, 3957, 266, 753, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx))^{4/3}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-b^2\int \frac{1}{(b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 3042$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-b^2\int\frac{1}{(b\tan(c+dx))^{2/3}}dx$$

$$\Big\downarrow \ 3957$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{b^3\int\frac{1}{(b\tan(c+dx))^{2/3}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \ 266$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-\frac{3b^3\int\frac{1}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \ 753$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\int\frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(\cdots}}{2\left(b^2\tan^2(c+dx)+\sqrt{3}b^{4\cdots}\right.}}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\int\frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan\cdots}}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\int\frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int\frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\arctan(b^{\cdots}}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \ 1142$$

3.17. $\int(b\tan(c+dx))^{4/3}\,dx$

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$3b^3\left(\frac{\frac{1}{2}\sqrt[3]{b}\int\frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int-\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\right.$$

$\downarrow$ 25

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$3b^3\left(\frac{\frac{1}{2}\sqrt[3]{b}\int\frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\right.$$

$\downarrow$ 1082

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\frac{\int\frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{\cdots}}{b^2\tan^2(c+dx)+\sqrt{\cdots}}\cdots\right.}{d}$$

$\downarrow$ 217

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan(\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan\cdots}\cdots\right.}{d}$$

$\downarrow$ 1103

$$\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}-$$
$$\frac{3b^3\left(\frac{\arctan\left(b^{2/3}\tan(c+dx)\right)}{3b^{5/3}}+\frac{-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6b^{5/3}}+\frac{\arctan\left(\sqrt{3}\left(\cdots\right.\right.}{\cdots}\right.}{d}$$

input
```
Int[(b*Tan[c + d*x])^(4/3),x]
```

3.17. $\int (b\tan(c+dx))^{4/3}\,dx$

output
```
(-3*b^3*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(5/3)) + (-ArcTan[Sqrt[3]*(1 -
(2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3
)*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3)) + (ArcTan[Sqrt[3]*(1
+ (2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4
/3)*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3))))/d + (3*b*(b*Tan[c
 + d*x])^(1/3))/d
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.17.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}+\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(d}{(b^2}\right.}{2d}$
default	$\frac{3b(b\tan(dx+c))^{\frac{1}{3}}}{d}+\frac{b\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{4d}-\frac{b(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(d}{(b^2}\right.}{2d}$

input
```
int((b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3*b*(b*tan(d*x+c))^(1/3)/d+1/4/d*b*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c))^(
2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))-1/2/d*b*(b^2)^(
1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/4/d*b*3^(1/2)*(b
^2)^(1/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)
+(b^2)^(1/3))-1/2/d*b*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6
)+3^(1/2))-1/d*b*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.28

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((b\tan\left(dx+c\right))^{\frac{1}{3}}b+\frac{1}{2}\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\right)-\left(-\frac{b^8}{d^6}\right)^{\frac{1}{6}}\left(\sqrt{-3}d+d\right)\log\left((b\tan\right.}{}$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
-1/4*((-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*log((b*tan(d*x + c))^(1/3)*b + 1/2
*(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)*lo
g((b*tan(d*x + c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d + d)) + (-b^
8/d^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d*x + c))^(1/3)*b + 1/2*(-b^8/d^6
)^(1/6)*(sqrt(-3)*d - d)) - (-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)*log((b*tan(d
*x + c))^(1/3)*b - 1/2*(-b^8/d^6)^(1/6)*(sqrt(-3)*d - d)) + 2*(-b^8/d^6)^(
1/6)*d*log((b*tan(d*x + c))^(1/3)*b + (-b^8/d^6)^(1/6)*d) - 2*(-b^8/d^6)^(
1/6)*d*log((b*tan(d*x + c))^(1/3)*b - (-b^8/d^6)^(1/6)*d) - 12*(b*tan(d*x
+ c))^(1/3)*b)/d
```

3.17.6 Sympy [F]

$$\int (b\tan(c+dx))^{4/3}\,dx = \int (b\tan(c+dx))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(4/3),x)
```

output
```
Integral((b*tan(c + d*x))**(4/3), x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{\sqrt{3}b^{\frac{7}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}} + (b\tan(dx+c))^{\frac{2}{3}} + b^{\frac{2}{3}}\right) - \sqrt{3}b^{\frac{7}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}} + (b\tan(dx+c))^{\frac{2}{3}} + b^{\frac{2}{3}}\right)\cdots}{\cdots}$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(3)*b^(7/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(
d*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(7/3)*log(-sqrt(3)*(b*tan(d*x + c))
^(1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(7/3)*arctan((sqr
t(3)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(7/3)*arctan(-(sqr
t(3)*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(7/3)*arctan((b*ta
n(d*x + c))^(1/3)/b^(1/3)) - 12*(b*tan(d*x + c))^(1/3)*b^2)/(b*d)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.86

$$\int (b\tan(c+dx))^{4/3}\,dx =$$

$$-\frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}} + (b\tan(dx+c))^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{d} - \frac{\sqrt{3}|b|^{\frac{1}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}\cdots\right)}{d}\cdots\right.$$

input
```
integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
-1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/
3) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/d - sqrt(3)*abs(b)^(1/3)*log(-
sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs
(b)^(2/3))/d + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x
+ c))^(1/3))/abs(b)^(1/3))/d + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3
) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/d + 4*abs(b)^(1/3)*arctan((b*t
an(d*x + c))^(1/3)/abs(b)^(1/3))/d - 12*(b*tan(d*x + c))^(1/3)/d)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^{4/3}\,dx = \frac{3\,b\,(b\tan(c+d\,x))^{1/3}}{d} - \frac{(-1)^{1/6}\,b^{4/3}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d}$$
$$-\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$
$$+\frac{(-1)^{1/6}\,b^{4/3}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(4/3),x)
```

output
```
(3*b*(b*tan(c + d*x))^(1/3))/d - ((-1)^(1/6)*b^(4/3)*atan(((-1)^(5/6)*(b*t
an(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d - ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)
*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2
)*1i)/2 + 1/2))/(2*d) - ((-1)^(1/6)*b^(4/3)*log(2*(b*tan(c + d*x))^(1/3) -
 (-1)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/
(2*d) + ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1
/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 + 1/4))/d + ((-1)^(1/6)*
b^(4/3)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(
1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))/d
```

3.18 $\int (b\tan(c+dx))^{2/3}\,dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 224

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{b^{2/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d} - \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

output
```
b^(2/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(-3^(1/2)
+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(3^(1/2)+2*(b*tan(d*x
+c))^(1/3)/b^(1/3))/d+1/4*b^(2/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c)
)^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(2/3)*ln(b^(2/3)+b^(1/3)*3^(
1/2)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.83

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)-i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)+\sqrt[6]{-1}\left(\log\left(1-\sqrt[6]{-1}\sqrt[6]{\tan^2(\right.\right.\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(2/3),x]
```

output
```
((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)
] + (-1)^(1/6)*(Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - Log[1 + (-1)^
(1/6)*(Tan[c + d*x]^2)^(1/6)] + (-1)^(2/3)*(Log[1 - (-1)^(5/6)*(Tan[c + d*
x]^2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)])))*(b*Tan[c + d*
x])^(5/3))/(2*b*d*(Tan[c + d*x]^2)^(5/6))
```

3.18.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 824, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(c+dx))^{2/3}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{b\int \frac{(b\tan(c+dx))^{2/3}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 266$$

$$\frac{3b\int \frac{b^4\tan^4(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$\downarrow$ 824

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}+\sqrt{3}}{2\left(b^2\tan^2(c+dx)\right.}}{}\right.}{d}$$

$\downarrow$ 27

$$\frac{3b\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}}}{}\right.}{d}$$

$\downarrow$ 216

$$\frac{3b\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}+\frac{\arctan(}{}\right.}{d}$$

$\downarrow$ 1142

$$3b\left(-\frac{-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}\right.$$

$\downarrow$ 25

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\right.$$

$\downarrow$ 1082

3.18. $\int (b\tan(c+dx))^{2/3}\,dx$

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}}{6\sqrt[3]{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(\right.}{\sqrt{3}}\right.$$
$$d$$

$\downarrow$ 217

$$3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan(}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\text{ta}}\right.$$
$$d$$

$\downarrow$ 1103

$$3b\left(\frac{\arctan\left(b^{2/3}\tan(c+dx)\right)}{3\sqrt[3]{b}}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}b^{4/3}\right.}{}\right.$$
$$d$$

input
```
Int[(b*Tan[c + d*x])^(2/3),x]
```

output
```
(3*b*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b
^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*Ta
n[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3)) - (-ArcTan[Sqrt[3]*(1 + (
2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)
*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3))))/d
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.18. $\int (b\tan(c+dx))^{2/3}\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.18.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(d\right.}{}\right.}{d}$
default	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(d\right.}{}\right.}{d}$

input
```
int((b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^
(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*
x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*
x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2
)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/(b^2)^(1/6)
*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.40

$$\int (b\tan(c+dx))^{2/3}\,dx =$$
$$-\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3+\frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right)$$
$$+\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3-\frac{1}{2}\left(\sqrt{-3}d^5+d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right)$$
$$-\frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3+\frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right)$$
$$+\frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b^3-\frac{1}{2}\left(\sqrt{-3}d^5-d^5\right)\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}\right)$$
$$+\frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}b^3\right)$$
$$-\frac{1}{2}\left(-\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(-d^5\left(-\frac{b^4}{d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}b^3\right)$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 + 1/2*
(sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) - 1)*(-b^4/d^6)^(1/
6)*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 + d^5)*(-b^4/d^6)^(5
/6)) - 1/4*(sqrt(-3) + 1)*(-b^4/d^6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3
+ 1/2*(sqrt(-3)*d^5 - d^5)*(-b^4/d^6)^(5/6)) + 1/4*(sqrt(-3) + 1)*(-b^4/d^
6)^(1/6)*log((b*tan(d*x + c))^(1/3)*b^3 - 1/2*(sqrt(-3)*d^5 - d^5)*(-b^4/d
^6)^(5/6)) + 1/2*(-b^4/d^6)^(1/6)*log(d^5*(-b^4/d^6)^(5/6) + (b*tan(d*x +
c))^(1/3)*b^3) - 1/2*(-b^4/d^6)^(1/6)*log(-d^5*(-b^4/d^6)^(5/6) + (b*tan(d
*x + c))^(1/3)*b^3)
```

3.18.6 Sympy [F]

$$\int (b\tan(c+dx))^{2/3}\,dx = \int (b\tan{(c+dx)})^{\frac{2}{3}}\,dx$$

input
```
integrate((b*tan(d*x+c))**(2/3),x)
```

output
```
Integral((b*tan(c + d*x))**(2/3), x)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.75

$$\int (b\tan(c+dx))^{2/3}\,dx =$$

$$-\frac{\left(\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\arctan\left(\frac{\sqrt{3}b^{\frac{1}{3}}+2\,(b\tan(\cdots}{b^{\frac{1}{3}}}\right.}{b^{\frac{1}{3}}}\cdots}{4\,d}$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c)
)^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b
^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(
1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(
1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x +
c))^(1/3)/b^(1/3))/b^(1/3))*b/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.92

$$\int (b\tan(c+dx))^{2/3}\,dx = -\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}|b|^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(-\frac{\sqrt{3}|b|^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd} + \frac{|b|^{\frac{5}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{bd}$$

input
```
integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
+ (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d) + 1/4*sqrt(3)*abs(b)^(5/3)*
log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3)
+ abs(b)^(2/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*
(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan(-(sq
rt(3)*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + abs(b
)^(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b*d)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{(-1)^{1/6}\,b^{2/3}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{d}$$
$$-\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$
$$+\frac{(-1)^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,(-1)^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,(b\tan(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(2/3),x)
```

output
```
((-1)^(1/6)*b^(2/3)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/
d - ((-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1
/2)*1i - 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 - 1/2))/(2*d) - (
(-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1
i + 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-1)^
(1/6)*b^(2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i - 1
)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d + ((-1)^(1/6)*b^(
2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i + 1)*(b*tan(
c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$

3.19.1 Optimal result

Integrand size = 12, antiderivative size = 131

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$

output
```
-1/2*b^(1/3)*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)-b^(
2/3)*(b*tan(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*(
b^(2/3)-2*(b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

3.19.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.81

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = -\frac{\left(\log\left(1+\sqrt[3]{\tan^2(c+dx)}\right)-\sqrt[3]{-1}\log\left(1-\sqrt[3]{-1}\sqrt[3]{\tan^2(c+dx)}\right)+(-1)^{2/3}\log\left(1+(-1)^{2/3}\sqrt[3]{\tan^2}\right.\right.}{2bd\tan^2(c+dx)^{2/3}}$$

input
```
Integrate[(b*Tan[c + d*x])^(1/3),x]
```

output
```
-1/2*((Log[1 + (Tan[c + d*x]^2)^(1/3)] - (-1)^(1/3)*Log[1 - (-1)^(1/3)*(Ta
n[c + d*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 + (-1)^(2/3)*(Tan[c + d*x]^2)^(1/3
)])*(b*Tan[c + d*x])^(4/3))/(b*d*(Tan[c + d*x]^2)^(2/3))
```

3.19.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 807, 821, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\int \frac{\sqrt[3]{b\tan(c+dx)}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b\int \frac{b^3\tan^3(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{807}$$

$$\frac{3b\int \frac{b^2\tan^2(c+dx)}{b^3\tan^3(c+dx)+b^2}d\left(b^2\tan^2(c+dx)\right)}{2d}$$

$$\Big\downarrow \text{821}$$

$$\frac{3b\left(\frac{\int \frac{b^2\tan^2(c+dx)+b^{2/3}}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}-\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$$\Big\downarrow \text{16}$$

$$\frac{3b\left(\frac{\int \frac{b^2\tan^2(c+dx)+b^{2/3}}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$\downarrow$ 1142

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))+\frac{1}{2}\int -\frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan}{3b^{2/3}}\right.}{2d}$$

$\downarrow$ 25

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan^2}{3b^{2/3}}\right.}{2d}$$

$\downarrow$ 1082

$$\frac{3b\left(\frac{3\int \frac{1}{2\sqrt[3]{b}\tan(c+dx)-4}d\left(1-2\sqrt[3]{b}\tan(c+dx)\right)-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$\downarrow$ 217

$$\frac{3b\left(\frac{-\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

$\downarrow$ 1103

$$\frac{3b\left(\frac{\frac{1}{2}\log(-b^{5/3}\tan(c+dx)+b^{4/3}+b^2\tan^2(c+dx))-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{2/3}}-\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{2/3}}\right)}{2d}$$

input
```
Int[(b*Tan[c + d*x])^(1/3),x]
```

3.19. $\int \sqrt[3]{b\tan(c+dx)}\,dx$

output
```
(3*b*(-1/3*Log[b^(2/3) + b^2*Tan[c + d*x]^2]/b^(2/3) + (-(Sqrt[3]*ArcTan[(
1 - 2*b^(1/3)*Tan[c + d*x])/Sqrt[3]]) + Log[b^(4/3) - b^(5/3)*Tan[c + d*x]
 + b^2*Tan[c + d*x]^2]/2)/(3*b^(2/3))))/(2*d)
```

3.19.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.19.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$
default	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-\right.}{3}\right.}{6(b^2)^{\frac{1}{3}}}\right.}{d}$

input
```
int((b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(-1/6/(b^2)^(1/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))+1/12/(b^2)^(1
/3)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+
1/6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(
1/3)-1)))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}b}{3b}\right) - (-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c) - (b\tan(dx+\right.}{4d}$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*tan(d*x + c))^(2/3)*(-b
)^(1/3) + sqrt(3)*b)/b) - (-b)^(1/3)*log((b*tan(d*x + c))^(1/3)*b*tan(d*x
+ c) - (b*tan(d*x + c))^(2/3)*(-b)^(2/3) - (-b)^(1/3)*b) + 2*(-b)^(1/3)*lo
g((b*tan(d*x + c))^(2/3) + (-b)^(2/3)))/d
```

3.19.6 Sympy [F]

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \int \sqrt[3]{b\tan(c+dx)}\,dx$$

input
```
integrate((b*tan(d*x+c))**(1/3),x)
```

output
```
Integral((b*tan(c + d*x))**(1/3), x)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{2\sqrt{3}b^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)+b^{\frac{4}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{4}{3}}-(b\tan\left(dx+c\right))^{\frac{2}{3}}\,b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)-2\,b^{\frac{4}{3}}\log}{4\,bd}$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*b^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2
/3))/b^(2/3)) + b^(4/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3
)*b^(2/3) + b^(4/3)) - 2*b^(4/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b
*d)
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{1}{4}\,b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}+\frac{|b|^{\frac{4}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{1}{3}}\,b\tan\left(dx+c\right)-(b\tan\left(dx+\right.\right.}{b^2d}\right.$$

input
```
integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
 - abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) + abs(b)^(4/3)*log((b*tan(d*x + c))
^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3)
)/(b^2*d) - 2*abs(b)^(4/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2
*d))
```

3.19.9 Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{b\tan(c+dx)}\,dx$$
$$= \frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\tan(c+d\,x))^{2/3}+81\,b^6\right)}{2\,d} - \frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-b)^{1/3}\ln\left(\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(b\tan(c+d\,x))^{2/3}}{d^4}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

input
```
int((b*tan(c + d*x))^(1/3),x)
```

output
```
((-b)^(1/3)*log(81*(-b)^(16/3)*(b*tan(c + d*x))^(2/3) + 81*b^6))/(2*d) - (
(-b)^(1/3)*log((81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*ta
n(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((8
1*b^6)/d^4 + (162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*tan(c + d*x))^(2/3
))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

3.20.1 Optimal result

Integrand size = 12, antiderivative size = 131

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

output
```
1/2*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/b^(1/3)/d-1/4*ln(b^(4/3)-b^(2/3)*(b*t
an(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/b^(1/3)/d-1/2*arctan(1/3*(b^(2/3)-2
*(b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\left(2\sqrt{3}\arctan\left(\frac{-1+2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)+2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)-\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)}{4d\sqrt[3]{b\tan(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x])^(-1/3),x]
```

output
```
((2*Sqrt[3]*ArcTan[(-1 + 2*Tan[c + d*x]^(2/3))/Sqrt[3]] + 2*Log[1 + Tan[c
+ d*x]^(2/3)] - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])*Tan[c +
d*x]^(1/3))/(4*d*(b*Tan[c + d*x])^(1/3))
```

3.20.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3957, 266, 807, 750, 16, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}dx$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{b\int \frac{1}{\sqrt[3]{b\tan(c+dx)}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3b\int \frac{\sqrt[3]{b\tan(c+dx)}}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3b\int \frac{1}{b^3\tan^3(c+dx)+b^2}d(b^2\tan^2(c+dx))}{2d}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d(b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\int \frac{2b^{2/3}-b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3b\left(\frac{\int \frac{2b^{2/3}-b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$\Big\downarrow$ 1142

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))-\frac{1}{2}\int -\frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$\Big\downarrow$ 25

$$\frac{3b\left(\frac{\frac{3}{2}b^{2/3}\int \frac{1}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))+\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$\Big\downarrow$ 1082

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))+3\int \frac{1}{2\sqrt[3]{b}\tan(c+dx)-4}d\left(1-2\sqrt[3]{b}\tan(c+dx)\right)}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$\Big\downarrow$ 217

$$\frac{3b\left(\frac{\frac{1}{2}\int \frac{b^{2/3}-2b^2\tan^2(c+dx)}{b^2\tan^2(c+dx)-b^{5/3}\tan(c+dx)+b^{4/3}}d(b^2\tan^2(c+dx))-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

$\Big\downarrow$ 1103

$$\frac{3b\left(\frac{-\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{b}\tan(c+dx)}{\sqrt{3}}\right)-\frac{1}{2}\log(-b^{5/3}\tan(c+dx)+b^{4/3}+b^2\tan^2(c+dx))}{3b^{4/3}}+\frac{\log(b^{2/3}+b^2\tan^2(c+dx))}{3b^{4/3}}\right)}{2d}$$

input
```
Int[(b*Tan[c + d*x])^(-1/3),x]
```

3.20. $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

output
```
(3*b*(Log[b^(2/3) + b^2*Tan[c + d*x]^2]/(3*b^(4/3)) + (-(Sqrt[3]*ArcTan[(1
 - 2*b^(1/3)*Tan[c + d*x])/Sqrt[3]]) - Log[b^(4/3) - b^(5/3)*Tan[c + d*x]
+ b^2*Tan[c + d*x]^2]/2)/(3*b^(4/3))))/(2*d)
```

3.20.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.20.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-1\right)}{3}\right)}{6(b^2)^{\frac{2}{3}}}\right.}{d}$
default	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}(b^2)^{\frac{1}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}-1\right)}{3}\right)}{6(b^2)^{\frac{2}{3}}}\right.}{d}$

input
```
int(1/(b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(1/6/(b^2)^(2/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))-1/12/(b^2)^(2/
3)*ln((b*tan(d*x+c))^(4/3)-(b*tan(d*x+c))^(2/3)*(b^2)^(1/3)+(b^2)^(2/3))+1
/6/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*tan(d*x+c))^(2/3)/(b^2)^(1
/3)-1)))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \left[\frac{\sqrt{3}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\tan(dx+c)+2b\tan(dx+c)^2-\sqrt{3}b^{\frac{4}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+(b\tan(dx+c))^{\frac{2}{3}}\left(\sqrt{3}b^{\frac{2}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{1}{3}}\right)-b}{\tan(dx+c)^2+1}\right)}{4\ldots}\right.$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*sqrt(3)*(b*tan(d*x + c))^(1/3)*b*s
qrt(-1/b^(2/3))*tan(d*x + c) + 2*b*tan(d*x + c)^2 - sqrt(3)*b^(4/3)*sqrt(-
1/b^(2/3)) + (b*tan(d*x + c))^(2/3)*(sqrt(3)*b^(2/3)*sqrt(-1/b^(2/3)) - 3*
b^(1/3)) - b)/(tan(d*x + c)^2 + 1)) - b^(2/3)*log((b*tan(d*x + c))^(1/3)*b
*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log(
(b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d), 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)*b^(2/3) - b^(4/3))/b^(4/3)) - b^(2/3)*
log((b*tan(d*x + c))^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3)
 + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)]
```

3.20.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\tan{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(1/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-1/3), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{2\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right) - b^{\frac{2}{3}}\log\left((b\tan{(dx+c)})^{\frac{4}{3}} - (b\tan{(dx+c)})^{\frac{2}{3}}\,b^{\frac{2}{3}} + b^{\frac{4}{3}}\right) + 2\,b^{\frac{2}{3}}\log}{4\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2
/3))/b^(2/3)) - b^(2/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3
)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b
*d)
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{2\,bd} - \frac{|b|^{\frac{2}{3}}\log\left((b\tan{(dx+c)})^{\frac{1}{3}}\,b\tan{(dx+c)} - (b\tan{(dx+c)})^{\frac{2}{3}}\,|b|^{\frac{2}{3}} + |b|^{\frac{4}{3}}\right)}{4\,bd} + \frac{|b|^{\frac{2}{3}}\log\left((b\tan{(dx+c)})^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{2\,bd}$$

input
```
integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - ab
s(b)^(2/3))/abs(b)^(2/3))/(b*d) - 1/4*abs(b)^(2/3)*log((b*tan(d*x + c))^(1
/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b*d) + 1/2*abs(b)^(2/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{\ln\left((b\tan(c+d\,x))^{2/3}+b^{2/3}\right)}{2\,b^{1/3}\,d} + \frac{\ln\left(\frac{81\,b^{11/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d} - \frac{\ln\left(\frac{81\,b^{11/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}-\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,b^{1/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(1/3),x)
```

output
```
log((b*tan(c + d*x))^(2/3) + b^(2/3))/(2*b^(1/3)*d) + (log((81*b^(11/3)*(3
^(1/2)*1i - 1))/d^3 + (162*b^3*(b*tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i -
1))/(4*b^(1/3)*d) - (log((81*b^(11/3)*(3^(1/2)*1i + 1))/d^3 - (162*b^3*(b*
tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*b^(1/3)*d)
```

3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 224

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

output
```
arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(-3^(1/2)+2*(b*ta
n(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/
3)/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)+
(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b*
tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d
```

3.21.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\left(i\log\left(1-i\sqrt[6]{\tan^2(c+dx)}\right)-i\log\left(1+i\sqrt[6]{\tan^2(c+dx)}\right)+\sqrt[6]{-1}\left((-1)^{2/3}\text{lo}\right.\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(-2/3),x]
```

output
```
((I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)] - I*Log[1 + I*(Tan[c + d*x]^2)^(1/6)
] + (-1)^(1/6)*((-1)^(2/3)*Log[1 - (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] - (-
1)^(2/3)*Log[1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)] + Log[1 - (-1)^(5/6)*(
Tan[c + d*x]^2)^(1/6)] - Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]))*(b*T
an[c + d*x])^(1/3))/(2*b*d*(Tan[c + d*x]^2)^(1/6))
```

3.21.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 753, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{(b\tan(c+dx))^{2/3}}dx$$
$$\Big\downarrow \ 3957$$
$$\frac{b\int \frac{1}{(b\tan(c+dx))^{2/3}(\tan^2(c+dx)b^2+b^2)}d(b\tan(c+dx))}{d}$$
$$\Big\downarrow \ 266$$
$$\frac{3b\int \frac{1}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{d}$$

$$\Big\downarrow \; 753$$

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan \cdots}}{2\left(b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\cdots\right.}\cdots}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{3b\left(\frac{\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{3b^{4/3}}+\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c\cdots}\cdots}{6\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{3b\left(\frac{\int \frac{2\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\int \frac{2\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\arctan(b^{2/\cdots}}{3\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \; 1142$$

$$3b\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\cdots\right.$$

$$\Big\downarrow \; 25$$

$$3b\left(\frac{\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{1}{2}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{3b\left(\frac{\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b\cdots}\cdots}{\cdots}\cdots\right.}{d}$$

$$\Big\downarrow \; 217$$

3.21. $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

$$3b\left(\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6b^{5/3}}+\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan(c}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c}\right.$$
$$\overline{\qquad d \qquad}$$

$\downarrow$ 1103

$$3b\left(\frac{\arctan(b^{2/3}\tan(c+dx))}{3b^{5/3}}+\frac{-\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6b^{5/3}}+\frac{\arctan\left(\sqrt{3}\left(\frac{2b}{}\right.\right.}{}\right.$$
$$\overline{\qquad d \qquad}$$

input
```
Int[(b*Tan[c + d*x])^(-2/3),x]
```

output
```
(3*b*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(5/3)) + (-ArcTan[Sqrt[3]*(1 - (2*
b^(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*T
an[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3)) + (ArcTan[Sqrt[3]*(1 + (
2*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)
*Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(5/3))))/d
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.21. $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

3.21.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.91

method	result
derivativedivides	$3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln\left(-\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\cdots$
default	$3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln\left(-\left(b\tan(dx+c)\right)^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\left(b\tan(dx+c)\right)^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\cdots$

input
```
int(1/(b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/d*b*(-1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln(-(b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2
)^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6/b^2*(b^2)^(1/6)*arctan(2*(b*
tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln((b*
tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/
6/b^2*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/b
^2*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.26

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd+bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}bd-bd\right)\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) + \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right) - \frac{1}{2}\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(-bd\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{6}} + (b\tan(dx+c))^{\frac{1}{3}}\right)$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-3) + 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b*d + b*d)*(-1/(
b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) + 1)*(-1/(b^4*d^
6))^(1/6)*log(-1/2*(sqrt(-3)*b*d + b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x
+ c))^(1/3)) + 1/4*(sqrt(-3) - 1)*(-1/(b^4*d^6))^(1/6)*log(1/2*(sqrt(-3)*b
*d - b*d)*(-1/(b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/4*(sqrt(-3) -
 1)*(-1/(b^4*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b*d - b*d)*(-1/(b^4*d^6))^(1/6
) + (b*tan(d*x + c))^(1/3)) + 1/2*(-1/(b^4*d^6))^(1/6)*log(b*d*(-1/(b^4*d^
6))^(1/6) + (b*tan(d*x + c))^(1/3)) - 1/2*(-1/(b^4*d^6))^(1/6)*log(-b*d*(-
1/(b^4*d^6))^(1/6) + (b*tan(d*x + c))^(1/3))
```

3.21.6 Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\tan{(c+dx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(2/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-2/3), x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.76

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{\sqrt{3}b^{\frac{1}{3}}\log\left(\sqrt{3}(b\tan{(dx+c)})^{\frac{1}{3}}\,b^{\frac{1}{3}} + (b\tan{(dx+c)})^{\frac{2}{3}} + b^{\frac{2}{3}}\right) - \sqrt{3}b^{\frac{1}{3}}\log\left(-\sqrt{3}(b\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
1/4*(sqrt(3)*b^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d
*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(1/3)*log(-sqrt(3)*(b*tan(d*x + c))^
(1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(1/3)*arctan((sqrt
(3)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(1/3)*arctan(-(sqrt
(3)*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(1/3)*arctan((b*tan
(d*x + c))^(1/3)/b^(1/3)))/(b*d)
```

3.21.8 Giac [F]

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\tan{(dx+c)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c))^(-2/3), x)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{2/3}\,d} - \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d} - \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d} + \frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d} + \frac{(-1)^{1/6}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(2/3),x)
```

output
```
((-1)^(1/6)*atan(((-1)^(5/6)*(b*tan(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(2
/3)*d) - ((-1)^(1/6)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) + (
-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3)*d) - ((-1)^(
1/6)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2
)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)*d) + ((-1)^(1/6)*log((-1)^(1
/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(
1/2)*1i)/4 + 1/4))/(b^(2/3)*d) + ((-1)^(1/6)*log(2*(b*tan(c + d*x))^(1/3)
- (-1)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))
/(b^(2/3)*d)
```

3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 245

$$\begin{aligned}\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx &= -\frac{\arctan\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} \\ &+ \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} \\ &- \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} \\ &+ \frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}\end{aligned}$$

output
```
-arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(-3^(1/2)+2*(b*t
an(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1
/3)/b^(1/3))/b^(4/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)
+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b
*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d-3/b/d/(b*tan(d*
x+c))^(1/3)
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.04

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{-6 - i\log\left(1 - i\sqrt[6]{\tan^2(c+dx)}\right)\sqrt[6]{\tan^2(c+dx)} + i\log\left(1 + i\sqrt[6]{\tan^2(c+dx)}\right)}{}$$

input
```
Integrate[(b*Tan[c + d*x])^(-4/3),x]
```

output
```
(-6 - I*Log[1 - I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + I*Log[1
 + I*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(1/6)*Log[1 - (
-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^(1/6)*Log[
1 + (-1)^(1/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) - (-1)^(5/6)
*Log[1 - (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6) + (-1)^
(5/6)*Log[1 + (-1)^(5/6)*(Tan[c + d*x]^2)^(1/6)]*(Tan[c + d*x]^2)^(1/6))/(
2*b*d*(b*Tan[c + d*x])^(1/3))
```

3.22.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 824, 27, 216, 1142, 25, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{3955}$$

$$-\frac{\int (b\tan(c+dx))^{2/3}dx}{b^2} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (b\tan(c+dx))^{2/3}dx}{b^2}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 3957

$$-\frac{\int \frac{(b\tan(c+dx))^{2/3}}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{bd}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 266

$$-\frac{3\int \frac{b^4\tan^4(c+dx)}{b^6\tan^6(c+dx)+b^2}d\sqrt[3]{b\tan(c+dx)}}{bd}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 824

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\frac{\int -\frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{2\left(b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}\right)}d\sqrt[3]{b\tan(c+dx)}}{3\sqrt[3]{b}}+\frac{\int -\frac{\sqrt[3]{b}+\sqrt{3}\cdots}{2\left(b^2\tan^2(c+dx\cdots\right.}\cdots}{\cdots}\cdots\right.}{bd}\cdots$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 27

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{b^2\tan^2(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan\cdots}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\cdots}\cdots}{\cdots}\cdots\right.}{bd}\cdots$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 216

$$-\frac{3\left(-\frac{\int \frac{\sqrt[3]{b}-\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{b}+\sqrt{3}\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}+\frac{\arctan\cdots}{\cdots}\cdots\right.}{bd}\cdots$$

$$\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

$\Big\downarrow$ 1142

3.22. $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

$$-\frac{3\left(-\frac{-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$\downarrow$ 25

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{1}{2}\sqrt[3]{b}\int \frac{1}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}}{6\sqrt[3]{b}}-\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

$\downarrow$ 1082

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)}{\sqrt{3}}}{6\sqrt[3]{b}}-\frac{\int \frac{1}{-b^2\tan^2(c+dx)-\frac{1}{3}}d(}{\sqrt{3}}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

bd

$\downarrow$ 217

$$-\frac{3\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}-2\sqrt[3]{b\tan(c+dx)}}{b^2\tan^2(c+dx)-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}}d\sqrt[3]{b\tan(c+dx)}+\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[3]{b}+2\sqrt[3]{b\tan}}{b^2\tan^2(c+dx)+\sqrt{3}b^{4/3}t}}{}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

bd

$\downarrow$ 1103

$$-\frac{3\left(\frac{\arctan(b^{2/3}\tan(c+dx))}{3\sqrt[3]{b}}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2b^{2/3}\tan(c+dx)}{\sqrt{3}}\right)\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}b^{4/3}\tan(c+dx)+b^{2/3}+b^2\tan^2(c+dx)\right)}{6\sqrt[3]{b}}-\frac{\frac{1}{2}\sqrt{3}\log\left(\sqrt{3}b^{4/3}\right.}{}\right.}{\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}}$$

bd

3.22. $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

input
```
Int[(b*Tan[c + d*x])^(-4/3),x]
```

output
```
(-3*(ArcTan[b^(2/3)*Tan[c + d*x]]/(3*b^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b^
(2/3)*Tan[c + d*x])/Sqrt[3])] - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(4/3)*Tan
[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3)) - (-ArcTan[Sqrt[3]*(1 + (2
*b^(2/3)*Tan[c + d*x])/Sqrt[3])] + (Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(4/3)*
Tan[c + d*x] + b^2*Tan[c + d*x]^2])/2)/(6*b^(1/3))))/(b*d) - 3/(b*d*(b*Tan
[c + d*x])^(1/3))
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.22. $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

3.22.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{3b\left(-\frac{1}{b^2(b\tan(dx+c))^{\frac{1}{3}}}-\frac{\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6\left(b^2\right)^{\frac{1}{6}}}}{}\right.}{}$
default	$\frac{3b\left(-\frac{1}{b^2(b\tan(dx+c))^{\frac{1}{3}}}-\frac{\frac{\sqrt{3}\left(b^2\right)^{\frac{5}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6\left(b^2\right)^{\frac{1}{6}}}}{}\right.}{}$

input

```
int(1/(b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

output

```
3/d*b*(-1/b^2/(b*tan(d*x+c))^(1/3)-(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln(-(b*ta
n(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6/
(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^
(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c
))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^
(1/6)+3^(1/2))+1/3/(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))/b
^2)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(187) = 374$.

Time = 0.25 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.76

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx =$$

$$-\frac{2\,b^2d\left(-\frac{1}{b^8d^6}\right)^{\frac{1}{6}}\log\left(b^7d^5\left(-\frac{1}{b^8d^6}\right)^{\frac{5}{6}}+(b\tan(dx+c))^{\frac{1}{3}}\right)\tan(dx+c)-2\,b^2d\left(-\frac{1}{b^8d^6}\right)^{\frac{1}{6}}\log\left(-b^7d^5\left(-\frac{1}{b^8d^6}\right)^{\frac{5}{6}}+\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(b^7*d^5*(-1/(b^8*d^6))^(5/6) + (b*t
an(d*x + c))^(1/3))*tan(d*x + c) - 2*b^2*d*(-1/(b^8*d^6))^(1/6)*log(-b^7*d
^5*(-1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)
*b^2*d - b^2*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*
(-1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^
2*d - b^2*d)*(-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 + b^7*d^5)*(-
1/(b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) - (sqrt(-3)*b^2*
d + b^2*d)*(-1/(b^8*d^6))^(1/6)*log(1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(
b^8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + (sqrt(-3)*b^2*d +
 b^2*d)*(-1/(b^8*d^6))^(1/6)*log(-1/2*(sqrt(-3)*b^7*d^5 - b^7*d^5)*(-1/(b^
8*d^6))^(5/6) + (b*tan(d*x + c))^(1/3))*tan(d*x + c) + 12*(b*tan(d*x + c))
^(2/3))/(b^2*d*tan(d*x + c))
```

3.22.6 Sympy [F]

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\tan{(c+dx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c))**(4/3),x)
```

output
```
Integral((b*tan(c + d*x))**(-4/3), x)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.74

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{\frac{\sqrt{3}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}} - \frac{\sqrt{3}\log\left(-\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right.}{b^{\frac{1}{3}}}}{}$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))
^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^
(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1
/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1
/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c
))^(1/3)/b^(1/3))/b^(1/3) - 12/(b*tan(d*x + c))^(1/3))/(b*d)
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = \frac{1}{4}b\left(\frac{\sqrt{3}|b|^{\frac{5}{3}}\log\left(\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^4d} - \frac{\sqrt{3}|b|^{\frac{5}{3}}\,\mathrm{lo}}{}\right.$$

input
```
integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^4*d) - sqrt(3)*abs(b)^(5/3)*
log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3)
+ abs(b)^(2/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*
(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan(-(sq
rt(3)*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 4*a
bs(b)^(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^4*d) - 12/((b*t
an(d*x + c))^(1/3)*b^2*d))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.13

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{3}{b\,d\,(b\tan{(c+d\,x)})^{1/3}} - \frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,(b\tan(c+d\,x))^{1/3}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{4/3}\,d}$$
$$-\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}$$
$$-\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}$$
$$+\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d}$$
$$+\frac{(-1)^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,(-1)^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\tan(c+d\,x))^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}\,d}$$

input
```
int(1/(b*tan(c + d*x))^(4/3),x)
```

output
```
((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/
4 - 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(4/3)*d) - ((-
1)^(1/6)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/(b^(4/3)*d)
 - ((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i
)/2 - 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(4/3)*d) -
 ((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/
2 + 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(4/3)*d) - 3
/(b*d*(b*tan(c + d*x))^(1/3)) + ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(
1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/4 + 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)
*1i)/4 + 1/4))/(b^(4/3)*d)
```

3.23 $\int (b\tan(c+dx))^n\,dx$

3.23.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (b\tan(c+dx))^n\,dx=\frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1+n)/b
/d/(1+n)
```

3.23.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (b\tan(c+dx))^n\,dx$$
$$=\frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(c+dx)\right)\tan(c+dx)(b\tan(c+dx))^n}{d(1+n)}$$

input
```
Integrate[(b*Tan[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*x]*
(b*Tan[c + d*x])^n)/(d*(1 + n))
```

3.23.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(c+dx))^n dx$$

$$\Big\downarrow \; 3957$$

$$\frac{b\int \frac{(b\tan(c+dx))^n}{\tan^2(c+dx)b^2+b^2}d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \; 278$$

$$\frac{(b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(c+dx)\right)}{bd(n+1)}$$

input
```
Int[(b*Tan[c + d*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*
x])^(1 + n))/(b*d*(1 + n))
```

3.23.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.23.4 Maple [F]

$$\int (b\tan(dx+c))^n\,dx$$

input
```
int((b*tan(d*x+c))^n,x)
```

output
```
int((b*tan(d*x+c))^n,x)
```

3.23.5 Fricas [F]

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(dx+c))^n\,dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c))^n, x)
```

3.23.6 Sympy [F]

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(c+dx))^n\,dx$$

input
```
integrate((b*tan(d*x+c))**n,x)
```

output
```
Integral((b*tan(c + d*x))**n, x)
```

3.23.7 Maxima [F]

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(dx+c))^n\,dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c))^n, x)
```

3.23.8 Giac [F]

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(dx+c))^n\,dx$$

input
```
integrate((b*tan(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c))^n, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (b\tan(c+dx))^n\,dx = \int (b\tan(c+d\,x))^n\,dx$$

input
```
int((b*tan(c + d*x))^n,x)
```

output
```
int((b*tan(c + d*x))^n, x)
```

3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}$$

output
```
-b^2*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d-1/2*b^2*(b*tan(d*x
+c)^2)^(1/2)*tan(d*x+c)/d+1/4*b^2*(b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

3.24.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = -\frac{\cot(c+dx)\left(-1+2\cot^2(c+dx)+4\cot^4(c+dx)\log(\cos(c+dx))\right)\left(b\tan^2(c+dx)\right)^{5/2}}{4d}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
-1/4*(Cot[c + d*x]*(-1 + 2*Cot[c + d*x]^2 + 4*Cot[c + d*x]^4*Log[Cos[c + d
*x]])*(b*Tan[c + d*x]^2)^(5/2))/d
```

3.24.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(c+dx)\right)^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan^5(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\int \tan(c+dx)^3dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\int \tan(c+dx)dx+\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3956}$$

$$b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(b*Tan[c + d*x]^2)^(5/2),x]
```

output
```
b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^2]*(-(Log[Cos[c + d*x]]/d) - Tan[c +
d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d))
```

3.24.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.24.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
default	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
risch	$\frac{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1}-\frac{2b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{4ib^2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)}{\left(e^{2i(dx+c)}+1\right)}}}{\left(e^{2i(dx+c}\right.}$

input
```
int((b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*(b*tan(d*x+c)^2)^(5/2)*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c
)^2))/tan(d*x+c)^5
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int (b\tan^2(c+dx))^{5/2}\,dx = \frac{\left(b^2\tan(dx+c)^4 - 2\,b^2\tan(dx+c)^2 - 2\,b^2\log\left(\frac{1}{\tan(dx+c)^2+1}\right) - 3\,b^2\right)\sqrt{b\tan(dx+c)^2}}{4\,d\tan(dx+c)}$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(b^2*tan(d*x + c)^4 - 2*b^2*tan(d*x + c)^2 - 2*b^2*log(1/(tan(d*x + c)
^2 + 1)) - 3*b^2)*sqrt(b*tan(d*x + c)^2)/(d*tan(d*x + c))
```

3.24.6 Sympy [F]

$$\int (b\tan^2(c+dx))^{5/2}\,dx = \int \left(b\tan^2(c+dx)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(5/2), x)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \frac{b^{\frac{5}{2}}\tan\left(dx+c\right)^4 - 2\,b^{\frac{5}{2}}\tan\left(dx+c\right)^2 + 2\,b^{\frac{5}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{4\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(b^(5/2)*tan(d*x + c)^4 - 2*b^(5/2)*tan(d*x + c)^2 + 2*b^(5/2)*log(tan
(d*x + c)^2 + 1))/d
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(88) = 176.

Time = 1.18 (sec) , antiderivative size = 646, normalized size of antiderivative = 6.59

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(2*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*sgn(tan(d*x + c))*tan(d*x)^4*tan(
c)^4 + 3*b^2*sgn(tan(d*x + c))*tan(d*x)^4*tan(c)^4 - 8*b^2*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*sgn(tan(d*x + c))*tan(d*x)^3*tan(c)^3 + 2*b^2*sgn(tan(d*x +
c))*tan(d*x)^4*tan(c)^2 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)^3*tan(c)^3 + 2*
b^2*sgn(tan(d*x + c))*tan(d*x)^2*tan(c)^4 + 12*b^2*log(4*(tan(d*x)^2*tan(c
)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2
+ 1))*sgn(tan(d*x + c))*tan(d*x)^2*tan(c)^2 - b^2*sgn(tan(d*x + c))*tan(d*
x)^4 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)^3*tan(c) + 4*b^2*sgn(tan(d*x + c))
*tan(d*x)^2*tan(c)^2 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)*tan(c)^3 - b^2*sgn
(tan(d*x + c))*tan(c)^4 - 8*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*sgn(tan(d*x +
 c))*tan(d*x)*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(d*x)^2 - 8*b^2*sgn(tan(
d*x + c))*tan(d*x)*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(c)^2 + 2*b^2*log(4
*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 + tan(c)^2 + 1))*sgn(tan(d*x + c)) + 3*b^2*sgn(tan(d*x + c)))*sqrt(
b)/(d*tan(d*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c
)^2 - 4*d*tan(d*x)*tan(c) + d)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{5/2} dx = \int \left(b\tan(c+d\,x)^2\right)^{5/2} dx$$

input
```
int((b*tan(c + d*x)^2)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(5/2), x)
```

3.25 $\int (b \tan^2(c+dx))^{3/2}\,dx$

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (b \tan^2(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} + \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}$$

output
```
b*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d+1/2*b*(b*tan(d*x+c)^2
)^(1/2)*tan(d*x+c)/d
```

3.25.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int (b\tan^2(c+dx))^{3/2}\,dx = \frac{\cot^3(c+dx)\,(b\tan^2(c+dx))^{3/2}\,(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(3/2),x]
```

output
```
(Cot[c + d*x]^3*(b*Tan[c + d*x]^2)^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c + d*
x]^2))/(2*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4141, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4141}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan^3(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\int \tan(c+dx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\left(\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}\right)$$

input
```
Int[(b*Tan[c + d*x]^2)^(3/2),x]
```

output
```
b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^2]*(Log[Cos[c + d*x]]/d + Tan[c + d*x]^
2/(2*d))
```

3.25.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.25.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{\left(b \left(\tan^{2}\left(dx+c\right)\right)\right)^{\frac{3}{2}} \left(-\left(\tan^{2}\left(dx+c\right)\right)+\ln\left(1+\tan^{2}\left(dx+c\right)\right)\right)}{2d \tan\left(dx+c\right)^{3}}$
default	$-\frac{\left(b \left(\tan^{2}\left(dx+c\right)\right)\right)^{\frac{3}{2}} \left(-\left(\tan^{2}\left(dx+c\right)\right)+\ln\left(1+\tan^{2}\left(dx+c\right)\right)\right)}{2d \tan\left(dx+c\right)^{3}}$
risch	$\frac{b \sqrt{-\frac{b \left(e^{2i(dx+c)}-1\right)^{2}}{\left(e^{2i(dx+c)}+1\right)^{2}}}\, \left(i e^{4i(dx+c)} \ln\left(e^{2i(dx+c)}+1\right)+e^{4i(dx+c)} dx+2\, e^{4i(dx+c)} c+2i e^{2i(dx+c)} \ln\left(e^{2i(dx+c)}+1\right)+2\, e^{2i(dx+c}\right.}{\left(e^{2i(dx+c)}-1\right) \left(e^{2i(dx+c)}+1\right) d}$

input
```
int((b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(b*tan(d*x+c)^2)^(3/2)*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/tan(d*x+c
)^3
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{\left(b\tan\left(dx+c\right)^2 + b\log\left(\frac{1}{\tan(dx+c)^2+1}\right) + b\right)\sqrt{b\tan\left(dx+c\right)^2}}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)) + b)*sqrt(b*tan(d*x
+ c)^2)/(d*tan(d*x + c))
```

3.25.6 Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \int \left(b\tan^2\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(3/2), x)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx = \frac{b^{\frac{3}{2}}\tan\left(dx+c\right)^2 - b^{\frac{3}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(b^(3/2)*tan(d*x + c)^2 - b^(3/2)*log(tan(d*x + c)^2 + 1))/d
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(55) = 110.

Time = 0.61 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.70

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx = \frac{\left(\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2+\tan(dx)^2\tan(c)^2-2\log\left(\frac{4\left(\tan\cdots\right.}{\tan(\cdots}\right.}{2\left(d\cdots\right.}$$

input
```
integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + tan(d*x)^2*tan(c)^
2 - 2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2
 + log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^
2 + tan(d*x)^2 + tan(c)^2 + 1)) + 1)*b^(3/2)*sgn(tan(d*x + c))/(d*tan(d*x)
^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx = \int \left(b\tan(c+d\,x)^2\right)^{3/2} dx$$

input
```
int((b*tan(c + d*x)^2)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(3/2), x)
```

3.26 $\int \sqrt{b \tan^2(c+dx)}\,dx$

3.26.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b \tan^2(c+dx)}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d
```

3.26.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{b \tan^2(c+dx)}\,dx = -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b \tan^2(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

3.26.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4141, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b \tan^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{b \tan(c+dx)^2}dx$$

$$\Big\downarrow \; 4141$$

$$\cot(c+dx)\sqrt{b \tan^2(c+dx)} \int \tan(c+dx)dx$$

$$\Big\downarrow \; 3042$$

$$\cot(c+dx)\sqrt{b \tan^2(c+dx)} \int \tan(c+dx)dx$$

$$\Big\downarrow \; 3956$$

$$-\frac{\cot(c+dx)\sqrt{b \tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

3.26.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.26.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\sqrt{b(\tan^2(dx+c))}\,\ln(1+\tan^2(dx+c))}{2d\tan(dx+c)}$
default	$\frac{\sqrt{b(\tan^2(dx+c))}\,\ln(1+\tan^2(dx+c))}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)x}{e^{2i(dx+c)}-1}-\frac{2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(dx+c)}+1)(dx+c)}{(e^{2i(dx+c)}-1)d}-\frac{i\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(e^{2i(d\ldots}{(e^{2i(dx+c\ldots}$

input
```
int((b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(b*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \sqrt{b\tan^2(c+dx)}\,dx=-\frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d\tan\left(dx+c\right)}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(d*x + c)^2)*log(1/(tan(d*x + c)^2 + 1))/(d*tan(d*x + c))
```

3.26.6 Sympy [F]

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \int \sqrt{b\tan^2\left(c+dx\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**2), x)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \frac{\sqrt{b}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)/d
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \sqrt{b\tan^2(c+dx)}\,dx = -\frac{\sqrt{b}\log\left(|\cos\left(dx+c\right)|\right)\operatorname{sgn}(\tan\left(dx+c\right))}{d}$$

input
```
integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b)*log(abs(cos(d*x + c)))*sgn(tan(d*x + c))/d
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^2(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^2}\,dx$$

input
```
int((b*tan(c + d*x)^2)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^2)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$

3.27.1 Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

output
```
ln(sin(d*x+c))*tan(d*x+c)/d/(b*tan(d*x+c)^2)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d
*x]^2])
```

3.27.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan(c+dx)\int \cot(c+dx)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx}{\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\tan(c+dx)\log(-\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^2],x]
```

output
```
(Log[-Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

3.27.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.27.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\right)}{2d\sqrt{b\left(\tan^2(dx+c)\right)}}$
default	$-\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\right)}{2d\sqrt{b\left(\tan^2(dx+c)\right)}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}-\frac{i\left(e^{2i(dx+c)}-1\right)\ln\left(e^{2i(dx+c)}-1\right)}{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\left(e^{2i(dx+c)}+1\right)d}$

input
```
int(1/(b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)-2*ln(tan(d*x+c)))/(b*tan(d*x+c)^2)^(
1/2)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\sqrt{b\tan\left(dx+c\right)^2}\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,bd\tan\left(dx+c\right)}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))/(b*d*t
an(d*x + c))
```

3.27.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^2\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**2), x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = -\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{b}} - \frac{2\log(\tan(dx+c))}{\sqrt{b}}}{2\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(log(tan(d*x + c)^2 + 1)/sqrt(b) - 2*log(tan(d*x + c))/sqrt(b))/d
```

3.27.8 Giac [F]

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan(dx+c)^2}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^2), x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{-b}\tan(c+dx)}{\sqrt{b\tan(c+dx)^2}}\right)}{\sqrt{-b}\,d}$$

input
```
int(1/(b*tan(c + d*x)^2)^(1/2),x)
```

output
```
atan(((-b)^(1/2)*tan(c + d*x))/(b*tan(c + d*x)^2)^(1/2))/((-b)^(1/2)*d)
```

3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$

3.28.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}} - \frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}$$

output
```
-1/2*cot(d*x+c)/b/d/(b*tan(d*x+c)^2)^(1/2)-ln(sin(d*x+c))*tan(d*x+c)/b/d/(
b*tan(d*x+c)^2)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)\tan^3(c+dx)}{2d\left(b\tan^2(c+dx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
-1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
 d*x]^3)/(d*(b*Tan[c + d*x]^2)^(3/2))
```

3.28.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(b\tan(c+dx)^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 4141$$

$$\frac{\tan(c+dx)\int \cot^3(c+dx)dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 3954$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int -\cot(c+dx)dx\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\tan(c+dx)\left(\int \cot(c+dx)dx+\frac{\cot^2(c+dx)}{2d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^2(c+dx)}{2d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \ 25$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}-\int\tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx\right)}{b\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^2(c+dx)}{2d}+\frac{\log(-\sin(c+dx))}{d}\right)}{b\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^2)^(-3/2),x]
```

output
```
-(((Cot[c + d*x]^2/(2*d) + Log[-Sin[c + d*x]]/d)*Tan[c + d*x])/(b*Sqrt[b*T
an[c + d*x]^2]))
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.28.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
default	$\frac{\tan(dx+c)\left(\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)-2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
risch	$\frac{i e^{4i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)+e^{4i(dx+c)}dx+2\,e^{4i(dx+c)}c-2i e^{2i(dx+c)}\ln\left(e^{2i(dx+c)}-1\right)-2\,e^{2i(dx+c)}dx-2i e^{2i(dx+c)}-4\,e^{2i(}}{b\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}$

input
```
int(1/(b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*tan(d*x+c)*(ln(1+tan(d*x+c)^2)*tan(d*x+c)^2-2*ln(tan(d*x+c))*tan(d*x
+c)^2-1)/(b*tan(d*x+c)^2)^(3/2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = -\frac{\sqrt{b\tan\left(dx+c\right)^2}\left(\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^2+\tan\left(dx+c\right)^2+1\right)}{2\,b^2d\tan\left(dx+c\right)^3}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*tan(d*x + c)^2)*(log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(
d*x + c)^2 + tan(d*x + c)^2 + 1)/(b^2*d*tan(d*x + c)^3)
```

3.28.6 Sympy [F]

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(-3/2), x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\,\log(\tan(dx+c))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\tan(dx+c)^2}}{2\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(log(tan(d*x + c)^2 + 1)/b^(3/2) - 2*log(tan(d*x + c))/b^(3/2) - 1/(b^
(3/2)*tan(d*x + c)^2))/d
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(60) = 120.

Time = 0.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.56

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \frac{\frac{\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{\sqrt{b}(\cos(dx+c)-1)\mathrm{sgn}(\tan(dx+c))} - \frac{4\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))} + \frac{8\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{\sqrt{b}\mathrm{sgn}(\tan(dx+c))} + \frac{\cdots}{\sqrt{b}(\cos(d\cdots}}{8\,bd}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*((4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(sqr
t(b)*(cos(d*x + c) - 1)*sgn(tan(d*x + c))) - 4*log(abs(-cos(d*x + c) + 1)/
abs(cos(d*x + c) + 1))/(sqrt(b)*sgn(tan(d*x + c))) + 8*log(abs(-(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1))/(sqrt(b)*sgn(tan(d*x + c))) + (cos(d*x +
 c) - 1)/(sqrt(b)*(cos(d*x + c) + 1)*sgn(tan(d*x + c))))/(b*d)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^2)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^2)^(3/2), x)
```

3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 97

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}$$

output
```
1/2*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/b^2/d/(b*tan(
d*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{2\cot(c+dx) - \cot^3(c+dx) + 4(\log(\cos(c+dx)) + \log(\tan(c+dx)))\tan(c+d}{4b^2d\sqrt{b\tan^2(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
(2*Cot[c + d*x] - Cot[c + d*x]^3 + 4*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]
])*Tan[c + d*x])/(4*b^2*d*Sqrt[b*Tan[c + d*x]^2])
```

3.29.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(b\tan(c+dx)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4141$$

$$\frac{\tan(c+dx)\int \cot^5(c+dx)dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan(c+dx)\int -\tan\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(c+dx)\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^5dx}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 3954$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int -\cot^3(c+dx)dx\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(c+dx)\left(\int \cot^3(c+dx)dx+\frac{\cot^4(c+dx)}{4d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tan(c+dx)\left(\int -\tan\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\cot^4(c+dx)}{4d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)^3dx\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$\Big\downarrow$ 3954

$$-\frac{\tan(c+dx)\left(\int -\cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$\Big\downarrow$ 25

$$-\frac{\tan(c+dx)\left(-\int \cot(c+dx)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\tan(c+dx)\left(-\int -\tan\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$\Big\downarrow$ 25

$$-\frac{\tan(c+dx)\left(\int \tan\left(\frac{1}{2}(2c+\pi)+dx\right)dx+\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

$\Big\downarrow$ 3956

$$-\frac{\tan(c+dx)\left(\frac{\cot^4(c+dx)}{4d}-\frac{\cot^2(c+dx)}{2d}-\frac{\log(-\sin(c+dx))}{d}\right)}{b^2\sqrt{b\tan^2(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^2)^(-5/2),x]
```

output
```
-(((-1/2*Cot[c + d*x]^2/d + Cot[c + d*x]^4/(4*d) - Log[-Sin[c + d*x]]/d)*T
an[c + d*x])/(b^2*Sqrt[b*Tan[c + d*x]^2]))
```

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.29.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)-4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\left(\tan^2(dx+c)\right)+1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
default	$-\frac{\tan(dx+c)\left(2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)-4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\left(\tan^2(dx+c)\right)+1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}-\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}+\frac{4i\left(e^{6i(dx+c)}-e^{4i(dx+c)}\right.}{b^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}$

input
```
int(1/(b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*tan(d*x+c)*(2*ln(1+tan(d*x+c)^2)*tan(d*x+c)^4-4*ln(tan(d*x+c))*tan(
d*x+c)^4-2*tan(d*x+c)^2+1)/(b*tan(d*x+c)^2)^(5/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \frac{\left(2\,\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)\tan\left(dx+c\right)^4+3\,\tan\left(dx+c\right)^4+2\,\tan\left(dx+c\right)^2-1\right)\sqrt{b}}{4\,b^3d\tan\left(dx+c\right)^5}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 - 1)*sqrt(b*tan(d*x + c)^2)/(b^3*d*tan(d*x + c)
^5)
```

3.29.6 Sympy [F]

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^2\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**2)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**2)**(-5/2), x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{2\,\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{5}{2}}}-\frac{4\,\log(\tan(dx+c))}{b^{\frac{5}{2}}}-\frac{2\,\sqrt{b}\tan(dx+c)^2-\sqrt{b}}{b^3\tan(dx+c)^4}}{4\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(2*log(tan(d*x + c)^2 + 1)/b^(5/2) - 4*log(tan(d*x + c))/b^(5/2) - (2
*sqrt(b)*tan(d*x + c)^2 - sqrt(b))/(b^3*tan(d*x + c)^4))/d
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(87) = 174$.

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.32

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx =$$
$$-\frac{\frac{\left(\frac{12\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{48\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^2}{b^{\frac{5}{2}}\,(\cos(dx+c)-1)^2\mathrm{sgn}(\tan(dx+c))}-\frac{32\,\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(dx+c))}+\frac{64\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}(\tan(dx+c))}+\frac{12\,b^{\frac{5}{2}}\,(\cos(dx+c)-1)\mathrm{sg}\cdots}{\cos(dx+c)\cdots}}{64\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/64*((12*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 48*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x + c) + 1)^2/(b^(5/2)*(cos(d*x + c) - 1
)^2*sgn(tan(d*x + c))) - 32*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
1))/(b^(5/2)*sgn(tan(d*x + c))) + 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 1))/(b^(5/2)*sgn(tan(d*x + c))) + (12*b^(5/2)*(cos(d*x + c) -
1)*sgn(tan(d*x + c))/(cos(d*x + c) + 1) + b^(5/2)*(cos(d*x + c) - 1)^2*sgn
(tan(d*x + c))/(cos(d*x + c) + 1)^2)/b^5)/d
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{5/2}} \,dx$$

input
```
int(1/(b*tan(c + d*x)^2)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^2)^(5/2), x)
```

3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 364

$$\begin{aligned}\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = &-\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}\\&-\frac{b^2\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\&+\frac{b^2\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\&-\frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\&+\frac{b^2\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\\&+\frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d}-\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d}\\&+\frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}\end{aligned}$$

output
```
-2*b^2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b^2*arctan(-1+2^(1/2)*tan(d
*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b^2*arc
tan(1+2^(1/2)*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c
)^(3/2)-1/4*b^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)
^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+ta
n(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/5*b^2*(b*tan
(d*x+c)^3)^(1/2)*tan(d*x+c)/d-2/9*b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^3/
d+2/13*b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^5/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.56

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{(b\tan^3(c+dx))^{5/2}\left(-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(\right.}{2\sqrt{2}}\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(5/2),x]
```

output
```
((b*Tan[c + d*x]^3)^(5/2)*(-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2
]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - Log[1 - Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[c +
d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) - 2*Sqrt[Tan[c + d*x]] + (2*Tan[c + d*x]
^(5/2))/5 - (2*Tan[c + d*x]^(9/2))/9 + (2*Tan[c + d*x]^(13/2))/13))/(d*Tan
[c + d*x]^(15/2))
```

3.30.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.64, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(b\tan(c+dx)^3\right)^{5/2}\,dx$$

$$\Big\downarrow \; 4141$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{15}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{15/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3954$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\int \tan^{\frac{11}{2}}(c+dx)dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\int \tan(c+dx)^{11/2}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3954$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \tan^{\frac{7}{2}}(c+dx)dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \tan(c+dx)^{7/2}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3954$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(-\int \tan^{\frac{3}{2}}(c+dx)dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

3.30. $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(-\int \tan(c+dx)^{3/2}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3954$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 3957$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 266$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(c+dx)}{5d}-\frac{2\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 755$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\frac{2\tan^{\frac{9}{2}}(c+dx)}{9d}+\frac{2\tan^{\frac{5}{2}}(\ldots}{5d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 1476$$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}\ldots\right)\right)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \ 1082$$

3.30. $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{2\tan^{\frac{13}{2}}(c+dx)}{13d}-\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan\right.}{\sqrt{2}}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\right.}{\sqrt{2}}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan\right.}{\sqrt{2}}\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1103

3.30. $\int\left(b\tan^3(c+dx)\right)^{5/2}\,dx$

$$\frac{b^2\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\cdots}\right.}{2\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^3)^(5/2),x]
```

output
```
(b^2*Sqrt[b*Tan[c + d*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]
/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[
1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (2*Sqrt[Tan[c + d*x
]])/d + (2*Tan[c + d*x]^(5/2))/(5*d) - (2*Tan[c + d*x]^(9/2))/(9*d) + (2*T
an[c + d*x]^(13/2))/(13*d)))/Tan[c + d*x]^(3/2)
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.30. $\int \left(b \tan^3(c+dx)\right)^{5/2} \, dx$

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.30.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{(b(\tan^3(dx+c)))^{\frac{5}{2}}\left(360(b\tan(dx+c))^{\frac{13}{2}}-520b^2(b\tan(dx+c))^{\frac{9}{2}}+585b^6(b^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{}}\right.\right.}{}$
default	$\frac{(b(\tan^3(dx+c)))^{\frac{5}{2}}\left(360(b\tan(dx+c))^{\frac{13}{2}}-520b^2(b\tan(dx+c))^{\frac{9}{2}}+585b^6(b^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{}}\right.\right.}{}$

input
```
int((b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/d*(b*tan(d*x+c)^3)^(5/2)*(360*(b*tan(d*x+c))^(13/2)-520*b^2*(b*tan(
d*x+c))^(9/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*
tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+1170*b^6*(b^2)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+936*
b^4*(b*tan(d*x+c))^(5/2)-4680*b^6*(b*tan(d*x+c))^(1/2))/tan(d*x+c)^5/(b*ta
n(d*x+c))^(5/2)/b^4
```

3.30.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.91

$$\int \left(b\tan^3(c+dx)\right)^{5/2} dx = \frac{585\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\frac{\sqrt{b\tan(dx+c)^3}b^2+\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\tan(dx+c)}{\tan(dx+c)}\right)\tan(dx+c)+585i\left(-\frac{b^{10}}{d^4}\right)^{\frac{1}{4}} d\log\left(\frac{\sqrt{b\cdots}}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/1170*(585*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 + (-b^10/d
^4)^(1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*x + c) + 585*I*(-b^10/d^4)^(
1/4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 + I*(-b^10/d^4)^(1/4)*d*tan(d*x + c
))/tan(d*x + c))*tan(d*x + c) - 585*I*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(
d*x + c)^3)*b^2 - I*(-b^10/d^4)^(1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*
x + c) - 585*(-b^10/d^4)^(1/4)*d*log((sqrt(b*tan(d*x + c)^3)*b^2 - (-b^10/
d^4)^(1/4)*d*tan(d*x + c))/tan(d*x + c))*tan(d*x + c) + 4*(45*b^2*tan(d*x
+ c)^6 - 65*b^2*tan(d*x + c)^4 + 117*b^2*tan(d*x + c)^2 - 585*b^2)*sqrt(b*
tan(d*x + c)^3))/(d*tan(d*x + c))
```

3.30.6 Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^{5/2} dx = \int \left(b\tan^3{(c+dx)}\right)^{\frac{5}{2}} dx$$

input
```
integrate((b*tan(d*x+c)**3)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(5/2), x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.49

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{360\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{13}{2}} - 520\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{9}{2}} + 936\,b^{\frac{5}{2}}\tan(dx+c)^{\frac{5}{2}} + 585\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(360*b^(5/2)*tan(d*x + c)^(13/2) - 520*b^(5/2)*tan(d*x + c)^(9/2) +
 936*b^(5/2)*tan(d*x + c)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2
)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2
)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan
(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(d*x + c)))/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.80

$$\int (b\tan^3(c+dx))^{5/2}\,dx = \frac{1}{2340}\left(\frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d} + \frac{1170\sqrt{2}b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|\right.}{}\right)}{d}\right.$$

input
```
integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b
)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 1170*sqrt(2)*b*sqrt(abs(b))
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(
abs(b)))/d + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(
b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 585*sqrt(2)*b*sqrt(abs(b))*log(
b*tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d + 8
*(45*sqrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^6 - 65*sqrt(b*tan(d*x + c
))*b^66*d^12*tan(d*x + c)^4 + 117*sqrt(b*tan(d*x + c))*b^66*d^12*tan(d*x +
 c)^2 - 585*sqrt(b*tan(d*x + c))*b^66*d^12)/(b^65*d^13))*b*sgn(tan(d*x + c
))
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx=\int \left(b\tan(c+d\,x)^3\right)^{5/2}dx$$

input
```
int((b*tan(c + d*x)^3)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(5/2), x)
```

3.31 $\int (b\tan^3(c+dx))^{3/2}\,dx$

3.31.1 Optimal result

Integrand size = 14, antiderivative size = 286

$$\begin{aligned}\int (b\tan^3(c+dx))^{3/2}\,dx &= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} \\ &- \frac{b\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{b\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{b\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}\end{aligned}$$

output
```
-2/3*b*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*
(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b*l
n(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/
tan(d*x+c)^(3/2)-1/4*b*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*
x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/7*b*(b*tan(d*x+c)^3)^(1/2)*tan(
d*x+c)^2/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.40

$$\int \left(b \tan^3(c+dx)\right)^{3/2} dx = \frac{b\sqrt{b\tan^3(c+dx)}\left(21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan(c+dx)} - 21\text{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\cdots\right)}{21d\tan^{\frac{7}{4}}(c+dx)}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(3/2),x]
```

output
```
(b*Sqrt[b*Tan[c + d*x]^3]*(21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*
x])^(1/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4) + 2*
Tan[c + d*x]^(7/4)*(-7 + 3*Tan[c + d*x]^2)))/(21*d*Tan[c + d*x]^(7/4))
```

3.31.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.214$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(c+dx)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(b\tan(c+dx)^3\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{9}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{9/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\int \tan^{\frac{5}{2}}(c+dx)dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\int \tan(c+dx)^{5/2}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3954}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\int \sqrt{\tan(c+dx)}dx+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\int \sqrt{\tan(c+dx)}dx+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{826}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\frac{2\tan^{\frac{3}{2}}(c+dx)}{3d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \text{1476}$$

3.31. $\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int\frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}\right.}{d}}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1082

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 217

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2\tan^{\frac{7}{2}}(c+dx)}{7d}-\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 1479

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c\right.}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 25

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+\right.}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$\downarrow$ 27

3.31. $\int\left(b\tan^3(c+dx)\right)^{3/2}\,dx$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan \cdots}\right.}{\sqrt{2}}\right.\cdots}{d}\cdots\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \color{blue}{1103}$$

$$\frac{b\sqrt{b\tan^3(c+dx)}\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{t\cdots}\right.}{2\sqrt{2}}\right.\cdots}{d}\cdots\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^3)^(3/2),x]
```

output
```
(b*Sqrt[b*Tan[c + d*x]^3]*((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/S
qrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqr
t[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqrt[2]))/2))/d - (2*Tan[c + d*x]^(3/2))
/(3*d) + (2*Tan[c + d*x]^(7/2))/(7*d)))/Tan[c + d*x]^(3/2)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.31. $\int (b\tan^3(c+dx))^{3/2}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.31.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\ \mathrm{arcta}}{84d\tan(dx+c)^3(b\tan(dx+}$
default	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42b^4\sqrt{2}\ \mathrm{arcta}}{84d\tan(dx+c)^3(b\tan(dx+}$

input
```
int((b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/d*(b*tan(d*x+c)^3)^(3/2)*(24*(b*tan(d*x+c))^(7/2)*(b^2)^(1/4)+21*b^4*
2^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(
1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1
/4))+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2
)^(1/4))-56*b^2*(b*tan(d*x+c))^(3/2)*(b^2)^(1/4))/tan(d*x+c)^3/(b*tan(d*x+
c))^(3/2)/b^2/(b^2)^(1/4)
```

3.31.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{21\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{3}{4}} d^3\tan(dx+c)+\sqrt{b\tan(dx+c)^3}b^4}{\tan(dx+c)}\right) - 21\left(-\frac{b^6}{d^4}\right)^{\frac{1}{4}} d\log\left(-\frac{\left(-\frac{b^6}{d^4}\right)^{\frac{3}{4}} d^3\tan(dx+c)-}{\tan(dx+}\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/42*(21*(-b^6/d^4)^(1/4)*d*log(((-b^6/d^4)^(3/4)*d^3*tan(d*x + c) + sqrt(
b*tan(d*x + c)^3)*b^4)/tan(d*x + c)) - 21*(-b^6/d^4)^(1/4)*d*log(-((-b^6/d
^4)^(3/4)*d^3*tan(d*x + c) - sqrt(b*tan(d*x + c)^3)*b^4)/tan(d*x + c)) - 2
1*I*(-b^6/d^4)^(1/4)*d*log((I*(-b^6/d^4)^(3/4)*d^3*tan(d*x + c) + sqrt(b*t
an(d*x + c)^3)*b^4)/tan(d*x + c)) + 21*I*(-b^6/d^4)^(1/4)*d*log((-I*(-b^6/
d^4)^(3/4)*d^3*tan(d*x + c) + sqrt(b*tan(d*x + c)^3)*b^4)/tan(d*x + c)) +
4*sqrt(b*tan(d*x + c)^3)*(3*b*tan(d*x + c)^2 - 7*b))/d
```

3.31.6 Sympy [F]

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \int \left(b\tan^3{(c+dx)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(3/2), x)
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.49

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{24\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{7}{2}} - 56\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}} + 21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(24*b^(3/2)*tan(d*x + c)^(7/2) - 56*b^(3/2)*tan(d*x + c)^(3/2) + 21*(
2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(
2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(
d*x + c)) + tan(d*x + c) + 1))*b^(3/2))/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.88

$$\int (b\tan^3(c+dx))^{3/2}\,dx = \frac{1}{84}b\left(\frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{bd} + \frac{42\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(d}\right.}{2\sqrt{|b|}}\right.}{bd}\right.$$

input
```
integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 42*sqrt(2)*abs(b)^(3/2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(
b)))/(b*d) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*t
an(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 21*sqrt(2)*abs(b)^(3/2)*log(b*
tan(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) +
 8*(3*sqrt(b*tan(d*x + c))*b^21*d^6*tan(d*x + c)^3 - 7*sqrt(b*tan(d*x + c)
)*b^21*d^6*tan(d*x + c))/(b^21*d^7))*sgn(tan(d*x + c))
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^{3/2} dx = \int \left(b\tan(c+d\,x)^3\right)^{3/2} dx$$

input
```
int((b*tan(c + d*x)^3)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(3/2), x)
```

3.32 $\int \sqrt{b \tan^3(c+dx)}\,dx$

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int \sqrt{b \tan^3(c+dx)}\,dx &= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} \\ &+ \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &+ \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)} \\ &- \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan^3(c+dx)}}{2\sqrt{2}d\tan^{\frac{3}{2}}(c+dx)}\end{aligned}$$

output

```
2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)
*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*l
n(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/
tan(d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+
c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.64

$$\int \sqrt{b\tan^3(c+dx)}\,dx$$
$$= \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+d\right.}{2\sqrt{2}}}{d\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[c + d*x]])*Sqrt[b*Tan[c + d*x]^3])/(d*Tan[c + d*x]^(3/2
))
```

3.32.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 4141, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{b\tan(c+dx)^3}dx$$
$$\Big\downarrow \text{4141}$$
$$\frac{\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)dx}{\tan^{\frac{3}{2}}(c+dx)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{b\tan^3(c+dx)}\int \tan(c+dx)^{3/2}dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3954$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\int \frac{1}{\sqrt{\tan(c+dx)}}dx\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 3957$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 266$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 755$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{}{\tan(c+dx)+}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\mathrm{ta}}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{\tan^{\frac{3}{2}}(c+dx)}$$

$$\Big\downarrow \; 217$$

3.32. $\int \sqrt{b\tan^3(c+dx)}\,dx$

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1479

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\cdots}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}\cdots}{}\right.\cdots}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\cdots}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

$\Big\downarrow$ 1103

$$\frac{\sqrt{b\tan^3(c+dx)}\left(\frac{2\sqrt{\tan(c+dx)}}{d}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\cdots}{}\right.\cdots}{d}\cdots\right)}{\tan^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^3],x]
```

3.32. $\int\sqrt{b\tan^3(c+dx)}\,dx$

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d
*x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c
+ d*x]]/(2*Sqrt[2]))/2))/d + (2*Sqrt[Tan[c + d*x]])/d)*Sqrt[b*Tan[c + d*x]
^3])/Tan[c + d*x]^(3/2)
```

3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.32. $\int \sqrt{b\tan^3(c+dx)}\,dx$

3.32.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\sqrt{b(\tan^3(dx+c))}\left((b^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}+(b}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$
default	$-\frac{\sqrt{b(\tan^3(dx+c))}\left((b^2)^{\frac{1}{4}}\sqrt{2}\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2(b^2)^{\frac{1}{4}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}+(b}{(b^2)^{\frac{1}{4}}}\right.\right.}{4d\tan(dx+c)\sqrt{b\tan(dx+c)}}$

input
```
int((b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(b*tan(d*x+c)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln((b*tan(d*x+c)+(b^2)^
(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2
)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(
d*x+c))^(1/2))/tan(d*x+c)/(b*tan(d*x+c))^(1/2)
```

3.32.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.07

$$\int\sqrt{b\tan^3(c+dx)}\,dx =$$

$$-\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan(dx+c)-d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\log\left(-\frac{d\left(-\frac{b^2}{d^4}\right)^{\frac{1}{4}}\tan(dx+c)-\sqrt{b\tan(d}{\tan(dx+c)}\right.}{}$$

input
```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(d*(-b^2/d^4)^(1/4)*log((d*(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(b*tan
(d*x + c)^3))/tan(d*x + c))*tan(d*x + c) - d*(-b^2/d^4)^(1/4)*log(-(d*(-b^
2/d^4)^(1/4)*tan(d*x + c) - sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x
+ c) + I*d*(-b^2/d^4)^(1/4)*log((I*d*(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(
b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c) - I*d*(-b^2/d^4)^(1/4)*log((
-I*d*(-b^2/d^4)^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))
*tan(d*x + c) - 4*sqrt(b*tan(d*x + c)^3))/(d*tan(d*x + c))
```

3.32.6 Sympy [F]

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan^3{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**3), x)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^3(c+dx)}\,dx = -\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan{(dx+c)}}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan{(dx+c)}}\right)\right)+$$

input
```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))
)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))
)) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) -
sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sq
rt(b)*sqrt(tan(d*x + c)))/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.76

$$\int \sqrt{b\tan^3(c+dx)}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\right.$$

$$+c))$$

input
```
integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*
sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d +
 sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sq
rt(abs(b)) + abs(b))/d - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqrt(2)
*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 8*sqrt(b*tan(d*x + c))/d)
*sgn(tan(d*x + c))
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^3(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^3}\,dx$$

input
```
int((b*tan(c + d*x)^3)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^3)^(1/2), x)
```

3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

3.33.1 Optimal result

Integrand size = 14, antiderivative size = 255

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}d\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}} + \frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}d\sqrt{b\tan^3(c+dx)}}$$

output
```
-2*tan(d*x+c)/d/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1
/2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(1+2^(1/2
)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*
ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*ta
n(d*x+c)^3)^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)
^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \frac{\tan(c+dx)\left(-2-\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(c+dx)}\right)\sqrt[4]{-\tan^2(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^3],x]
```

output
```
(Tan[c + d*x]*(-2 - ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4
) + ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(1/4)))/(d*Sqrt[b*T
an[c + d*x]^3])
```

3.33.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.70, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.071$, Rules used = {3042, 4141, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^3}}dx$$

$$\Big\downarrow \ 4141$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}dx}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{3/2}}dx}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \sqrt{\tan(c+dx)}dx-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\right.}{\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{217}$$

3.33. $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{\sqrt{b\tan^3(c+dx)}}$$

$\big\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx}\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\big\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\big\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c-\right.}{\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

$\big\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(\right.}{2\sqrt{2}}\right.}{d}\right.}{\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^3],x]
```

3.33. $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

output
```
(((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqr
t[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d - 2/(d*Sqrt[Tan[c + d*x]]))*Tan[c + d*x]^(3/2))/
Sqrt[b*Tan[c + d*x]^3]
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.33. $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

3.33.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b}}{}\right.}{4d\sqrt{b(\tan^3(dx+c))}\,(b^2)^{\frac{1}{4}}}$
default	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{b}}{}\right.}{4d\sqrt{b(\tan^3(dx+c))}\,(b^2)^{\frac{1}{4}}}$

input
```
int(1/(b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*tan(d*x+c)*(2^(1/2)*(b*tan(d*x+c))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b
*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(d*x+c))^(1/2)*ar
ctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*
tan(d*x+c))^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^
(1/4))+8*(b^2)^(1/4))/(b*tan(d*x+c)^3)^(1/2)/(b^2)^(1/4)
```

3.33.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx =$$

$$-\frac{bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2d^3\left(-\frac{1}{b^2d^4}\right)^{\frac{3}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan(dx+c)^2-bd\left(-\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2d^3\left(-\frac{1}{b^2d^4}\right)^{\frac{3}{4}}\tan(\right.}{\tan(}}{}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(b*d*(-1/(b^2*d^4))^(1/4)*log((b^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x +
 c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^2 - b*d*(-1/(b^2*
d^4))^(1/4)*log(-(b^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x + c) - sqrt(b*tan(d
*x + c)^3))/tan(d*x + c))*tan(d*x + c)^2 - I*b*d*(-1/(b^2*d^4))^(1/4)*log(
(I*b^2*d^3*(-1/(b^2*d^4))^(3/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan
(d*x + c))*tan(d*x + c)^2 + I*b*d*(-1/(b^2*d^4))^(1/4)*log((-I*b^2*d^3*(-1
/(b^2*d^4))^(3/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan
(d*x + c)^2 + 4*sqrt(b*tan(d*x + c)^3))/(b*d*tan(d*x + c)^2)
```

3.33.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^3{\left(c+dx \right)}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**3), x)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\right.}{\sqrt{b}}}{4\,d}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*lo
g(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(d*x +
c))))/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = -\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\sqrt{\vphantom{2}}\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) + 2*sqrt
(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(
d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) - sqrt(2)*abs(b)^(3/2)*
log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(
b^4*d*sgn(tan(d*x + c))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(
2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d*sgn(tan(d*x + c))) +
 8/(sqrt(b*tan(d*x + c))*b^2*d*sgn(tan(d*x + c))))
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^3}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(1/2), x)
```

3.34 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$

3.34.1 Optimal result

Integrand size = 14, antiderivative size = 298

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx =&\ \frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}\\&-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\&+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\&-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\\&+\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}bd\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

output
```
2/3/b/d/(b*tan(d*x+c)^3)^(1/2)-2/7*cot(d*x+c)^2/b/d/(b*tan(d*x+c)^3)^(1/2)
+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*t
an(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)
/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(
d*x+c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4*ln(1+2^(1/
2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)
^3)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.33

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \frac{14-6\cot^2(c+dx)-21\arctan\left(\sqrt[4]{-\tan^2(c+dx)}\right)(-\tan^2(c+dx))^{3/4}-21\text{ar}}{21bd\sqrt{b\tan^3(c+dx)}}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(-3/2),x]
```

output
```
(14 - 6*Cot[c + d*x]^2 - 21*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]
^2)^(3/4) - 21*ArcTanh[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x]^2)^(3/4))/(
21*b*d*Sqrt[b*Tan[c + d*x]^3])
```

3.34.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.67, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(b\tan(c+dx)^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{9}{2}}(c+dx)}dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{9/2}}dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{ 3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}dx-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{5/2}}dx-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 3955$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\sqrt{\tan(c+dx)}}dx+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 3957$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{\int \frac{1}{\sqrt{\tan(c+dx)}\left(\tan^2(c+dx)+1\right)}d\tan(c+dx)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 266$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\int \frac{1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 755$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{2}{7d\tan^{\frac{7}{2}}(c+dx)}\right)}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 1476$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \; 1082$$

3.34. $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)\right)}{d}+\frac{}{3d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 217

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)\right)}{d}+\frac{2}{3d\tan^{\frac{3}{2}}(c+dx)}-\frac{}{7d\tan}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+da}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1103

3.34. $\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+\right.}{2\sqrt{2}}\right.}{d}\right.}{b\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^3)^(-3/2),x]
```

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]] + Tan[c + d*x]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]/(2*Sqrt[2]))/2))/d - 2/(7*d*Tan[c + d*x]^(7/2)) + 2/(3*d*Tan[c + d*
x]^(3/2)))*Tan[c + d*x]^(3/2))/(b*Sqrt[b*Tan[c + d*x]^3])
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.34. $\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx$

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.34.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\text{ar}}{84d\,b^4(b(}$
default	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(b\tan(dx+c)\right)^{\frac{7}{2}}\text{ar}}{84d\,b^4(b(}$

input
```
int(1/(b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/84/d*tan(d*x+c)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*ln((b*t
an(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x
+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42*(b^2)^(1/4)*
2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1
/4))/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1
/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(d*x+c)^2-24*
b^4)/(b*tan(d*x+c)^3)^(3/2)
```

3.34.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07

$$\int\frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx=\frac{21\,b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2d\left(-\frac{1}{b^6d^4}\right)^{\frac{1}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan\left(dx+c\right)^5-21\,b^2d\left(-\right.}{}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/42*(21*b^2*d*(-1/(b^6*d^4))^(1/4)*log((b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*
x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^5 - 21*b^2*d*(
-1/(b^6*d^4))^(1/4)*log(-(b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) - sqrt(b
*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^5 + 21*I*b^2*d*(-1/(b^6*d^4))
^(1/4)*log((I*b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c
)^3))/tan(d*x + c))*tan(d*x + c)^5 - 21*I*b^2*d*(-1/(b^6*d^4))^(1/4)*log((
-I*b^2*d*(-1/(b^6*d^4))^(1/4)*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d
*x + c))*tan(d*x + c)^5 + 4*sqrt(b*tan(d*x + c)^3)*(7*tan(d*x + c)^2 - 3))
/(b^2*d*tan(d*x + c)^5)
```

3.34.6 Sympy [F]

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(-3/2), x)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx = \frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\cdots}\right.\right.}{b^{\frac{3}{2}}\cdots}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)
*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(3/2) + 8*(21*sqrt(tan(d*x + c)
) + 7/tan(d*x + c)^(3/2) - 3/tan(d*x + c)^(7/2))/b^(3/2) - 168*sqrt(tan(d*
x + c))/b^(3/2))/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.94

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \frac{1}{84}b^4\left(\frac{42\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^6 d\text{sgn}\left(\tan\left(dx+c\right)\right)} + \frac{42\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}}{}\right.}{b^6 d\text{sgn}\left(\tan\right.}\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

output
```
1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
 + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 42*sq
rt(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*ta
n(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 21*sqrt(2)*sqrt(abs
(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(
b))/(b^6*d*sgn(tan(d*x + c))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c)
 - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d*sgn(tan(d*x
+ c))) + 8*(7*b^2*tan(d*x + c)^2 - 3*b^2)/(sqrt(b*tan(d*x + c))*b^7*d*sgn(
tan(d*x + c))*tan(d*x + c)^3))
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(3/2), x)
```

3.35 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 364

$$\begin{aligned}\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = &-\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} \\ &+\frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}} \\ &-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &+\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}} \\ &-\frac{\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx)}{2\sqrt{2}b^2d\sqrt{b\tan^3(c+dx)}}\end{aligned}$$

output
```
-2/5*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2/9*cot(d*x+c)^3/b^2/d/(b*tan
(d*x+c)^3)^(1/2)-2/13*cot(d*x+c)^5/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2*tan(d*x+
c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*ta
n(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4
*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/
(b*tan(d*x+c)^3)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d
*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{-234\cot(c+dx)+130\cot^3(c+dx)-90\cot^5(c+dx)+585\text{arctanh}\left(\sqrt[4]{-\tan^2}\right.}{}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^(-5/2),x]
```

output
```
(-234*Cot[c + d*x] + 130*Cot[c + d*x]^3 - 90*Cot[c + d*x]^5 + 585*ArcTanh[
(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(5/4)*Tan[c + d*x]^(1/4) + 1170*T
an[c + d*x] + 585*ArcTan[(-Tan[c + d*x]^2)^(1/4)]*(-Tan[c + d*x])^(1/4)*Ta
n[c + d*x]^(5/4))/(585*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

3.35.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.64, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 4141, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{15}{2}}(c+dx)}dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan(c+dx)^{15/2}}dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{11}{2}}(c+dx)}dx-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{11/2}}dx-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}dx+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \frac{1}{\tan(c+dx)^{7/2}}dx+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(-\int \frac{1}{\tan(c+dx)^{3/2}}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$\Big\downarrow \text{3955}$$

3.35. $\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \sqrt{\tan(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 3042

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\int \sqrt{\tan(c+dx)}dx-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 3957

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{\int \frac{\sqrt{\tan(c+dx)}}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 266

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\int \frac{\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c+dx)}+\frac{2}{d\sqrt{\tan(c+dx)}}\right)}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 826

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\int \frac{\tan(c+dx)+1}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{2}{9d\tan^{\frac{9}{2}}(c+dx)}-\frac{2}{13d\tan^{\frac{13}{2}}(c-}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 1476

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}+\frac{1}{2}\int \frac{1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{t}\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 1082

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(c+dx)-1}d\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{}{5d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\Big\downarrow$ 217

3.35. $\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx$

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(c+dx)}{\tan^2(c+dx)+1}d\sqrt{\tan(c+dx)}\right)}{d}-\frac{2}{5d\tan^{\frac{5}{2}}(c+dx)}+\frac{}{9d\tan}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1479

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}-\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 25

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right.}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 27

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{\tan(c+dx)}}{\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{\tan(c+dx)}+1}{\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1}d\sqrt{\tan(c+dx)}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx\right.}}{\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

$\downarrow$ 1103

$$\frac{\tan^{\frac{3}{2}}(c+dx)\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+\right.}{2\sqrt{2}}\right.\right.}{d}\right.}{b^2\sqrt{b\tan^3(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^3)^(-5/2),x]
```

3.35. $\int\frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

output
```
(((2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2]) + ArcTan[1 + Sqrt
[2]*Sqrt[Tan[c + d*x]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] +
 Tan[c + d*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
d*x]]/(2*Sqrt[2]))/2))/d - 2/(13*d*Tan[c + d*x]^(13/2)) + 2/(9*d*Tan[c + d
*x]^(9/2)) - 2/(5*d*Tan[c + d*x]^(5/2)) + 2/(d*Sqrt[Tan[c + d*x]]))*Tan[c
+ d*x]^(3/2))/(b^2*Sqrt[b*Tan[c + d*x]^3])
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.35. $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

3.35.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\arctan}{}$
default	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\arctan}{}$

input
```
int(1/(b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2340/d*tan(d*x+c)/b^6*(585*2^(1/2)*(b*tan(d*x+c))^(13/2)*ln(-((b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(d*
x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4)
)+1170*2^(1/2)*(b*tan(d*x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-
(b^2)^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*b^6*tan(d*x+c)^6-936*b^6*(b^2)^
(1/4)*tan(d*x+c)^4+520*b^6*(b^2)^(1/4)*tan(d*x+c)^2-360*b^6*(b^2)^(1/4))/(
b*tan(d*x+c)^3)^(5/2)/(b^2)^(1/4)
```

3.35.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{585\,b^3d\left(-\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^8d^3\left(-\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}\tan(dx+c)+\sqrt{b\tan(dx+c)^3}}{\tan(dx+c)}\right)\tan(dx+c)^8-585\,b^3}{}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/1170*(585*b^3*d*(-1/(b^10*d^4))^(1/4)*log((b^8*d^3*(-1/(b^10*d^4))^(3/4)
*tan(d*x + c) + sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 - 585
*b^3*d*(-1/(b^10*d^4))^(1/4)*log(-(b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x +
 c) - sqrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 - 585*I*b^3*d*(
-1/(b^10*d^4))^(1/4)*log((I*b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x + c) + s
qrt(b*tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 + 585*I*b^3*d*(-1/(b^1
0*d^4))^(1/4)*log((-I*b^8*d^3*(-1/(b^10*d^4))^(3/4)*tan(d*x + c) + sqrt(b*
tan(d*x + c)^3))/tan(d*x + c))*tan(d*x + c)^8 + 4*(585*tan(d*x + c)^6 - 11
7*tan(d*x + c)^4 + 65*tan(d*x + c)^2 - 45)*sqrt(b*tan(d*x + c)^3))/(b^3*d*
tan(d*x + c)^8)
```

3.35.6 Sympy [F]

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan^3(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**3)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**3)**(-5/2), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.47

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\right.\right.}{b^{\frac{5}{2}}}}{}$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

output
```
1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt
(2)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt
(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(
tan(d*x + c)) - 117*sqrt(b)/tan(d*x + c)^(5/2) + 65*sqrt(b)/tan(d*x + c)^(
9/2) - 45*sqrt(b)/tan(d*x + c)^(13/2))/b^3)/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.84

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \frac{1}{2340}\,b^6\left(\frac{1170\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\,\sqrt{b\tan(dx+c)}\right)}{2\,\sqrt{|b|}}\right)}{b^{10}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)} + \frac{1170\,\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\right.}{b^{10}d\mathrm{sgn}}\right.$$

input
```
integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

output
```
1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs
(b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) +
1170*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sq
rt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) - 585*sqrt(2)
*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b
)) + abs(b))/(b^10*d*sgn(tan(d*x + c))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*t
an(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^10*d*
sgn(tan(d*x + c))) + 8*(585*b^6*tan(d*x + c)^6 - 117*b^6*tan(d*x + c)^4 +
65*b^6*tan(d*x + c)^2 - 45*b^6)/(sqrt(b*tan(d*x + c))*b^14*d*sgn(tan(d*x +
 c))*tan(d*x + c)^6))
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^3)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^3)^(5/2), x)
```

3.36 $\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx$

3.36.1 Optimal result

Integrand size = 14, antiderivative size = 182

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d}$$

output
```
b^2*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b^2*x*cot(d*x+c)^2*(tan(d*x+c)^4*b
)^(1/2)-1/3*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b^2*(tan(d*x+c)^4*
b)^(1/2)*tan(d*x+c)^3/d-1/7*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^5/d+1/9*
b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^7/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.47

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \frac{\cot(c+dx)\left(35-45\cot^2(c+dx)+63\cot^4(c+dx)-105\cot^6(c+dx)+315\cot^8(c+dx)-\right.}{315d}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(5/2),x]
```

output
```
(Cot[c + d*x]*(35 - 45*Cot[c + d*x]^2 + 63*Cot[c + d*x]^4 - 105*Cot[c + d*
x]^6 + 315*Cot[c + d*x]^8 - 315*ArcTan[Tan[c + d*x]]*Cot[c + d*x]^9)*(b*Ta
n[c + d*x]^4)^(5/2))/(315*d)
```

3.36.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^{5/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan^{10}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan(c+dx)^{10}dx$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^9(c+dx)}{9d}-\int \tan^8(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^9(c+dx)}{9d}-\int \tan(c+dx)^8dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^6(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^6dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}\right)$$

$$\Big\downarrow \; 3954$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int \tan^4(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int \tan(c+dx)^4dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}\right)$$

$$\Big\downarrow \; 3954$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^2(c+dx)dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 3042$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^2dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \; 3954$$

$$b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int 1dx+\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

$$\Big\downarrow \; 24$$

$$b^2\left(\frac{\tan^9(c+dx)}{9d}-\frac{\tan^7(c+dx)}{7d}+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^4)^(5/2),x]
```

output
```
b^2*Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d - Tan[c + d
*x]^3/(3*d) + Tan[c + d*x]^5/(5*d) - Tan[c + d*x]^7/(7*d) + Tan[c + d*x]^9
/(9*d))
```

3.36. $\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx$

3.36.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.36.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

method	result
derivativedivides	$-\frac{\left(\left(\tan^{4}(dx+c)\right)b\right)^{\frac{5}{2}}\left(-35\left(\tan^{9}(dx+c)\right)+45\left(\tan^{7}(dx+c)\right)-63\left(\tan^{5}(dx+c)\right)+105\left(\tan^{3}(dx+c)\right)+315\arctan(\tan(dx+c\right.}{315d\tan(dx+c)^{10}}$
default	$-\frac{\left(\left(\tan^{4}(dx+c)\right)b\right)^{\frac{5}{2}}\left(-35\left(\tan^{9}(dx+c)\right)+45\left(\tan^{7}(dx+c)\right)-63\left(\tan^{5}(dx+c)\right)+105\left(\tan^{3}(dx+c)\right)+315\arctan(\tan(dx+c\right.}{315d\tan(dx+c)^{10}}$
risch	$\frac{b^{2}\left(e^{2i(dx+c)}+1\right)^{2}\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^{4}b}{\left(e^{2i(dx+c)}+1\right)^{4}}}\,x}{\left(e^{2i(dx+c)}-1\right)^{2}}-\frac{2ib^{2}\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^{4}b}{\left(e^{2i(dx+c)}+1\right)^{4}}}\left(1575\,e^{16i(dx+c)}+6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}\right.}{315\left(e^{2i(d\right.}}$

input
```
int((tan(d*x+c)^4*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/d*(tan(d*x+c)^4*b)^(5/2)*(-35*tan(d*x+c)^9+45*tan(d*x+c)^7-63*tan(d
*x+c)^5+105*tan(d*x+c)^3+315*arctan(tan(d*x+c))-315*tan(d*x+c))/tan(d*x+c)
^10
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.53

$$\int (b\tan^4(c+dx))^{5/2}\,dx = \frac{(35\,b^2\tan(dx+c)^9 - 45\,b^2\tan(dx+c)^7 + 63\,b^2\tan(dx+c)^5 - 105\,b^2\tan(dx+c)^3 - 315\,b^2}{315\,d\tan(dx+c)^2}$$

input
```
integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^2*tan(d*x + c)^9 - 45*b^2*tan(d*x + c)^7 + 63*b^2*tan(d*x + c)
^5 - 105*b^2*tan(d*x + c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x + c))*sqrt(b*t
an(d*x + c)^4)/(d*tan(d*x + c)^2)
```

3.36.6 Sympy [F]

$$\int (b\tan^4(c+dx))^{5/2}\,dx = \int (b\tan^4(c+dx))^{\frac{5}{2}}\,dx$$

input
```
integrate((tan(d*x+c)**4*b)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(5/2), x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.43

$$\int (b\tan^4(c+dx))^{5/2}\,dx = \frac{35\,b^{\frac{5}{2}}\tan(dx+c)^9 - 45\,b^{\frac{5}{2}}\tan(dx+c)^7 + 63\,b^{\frac{5}{2}}\tan(dx+c)^5 - 105\,b^{\frac{5}{2}}\tan(dx+c)^3 - 315\,(}{315\,d}$$

input
```
integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

output
```
1/315*(35*b^(5/2)*tan(d*x + c)^9 - 45*b^(5/2)*tan(d*x + c)^7 + 63*b^(5/2)*
tan(d*x + c)^5 - 105*b^(5/2)*tan(d*x + c)^3 - 315*(d*x + c)*b^(5/2) + 315*
b^(5/2)*tan(d*x + c))/d
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(162) = 324$.

Time = 6.98 (sec) , antiderivative size = 960, normalized size of antiderivative = 5.27

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

output
```
-1/315*(315*b^2*d*x*tan(d*x)^9*tan(c)^9 - 2835*b^2*d*x*tan(d*x)^8*tan(c)^8
 + 315*b^2*tan(d*x)^9*tan(c)^8 + 315*b^2*tan(d*x)^8*tan(c)^9 + 11340*b^2*d
*x*tan(d*x)^7*tan(c)^7 - 105*b^2*tan(d*x)^9*tan(c)^6 - 2835*b^2*tan(d*x)^8
*tan(c)^7 - 2835*b^2*tan(d*x)^7*tan(c)^8 - 105*b^2*tan(d*x)^6*tan(c)^9 - 2
6460*b^2*d*x*tan(d*x)^6*tan(c)^6 + 63*b^2*tan(d*x)^9*tan(c)^4 + 945*b^2*ta
n(d*x)^8*tan(c)^5 + 11340*b^2*tan(d*x)^7*tan(c)^6 + 11340*b^2*tan(d*x)^6*t
an(c)^7 + 945*b^2*tan(d*x)^5*tan(c)^8 + 63*b^2*tan(d*x)^4*tan(c)^9 + 39690
*b^2*d*x*tan(d*x)^5*tan(c)^5 - 45*b^2*tan(d*x)^9*tan(c)^2 - 567*b^2*tan(d*
x)^8*tan(c)^3 - 3780*b^2*tan(d*x)^7*tan(c)^4 - 26460*b^2*tan(d*x)^6*tan(c)
^5 - 26460*b^2*tan(d*x)^5*tan(c)^6 - 3780*b^2*tan(d*x)^4*tan(c)^7 - 567*b^
2*tan(d*x)^3*tan(c)^8 - 45*b^2*tan(d*x)^2*tan(c)^9 - 39690*b^2*d*x*tan(d*x
)^4*tan(c)^4 + 35*b^2*tan(d*x)^9 + 405*b^2*tan(d*x)^8*tan(c) + 2268*b^2*ta
n(d*x)^7*tan(c)^2 + 8820*b^2*tan(d*x)^6*tan(c)^3 + 39690*b^2*tan(d*x)^5*ta
n(c)^4 + 39690*b^2*tan(d*x)^4*tan(c)^5 + 8820*b^2*tan(d*x)^3*tan(c)^6 + 22
68*b^2*tan(d*x)^2*tan(c)^7 + 405*b^2*tan(d*x)*tan(c)^8 + 35*b^2*tan(c)^9 +
 26460*b^2*d*x*tan(d*x)^3*tan(c)^3 - 45*b^2*tan(d*x)^7 - 567*b^2*tan(d*x)^
6*tan(c) - 3780*b^2*tan(d*x)^5*tan(c)^2 - 26460*b^2*tan(d*x)^4*tan(c)^3 -
26460*b^2*tan(d*x)^3*tan(c)^4 - 3780*b^2*tan(d*x)^2*tan(c)^5 - 567*b^2*tan
(d*x)*tan(c)^6 - 45*b^2*tan(c)^7 - 11340*b^2*d*x*tan(d*x)^2*tan(c)^2 + 63*
b^2*tan(d*x)^5 + 945*b^2*tan(d*x)^4*tan(c) + 11340*b^2*tan(d*x)^3*tan(c...
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \int \left(b\tan(c+d\,x)^4\right)^{5/2}\,dx$$

input
```
int((b*tan(c + d*x)^4)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(5/2), x)
```

3.37 $\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx$

3.37.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d}+\frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d}$$

output
```
b*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1
/2)-1/3*b*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b*(tan(d*x+c)^4*b)^(1/2)
*tan(d*x+c)^3/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.60

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{\cot(c+dx)\left(3-5\cot^2(c+dx)+15\cot^4(c+dx)-15\arctan(\tan(c+dx))\cot^5(c+dx)\right)(b\,\mathrm{t}}{15d}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(3/2),x]
```

output
```
(Cot[c + d*x]*(3 - 5*Cot[c + d*x]^2 + 15*Cot[c + d*x]^4 - 15*ArcTan[Tan[c
+ d*x]]*Cot[c + d*x]^5)*(b*Tan[c + d*x]^4)^(3/2))/(15*d)
```

3.37.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{4141}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\int \tan(c+dx)^6dx$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\frac{\tan^5(c+dx)}{5d}-\int \tan(c+dx)^4dx\right)$$

$$\Big\downarrow \text{3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan^2(c+dx)dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(\int \tan(c+dx)^2dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}\right)$$

$$\downarrow \text{3954}$$

$$b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\left(-\int 1dx+\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}\right)$$

$$\downarrow \text{24}$$

$$b\left(\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

input
```
Int[(b*Tan[c + d*x]^4)^(3/2),x]
```

output
```
b*Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d - Tan[c + d*x
]^3/(3*d) + Tan[c + d*x]^5/(5*d))
```

3.37.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.37.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
default	$-\frac{\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
risch	$\frac{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\, x}{\left(e^{2i(dx+c)}-1\right)^2}-\frac{2ib\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+}\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)^3 d}$

input
```
int((tan(d*x+c)^4*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*(tan(d*x+c)^4*b)^(3/2)*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*arctan(t
an(d*x+c))-15*tan(d*x+c))/tan(d*x+c)^6
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{\left(3\,b\tan(dx+c)^5-5\,b\tan(dx+c)^3-15\,bdx+15\,b\tan(dx+c)\right)\sqrt{b\tan(dx+c)^4}}{15\,d\tan(dx+c)^2}$$

input
```
integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*b*tan(d*x + c)^5 - 5*b*tan(d*x + c)^3 - 15*b*d*x + 15*b*tan(d*x +
c))*sqrt(b*tan(d*x + c)^4)/(d*tan(d*x + c)^2)
```

3.37.6 Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\int \left(b\tan^4{(c+dx)}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((tan(d*x+c)**4*b)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(3/2), x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\frac{3\,b^{\frac{3}{2}}\tan{(dx+c)}^5-5\,b^{\frac{3}{2}}\tan{(dx+c)}^3-15\,(dx+c)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan{(dx+c)}}{15\,d}$$

input
```
integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

output
```
1/15*(3*b^(3/2)*tan(d*x + c)^5 - 5*b^(3/2)*tan(d*x + c)^3 - 15*(d*x + c)*b
^(3/2) + 15*b^(3/2)*tan(d*x + c))/d
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(98) = 196$.

Time = 2.94 (sec) , antiderivative size = 992, normalized size of antiderivative = 9.02

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

output
```
1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*
tan(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))
*tan(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)
)*tan(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)
^4 + 75*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x
) + tan(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^
4*tan(c)^5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c)
+ 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*ta
n(d*x)^5*tan(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(
c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) +
tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)
^3 + 300*tan(d*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2
*tan(c)^2 + 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d
*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c
) + 150*pi*tan(d*x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x
) + tan(c)))*tan(d*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(...
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx=\int \left(b\tan(c+d\,x)^4\right)^{3/2}\,dx$$

input
```
int((b*tan(c + d*x)^4)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(3/2), x)
```

3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b\tan^4(c+dx)}\,dx=\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

output
```
cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\cot(c+dx)(-1+\arctan(\tan(c+dx))\cot(c+dx))\sqrt{b\tan^4(c+dx)}}{d}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
-((Cot[c + d*x]*(-1 + ArcTan[Tan[c + d*x]]*Cot[c + d*x])*Sqrt[b*Tan[c + d*
x]^4])/d)
```

3.38.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b \tan^4(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b \tan(c+dx)^4}dx$$

$$\Big\downarrow \text{4141}$$

$$\cot^2(c+dx)\sqrt{b \tan^4(c+dx)}\int \tan^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cot^2(c+dx)\sqrt{b \tan^4(c+dx)}\int \tan(c+dx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\cot^2(c+dx)\sqrt{b \tan^4(c+dx)}\left(\frac{\tan(c+dx)}{d}-\int 1dx\right)$$

$$\Big\downarrow \text{24}$$

$$\left(\frac{\tan(c+dx)}{d}-x\right)\cot^2(c+dx)\sqrt{b \tan^4(c+dx)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
Cot[c + d*x]^2*Sqrt[b*Tan[c + d*x]^4]*(-x + Tan[c + d*x]/d)
```

3.38.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.38.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{\left(\tan^{4}\left(dx+c\right)\right)b}\,\left(-\tan\left(dx+c\right)+\arctan\left(\tan\left(dx+c\right)\right)\right)}{d\tan\left(dx+c\right)^{2}}$	42
default	$-\frac{\sqrt{\left(\tan^{4}\left(dx+c\right)\right)b}\,\left(-\tan\left(dx+c\right)+\arctan\left(\tan\left(dx+c\right)\right)\right)}{d\tan\left(dx+c\right)^{2}}$	42
risch	$\frac{\sqrt{\frac{\left(\mathrm{e}^{2i(dx+c)}-1\right)^{4}b}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}x}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}}-\frac{2i\sqrt{\frac{\left(\mathrm{e}^{2i(dx+c)}-1\right)^{4}b}{\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^{2}d}$	120

input
```
int((tan(d*x+c)^4*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(tan(d*x+c)^4*b)^(1/2)*(-tan(d*x+c)+arctan(tan(d*x+c)))/tan(d*x+c)^2
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{\sqrt{b\tan(dx+c)^4}(dx-\tan(dx+c))}{d\tan(dx+c)^2}$$

input
```
integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(d*x + c)^4)*(d*x - tan(d*x + c))/(d*tan(d*x + c)^2)
```

3.38.6 Sympy [F]

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\tan^4(c+dx)}\,dx$$

input
```
integrate((tan(d*x+c)**4*b)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**4), x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \sqrt{b\tan^4(c+dx)}\,dx = -\frac{(dx+c)\sqrt{b}-\sqrt{b}\tan(dx+c)}{d}$$

input
```
integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

output
```
-((d*x + c)*sqrt(b) - sqrt(b)*tan(d*x + c))/d
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(46) = 92$.

Time = 0.46 (sec) , antiderivative size = 229, normalized size of antiderivative = 4.58

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \frac{\left(\pi - 4\,dx\tan\left(dx\right)\tan\left(c\right) - \pi\mathrm{sgn}\left(2\tan\left(dx\right)^2\tan\left(c\right) + 2\tan\left(dx\right)\tan\left(c\right)^2 - 2\tan\left(dx\right) - 2\tan\left(c\right)\right)\tan\left(\right.}{}$$

input
```
integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

output
```
1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*
tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2
*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arc
tan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + p
i*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) -
 2*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x)
+ tan(c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*sqrt(b)/(d*tan(d
*x)*tan(c) - d)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^4}\,dx$$

input
```
int((b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^4)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$

3.39.1 Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

output
```
-tan(d*x+c)/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/(tan(d*x+c)^4*b)^(1/2)
```

3.39.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
-((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[
b*Tan[c + d*x]^4]))
```

3.39.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4141, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^4}}dx$$

$$\Big\downarrow \text{4141}$$

$$\frac{\tan^2(c+dx)\int \cot^2(c+dx)dx}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx-\frac{\cot(c+dx)}{d}\right)}{\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot(c+dx)}{d}-x\right)}{\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^4],x]
```

output
```
((-x - Cot[c + d*x]/d)*Tan[c + d*x]^2)/Sqrt[b*Tan[c + d*x]^4]
```

3.39.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.39.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{(\tan^4(dx+c))b}}$	40
default	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{(\tan^4(dx+c))b}}$	40
risch	$\frac{(\mathrm{e}^{2i(dx+c)}-1)^2 x}{\sqrt{\frac{(\mathrm{e}^{2i(dx+c)}-1)^4 b}{(\mathrm{e}^{2i(dx+c)}+1)^4}}\,(\mathrm{e}^{2i(dx+c)}+1)^2}+\frac{2i(\mathrm{e}^{2i(dx+c)}-1)}{\sqrt{\frac{(\mathrm{e}^{2i(dx+c)}-1)^4 b}{(\mathrm{e}^{2i(dx+c)}+1)^4}}\,(\mathrm{e}^{2i(dx+c)}+1)^2 d}$	120

input
```
int(1/(tan(d*x+c)^4*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*tan(d*x+c)*(arctan(tan(d*x+c))*tan(d*x+c)+1)/(tan(d*x+c)^4*b)^(1/2)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\sqrt{b\tan\left(dx+c\right)^4}(dx\tan\left(dx+c\right)+1)}{bd\tan\left(dx+c\right)^3}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*tan(d*x + c)^4)*(d*x*tan(d*x + c) + 1)/(b*d*tan(d*x + c)^3)
```

3.39.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^4\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(tan(d*x+c)**4*b)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**4), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(dx+c)}}{d}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

output
```
-((d*x + c)/sqrt(b) + 1/(sqrt(b)*tan(d*x + c)))/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = -\frac{\frac{2\,(dx+c)}{\sqrt{b}} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{b}} + \frac{1}{\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)/sqrt(b) - tan(1/2*d*x + 1/2*c)/sqrt(b) + 1/(sqrt(b)*tan(
1/2*d*x + 1/2*c)))/d
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^4}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(1/2), x)
```

3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$

3.40.1 Optimal result

Integrand size = 14, antiderivative size = 119

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}$$

output
```
1/3*cot(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b/d/(tan(d*x+c)
^4*b)^(1/2)-tan(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/b/(tan(d*
x+c)^4*b)^(1/2)
```

3.40.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{5d\left(b\tan^4(c+dx)\right)^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(-3/2),x]
```

output
```
-1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(
b*Tan[c + d*x]^4)^(3/2))
```

3.40.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan^2(c+dx)\int \cot^6(c+dx)dx}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^4(c+dx)dx-\frac{\cot^5(c+dx)}{5d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^5(c+dx)}{5d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^2(c+dx)dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}\right)}{b\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}-x\right)}{b\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^4)^(-3/2),x]
```

output
```
((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d))*Tan[c
 + d*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

3.40.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.40.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\left(\tan^5(dx+c)\right)+15\left(\tan^4(dx+c)\right)-5\left(\tan^2(dx+c)\right)+3\right)}{15d\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}}$	63
default	$-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\left(\tan^5(dx+c)\right)+15\left(\tan^4(dx+c)\right)-5\left(\tan^2(dx+c)\right)+3\right)}{15d\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{3}{2}}}$	63
risch	$\frac{\left(e^{2i(dx+c)}-1\right)^2 x}{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}}+\frac{2i\left(45\,e^{8i(dx+c)}-90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}-70\,e^{2i(dx+c)}+23\right)}{15b\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}\,d}$	174

input
```
int(1/(tan(d*x+c)^4*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/d*tan(d*x+c)*(15*arctan(tan(d*x+c))*tan(d*x+c)^5+15*tan(d*x+c)^4-5*t
an(d*x+c)^2+3)/(tan(d*x+c)^4*b)^(3/2)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = -\frac{\left(15\,dx\tan\left(dx+c\right)^5+15\,\tan\left(dx+c\right)^4-5\,\tan\left(dx+c\right)^2+3\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,b^2d\tan\left(dx+c\right)^7}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(15*d*x*tan(d*x + c)^5 + 15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)*s
qrt(b*tan(d*x + c)^4)/(b^2*d*tan(d*x + c)^7)
```

3.40.6 Sympy [F]

$$\int \frac{1}{(b\tan^4(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan^4{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(tan(d*x+c)**4*b)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(-3/2), x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

$$\int \frac{1}{(b\tan^4(c+dx))^{3/2}}\,dx = -\frac{\frac{15\,(dx+c)}{b^{\frac{3}{2}}} + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{b^{\frac{3}{2}}\tan(dx+c)^5}}{15\,d}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

output
```
-1/15*(15*(d*x + c)/b^(3/2) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(
b^(3/2)*tan(d*x + c)^5))/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{1}{(b\tan^4(c+dx))^{3/2}}\,dx = -\frac{\frac{480\,(dx+c)}{\sqrt{b}} - \frac{3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 35\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 330\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{b^{\frac{5}{2}}} + \frac{330\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 3}{\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,bd}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

output
```
-1/480*(480*(d*x + c)/sqrt(b) - (3*b^2*tan(1/2*d*x + 1/2*c)^5 - 35*b^2*tan
(1/2*d*x + 1/2*c)^3 + 330*b^2*tan(1/2*d*x + 1/2*c))/b^(5/2) + (330*tan(1/2
*d*x + 1/2*c)^4 - 35*tan(1/2*d*x + 1/2*c)^2 + 3)/(sqrt(b)*tan(1/2*d*x + 1/
2*c)^5))/(b*d)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(3/2), x)
```

3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 183

$$\begin{aligned}\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx &= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} \\ &- \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} \\ &- \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}\end{aligned}$$

output
```
1/3*cot(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b^2/d/(tan(d*
x+c)^4*b)^(1/2)+1/7*cot(d*x+c)^5/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/9*cot(d*x+
c)^7/b^2/d/(tan(d*x+c)^4*b)^(1/2)-tan(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-
x*tan(d*x+c)^2/b^2/(tan(d*x+c)^4*b)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.25

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},-\tan^2(c+dx)\right)\tan(c+dx)}{9d\left(b\tan^4(c+dx)\right)^{5/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^(-5/2),x]
```

output
```
-1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(
b*Tan[c + d*x]^4)^(5/2))
```

3.41.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 4141, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(b\tan(c+dx)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 4141}$$

$$\frac{\tan^2(c+dx)\int \cot^{10}(c+dx)dx}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\int \tan\left(c+dx+\frac{\pi}{2}\right)^{10}dx}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^8(c+dx)dx-\frac{\cot^9(c+dx)}{9d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^8dx-\frac{\cot^9(c+dx)}{9d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^6(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^6dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(-\int \cot^4(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(-\int \tan\left(c+dx+\frac{\pi}{2}\right)^4dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(\int \cot^2(c+dx)dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(c+dx)\left(\int \tan\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(c+dx)\left(-\int 1dx-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan^2(c+dx)\left(-\frac{\cot^9(c+dx)}{9d}+\frac{\cot^7(c+dx)}{7d}-\frac{\cot^5(c+dx)}{5d}+\frac{\cot^3(c+dx)}{3d}-\frac{\cot(c+dx)}{d}-x\right)}{b^2\sqrt{b\tan^4(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^4)^(-5/2),x]
```

output
```
((-x - Cot[c + d*x]/d + Cot[c + d*x]^3/(3*d) - Cot[c + d*x]^5/(5*d) + Cot[
c + d*x]^7/(7*d) - Cot[c + d*x]^9/(9*d))*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c
 + d*x]^4])
```

3.41. $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

3.41.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.41.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(315\arctan(\tan(dx+c))\left(\tan^9(dx+c)\right)+315\left(\tan^8(dx+c)\right)-105\left(\tan^6(dx+c)\right)+63\left(\tan^4(dx+c)\right)-45\left(\tan^2(\right.\right.}{315d\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{5}{2}}}$
default	$-\frac{\tan(dx+c)\left(315\arctan(\tan(dx+c))\left(\tan^9(dx+c)\right)+315\left(\tan^8(dx+c)\right)-105\left(\tan^6(dx+c)\right)+63\left(\tan^4(dx+c)\right)-45\left(\tan^2(\right.\right.}{315d\left(\left(\tan^4(dx+c)\right)b\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)^2 x}{b^2\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{\left(e^{2i(dx+c)}-1\right)^4 b}{\left(e^{2i(dx+c)}+1\right)^4}}}+\frac{2i\left(1575\,e^{16i(dx+c)}-6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}-31500\,e^{10i(dx+c)}+394\right.}{315b^2\left(e^{2i(dx+c)}-1\right)^7\left(e^{2i(dx+c)}+\right.}$

input
```
int(1/(tan(d*x+c)^4*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315/d*tan(d*x+c)*(315*arctan(tan(d*x+c))*tan(d*x+c)^9+315*tan(d*x+c)^8-
105*tan(d*x+c)^6+63*tan(d*x+c)^4-45*tan(d*x+c)^2+35)/(tan(d*x+c)^4*b)^(5/2
)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx =$$

$$-\frac{\left(315\,dx\tan\left(dx+c\right)^{9}+315\,\tan\left(dx+c\right)^{8}-105\,\tan\left(dx+c\right)^{6}+63\,\tan\left(dx+c\right)^{4}-45\,\tan\left(dx+c\right)^{2}+35\right.}{315\,b^{3}d\tan\left(dx+c\right)^{11}}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

output
```
-1/315*(315*d*x*tan(d*x + c)^9 + 315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)*sqrt(b*tan(d*x + c)^4)/(b^3*d
*tan(d*x + c)^11)
```

3.41.6 Sympy [F]

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan^{4}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(tan(d*x+c)**4*b)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**4)**(-5/2), x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{315\,(dx+c)}{b^{\frac{5}{2}}}+\frac{315\,\tan(dx+c)^{8}-105\,\tan(dx+c)^{6}+63\,\tan(dx+c)^{4}-45\,\tan(dx+c)^{2}+35}{b^{\frac{5}{2}}\tan(dx+c)^{9}}}{315\,d}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

output
```
-1/315*(315*(d*x + c)/b^(5/2) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
 63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b^(5/2)*tan(d*x + c)^9))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx =$$

$$-\frac{\frac{161280\,(dx+c)}{b^{\frac{5}{2}}} + \frac{121590\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-18480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3528\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-495\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35}{b^{\frac{5}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}} - \frac{35\,b^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}{}}{161280\,d}$$

input
```
integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

output
```
-1/161280*(161280*(d*x + c)/b^(5/2) + (121590*tan(1/2*d*x + 1/2*c)^8 - 184
80*tan(1/2*d*x + 1/2*c)^6 + 3528*tan(1/2*d*x + 1/2*c)^4 - 495*tan(1/2*d*x
+ 1/2*c)^2 + 35)/(b^(5/2)*tan(1/2*d*x + 1/2*c)^9) - (35*b^20*tan(1/2*d*x +
 1/2*c)^9 - 495*b^20*tan(1/2*d*x + 1/2*c)^7 + 3528*b^20*tan(1/2*d*x + 1/2*
c)^5 - 18480*b^20*tan(1/2*d*x + 1/2*c)^3 + 121590*b^20*tan(1/2*d*x + 1/2*c
))/b^(45/2))/d
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^4\right)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^4)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^4)^(5/2), x)
```

3.42 $\int (b\tan^p(c+dx))^n \, dx$

3.42.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^p(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(
d*x+c)^p)^n/d/(n*p+1)
```

3.42.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int (b\tan^p(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d(1+np)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

3.42.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)^p)^n dx$$

$$\Big\downarrow \text{4142}$$

$$\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \tan^{np}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \tan(c+dx)^{np}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\int \frac{\tan^{np}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(np+1),\frac{1}{2}(np+3),-\tan^2(c+dx)\right)}{d(np+1)}$$

input
```
Int[(b*Tan[c + d*x]^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

3.42.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.42.4 Maple [F]

$$\int (b(\tan^p(dx+c)))^n dx$$

input
```
int((b*tan(d*x+c)^p)^n,x)
```

output
```
int((b*tan(d*x+c)^p)^n,x)
```

3.42.5 Fricas [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)^p})^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^p)^n, x)
```

3.42.6 Sympy [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan^p{(c+dx)})^n \, dx$$

input
```
integrate((b*tan(d*x+c)**p)**n,x)
```

output
```
Integral((b*tan(c + d*x)**p)**n, x)
```

3.42.7 Maxima [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)^p})^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^n, x)
```

3.42.8 Giac [F]

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan{(dx+c)^p})^n \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^n, x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^n \, dx = \int (b\tan(c+d\,x)^p)^n \, dx$$

input
```
int((b*tan(c + d*x)^p)^n,x)
```

output
```
int((b*tan(c + d*x)^p)^n, x)
```

3.43 $\int (b\tan^2(c+dx))^n \,dx$

3.43.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^2(c+dx))^n \,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+2n),\frac{1}{2}(3+2n),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

output
```
hypergeom([1, 1/2+n],[3/2+n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c)^2)^n/
d/(1+2*n)
```

3.43.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int (b\tan^2(c+dx))^n \,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+n,\frac{3}{2}+n,-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

input
```
Integrate[(b*Tan[c + d*x]^2)^n,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + n, 3/2 + n, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*T
an[c + d*x]^2)^n)/(d*(1 + 2*n))
```

3.43.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b \tan^2(c+dx)\right)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b \tan(c+dx)^2\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-2n}(c+dx) \left(b \tan^2(c+dx)\right)^n \int \tan^{2n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-2n}(c+dx) \left(b \tan^2(c+dx)\right)^n \int \tan(c+dx)^{2n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-2n}(c+dx) \left(b \tan^2(c+dx)\right)^n \int \frac{\tan^{2n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx) \left(b \tan^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(2n+1),\frac{1}{2}(2n+3),-\tan^2(c+dx)\right)}{d(2n+1)}$$

input
```
Int[(b*Tan[c + d*x]^2)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

3.43.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.43.4 Maple [F]

$$\int \left(b \left(\tan^{2}\left(d x +c \right)\right)\right)^{n}d x$$

input
```
int((b*tan(d*x+c)^2)^n,x)
```

output
```
int((b*tan(d*x+c)^2)^n,x)
```

3.43.5 Fricas [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan\left(dx+c\right)^2\right)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^2)^n, x)
```

3.43.6 Sympy [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan^2\left(c+dx\right)\right)^n \, dx$$

input
```
integrate((b*tan(d*x+c)**2)**n,x)
```

output
```
Integral((b*tan(c + d*x)**2)**n, x)
```

3.43.7 Maxima [F]

$$\int \left(b\tan^2(c+dx)\right)^n \, dx = \int \left(b\tan\left(dx+c\right)^2\right)^n \, dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^2)^n, x)
```

3.43.8 Giac [F]

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^2\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^2)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^2)^n, x)
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^2(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^2\right)^n\,dx$$

input
```
int((b*tan(c + d*x)^2)^n,x)
```

output
```
int((b*tan(c + d*x)^2)^n, x)
```

3.44 $\int (b\tan^3(c+dx))^n \, dx$

3.44.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (b\tan^3(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

output
```
hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+
c)^3)^n/d/(1+3*n)
```

3.44.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (b\tan^3(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+3n), \frac{3(1+n)}{2}, -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

input
```
Integrate[(b*Tan[c + d*x]^3)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

3.44.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^3(c+dx)\right)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^3\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \tan^{3n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \tan(c+dx)^{3n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\int \frac{\tan^{3n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(3n+1),\frac{3(n+1)}{2},-\tan^2(c+dx)\right)}{d(3n+1)}$$

input
```
Int[(b*Tan[c + d*x]^3)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

3.44.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.44.4 Maple [F]

$$\int \left(b\left(\tan^{3}\left(dx+c\right)\right)\right)^{n}dx$$

input
```
int((b*tan(d*x+c)^3)^n,x)
```

output
```
int((b*tan(d*x+c)^3)^n,x)
```

3.44.5 Fricas [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^3)^n, x)
```

3.44.6 Sympy [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan^3\left(c+dx\right)\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)**3)**n,x)
```

output
```
Integral((b*tan(c + d*x)**3)**n, x)
```

3.44.7 Maxima [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^3)^n, x)
```

3.44.8 Giac [F]

$$\int \left(b\tan^3(c+dx)\right)^n\,dx=\int \left(b\tan\left(dx+c\right)^3\right)^n\,dx$$

input
```
integrate((b*tan(d*x+c)^3)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^3)^n, x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^3(c+dx)\right)^n\,dx=\int \left(b\tan(c+d\,x)^3\right)^n\,dx$$

input
```
int((b*tan(c + d*x)^3)^n,x)
```

output
```
int((b*tan(c + d*x)^3)^n, x)
```

3.45 $\int (b\tan^4(c+dx))^n \, dx$

3.45.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int (b\tan^4(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4n),\frac{1}{2}(3+4n),-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

output
```
hypergeom([1, 1/2+2*n],[3/2+2*n],-tan(d*x+c)^2)*tan(d*x+c)*(tan(d*x+c)^4*b
)^n/d/(1+4*n)
```

3.45.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (b\tan^4(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+2n,\frac{3}{2}+2n,-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

input
```
Integrate[(b*Tan[c + d*x]^4)^n,x]
```

output
```
(Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 2*n, -Tan[c + d*x]^2]*Tan[c + d*x]*
(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

3.45.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4141, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^4(c+dx)\right)^n\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\tan(c+dx)^4\right)^n dx$$

$$\Big\downarrow \text{4141}$$

$$\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n \int \tan^{4n}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n \int \tan(c+dx)^{4n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n \int \frac{\tan^{4n}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4n+1),\frac{1}{2}(4n+3),-\tan^2(c+dx)\right)}{d(4n+1)}$$

input
```
Int[(b*Tan[c + d*x]^4)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

3.45.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.45.4 Maple [F]

$$\int \left(\left(\tan^{4}\left(dx+c\right)\right)b\right)^{n}dx$$

input
```
int((tan(d*x+c)^4*b)^n,x)
```

output
```
int((tan(d*x+c)^4*b)^n,x)
```

3.45.5 Fricas [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((tan(d*x+c)^4*b)^n,x, algorithm="fricas")
```

output
```
integral((b*tan(d*x + c)^4)^n, x)
```

3.45.6 Sympy [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan^4\left(c+dx\right)\right)^n\,dx$$

input
```
integrate((tan(d*x+c)**4*b)**n,x)
```

output
```
Integral((b*tan(c + d*x)**4)**n, x)
```

3.45.7 Maxima [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((tan(d*x+c)^4*b)^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^4)^n, x)
```

3.45.8 Giac [F]

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan\left(dx+c\right)^4\right)^n\,dx$$

input
```
integrate((tan(d*x+c)^4*b)^n,x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^4)^n, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \left(b\tan^4(c+dx)\right)^n\,dx = \int \left(b\tan(c+d\,x)^4\right)^n\,dx$$

input
```
int((b*tan(c + d*x)^4)^n,x)
```

output
```
int((b*tan(c + d*x)^4)^n, x)
```

3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 71

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \frac{2b^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+}{d(2+5p)}$$

output
```
2*b^2*hypergeom([1, 1/2+5/4*p],[3/2+5/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)
^(1/2)*tan(d*x+c)^(1+2*p)/d/(2+5*p)
```

3.46.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+5p),\frac{1}{4}(6+5p),-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{5/2}}{d\left(1+\frac{5p}{2}\right)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d
*x]*(b*Tan[c + d*x]^p)^(5/2))/(d*(1 + (5*p)/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)^p)^{5/2}dx$$

$$\Big\downarrow \text{4142}$$

$$b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{5p}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{5p/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{5p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(5p+2),\frac{1}{4}(5p+6),-\tan^2(c+dx)\right)}{d(5p+2)}$$

input
```
Int[(b*Tan[c + d*x]^p)^(5/2),x]
```

output
```
(2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan
[c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

3.46.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.46.4 Maple [F]

$$\int (b(\tan^p(dx+c)))^{\frac{5}{2}}dx$$

input
```
int((b*tan(d*x+c)^p)^(5/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(5/2),x)
```

3.46.5 Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.46.6 Sympy [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan^p{(c+dx)})^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(5/2), x)
```

3.46.7 Maxima [F]

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \int (b\tan{(dx+c)}^p)^{\frac{5}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(5/2), x)
```

3.46.8 Giac [F]

$$\int (b \tan^p(c+dx))^{5/2} \, dx = \int (b \tan(dx+c)^p)^{\frac{5}{2}} \, dx$$

input
```
integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(5/2), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int (b \tan^p(c+dx))^{5/2} \, dx = \int (b \tan(c+d\,x)^p)^{5/2} \, dx$$

input
```
int((b*tan(c + d*x)^p)^(5/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(5/2), x)
```

3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$

3.47.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \frac{2b\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

output
```
2*b*hypergeom([1, 1/2+3/4*p],[3/2+3/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(
1/2)*tan(d*x+c)^(p+1)/d/(2+3*p)
```

3.47.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3p),\frac{3(2+p)}{4},-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{3/2}}{d\left(1+\frac{3p}{2}\right)}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[c +
 d*x]*(b*Tan[c + d*x]^p)^(3/2))/(d*(1 + (3*p)/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan^p(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(c+dx)^p)^{3/2}\,dx$$

$$\Big\downarrow \text{4142}$$

$$b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{3p}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{3p/2}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{3p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3p+2),\frac{3(p+2)}{4},-\tan^2(c+dx)\right)}{d(3p+2)}$$

input
```
Int[(b*Tan[c + d*x]^p)^(3/2),x]
```

output
```
(2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan
[c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

3.47.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.47.4 Maple [F]

$$\int (b(\tan^p(dx+c)))^{\frac{3}{2}}dx$$

input
```
int((b*tan(d*x+c)^p)^(3/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(3/2),x)
```

3.47.5 Fricas [F(-2)]

Exception generated.

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.47.6 Sympy [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan^p{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(3/2), x)
```

3.47.7 Maxima [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan{(dx+c)}^p)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(3/2), x)
```

3.47.8 Giac [F]

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan(dx+c)^p)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(3/2), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \int (b\tan(c+d\,x)^p)^{3/2}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(3/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(3/2), x)
```

3.48 $\int \sqrt{b\tan^p(c+dx)}\,dx$

3.48.1 Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

output
```
2*hypergeom([1, 1/2+1/4*p],[3/2+1/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(1/
2)*tan(d*x+c)/d/(2+p)
```

3.48.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2+p}{4},\frac{6+p}{4},-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

input
```
Integrate[Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

3.48.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan^p(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{b\tan(c+dx)^p}dx$$

$$\Big\downarrow \; 4142$$

$$\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan^{\frac{p}{2}}(c+dx)dx$$

$$\Big\downarrow \; 3042$$

$$\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \tan(c+dx)^{p/2}dx$$

$$\Big\downarrow \; 3957$$

$$\frac{\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\int \frac{\tan^{\frac{p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 278$$

$$\frac{2\tan^{\frac{p+2}{2}-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{p+2}{4},\frac{p+6}{4},-\tan^2(c+dx)\right)}{d(p+2)}$$

input
```
Int[Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
]^(-1/2*p + (2 + p)/2)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

3.48.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.48.4 Maple [F]

$$\int \sqrt{b\left(\tan^{p}\left(dx+c\right)\right)}dx$$

input
```
int((b*tan(d*x+c)^p)^(1/2),x)
```

output
```
int((b*tan(d*x+c)^p)^(1/2),x)
```

3.48.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.48.6 Sympy [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan^p{(c+dx)}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(1/2),x)
```

output
```
Integral(sqrt(b*tan(c + d*x)**p), x)
```

3.48.7 Maxima [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan{(dx+c)}^p}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(d*x + c)^p), x)
```

3.48.8 Giac [F]

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan\left(dx+c\right)^p}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(d*x + c)^p), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \int \sqrt{b\tan\left(c+d\,x\right)^p}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(1/2),x)
```

output
```
int((b*tan(c + d*x)^p)^(1/2), x)
```

3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$

3.49.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-1/4*p],[3/2-1/4*p],-tan(d*x+c)^2)*tan(d*x+c)/d/(2-p)/(
b*tan(d*x+c)^p)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(-2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*
x])/(d*(-2 + p)*Sqrt[b*Tan[c + d*x]^p])
```

3.49.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(c+dx)^p}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{p}{2}}(c+dx)dx}{\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-p/2}dx}{\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-p}{4},\frac{6-p}{4},-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Int[1/Sqrt[b*Tan[c + d*x]^p],x]
```

output
```
(2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
])/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

3.49.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.49.4 Maple [F]

$$\int \frac{1}{\sqrt{b\left(\tan^{p}\left(dx+c\right)\right)}}dx$$

input
```
int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

3.49.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.49.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan^p\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(1/2),x)
```

output
```
Integral(1/sqrt(b*tan(c + d*x)**p), x)
```

3.49.7 Maxima [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^p}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

3.49.8 Giac [F]

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(dx+c\right)^p}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^p}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(1/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(1/2), x)
```

3.50 $\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx$

3.50.1 Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-3/4*p],[3/2-3/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-p)/b/d
/(2-3*p)/(b*tan(d*x+c)^p)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),-\frac{3}{4}(-2+p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{3p}{2}\right)(b\tan^p(c+dx))^{3/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(-3/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 3*p)/4, (-3*(-2 + p))/4, -Tan[c + d*x]^2]*Tan[c
 + d*x])/(d*(1 - (3*p)/2)*(b*Tan[c + d*x]^p)^(3/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(c+dx)^p)^{3/2}}dx$$

$$\Big\downarrow \text{4142}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{3p}{2}}(c+dx)dx}{b\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-3p/2}dx}{b\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{3p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{bd\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \text{278}$$

$$\frac{2\tan^{1-p}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3p),\frac{3(2-p)}{4},-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

input
```
Int[(b*Tan[c + d*x]^p)^(-3/2),x]
```

output
```
(2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c
 + d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])
```

3.50.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.50.4 Maple [F]

$$\int \frac{1}{\left(b\left(\tan^{p}\left(dx+c\right)\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

3.50.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.50.6 Sympy [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan^p{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(3/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(-3/2), x)
```

3.50.7 Maxima [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

3.50.8 Giac [F]

$$\int \frac{1}{\left(b\tan^p(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(dx+c\right)^p\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\tan^p(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(c+d\,x\right)^p\right)^{3/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(3/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(3/2), x)
```

3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx$

3.51.1 Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

output
```
2*hypergeom([1, 1/2-5/4*p],[3/2-5/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-2*p)/b
^2/d/(2-5*p)/(b*tan(d*x+c)^p)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)\tan(c+dx)}{d\left(1-\frac{5p}{2}\right)(b\tan^p(c+dx))^{5/2}}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^(-5/2),x]
```

output
```
(Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d
*x])/(d*(1 - (5*p)/2)*(b*Tan[c + d*x]^p)^(5/2))
```

3.51.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(b\tan(c+dx)^p)^{5/2}}dx$$

$$\Big\downarrow \; 4142$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{5p}{2}}(c+dx)dx}{b^2\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \tan(c+dx)^{-5p/2}dx}{b^2\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \; 3957$$

$$\frac{\tan^{\frac{p}{2}}(c+dx)\int \frac{\tan^{-\frac{5p}{2}}(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{b^2d\sqrt{b\tan^p(c+dx)}}$$

$$\Big\downarrow \; 278$$

$$\frac{2\tan^{1-2p}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-5p),\frac{1}{4}(6-5p),-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

input

```
Int[(b*Tan[c + d*x]^p)^(-5/2),x]
```

output

```
(2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
 d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

3.51.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.51.4 Maple [F]

$$\int \frac{1}{\left(b \left(\tan^{p}\left(d x +c \right)\right)\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

output
```
int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

3.51.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.51.6 Sympy [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan^p(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)**p)**(5/2),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(-5/2), x)
```

3.51.7 Maxima [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan(dx+c)^p)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

3.51.8 Giac [F]

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(dx+c)}^p)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\tan{(c+d\,x)}^p)^{5/2}}\,dx$$

input
```
int(1/(b*tan(c + d*x)^p)^(5/2),x)
```

output
```
int(1/(b*tan(c + d*x)^p)^(5/2), x)
```

3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$

3.52.1 Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx=-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

output
```
-cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^p)^(1/p)/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx=-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

input
```
Integrate[(b*Tan[c + d*x]^p)^p^(-1),x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

3.52.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4142, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(b\tan(c+dx)^p\right)^{\frac{1}{p}}\,dx$$

$$\Big\downarrow \text{ 4142}$$

$$\cot(c+dx)\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cot(c+dx)\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}\int \tan(c+dx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}}{d}$$

input
```
Int[(b*Tan[c + d*x]^p)^p^(-1),x]
```

output
```
-((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

3.52.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.52.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.38 (sec) , antiderivative size = 5979, normalized size of antiderivative = 186.84

method	result	size
risch	Expression too large to display	5979

input
```
int((b*tan(d*x+c)^p)^(1/p),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = -\frac{b^{\left(\frac{1}{p}\right)}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="fricas")
```

output
```
-1/2*b^(1/p)*log(1/(tan(d*x + c)^2 + 1))/d
```

3.52.6 Sympy [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan^p{(c+dx)})^{\frac{1}{p}}\,dx$$

input
```
integrate((b*tan(d*x+c)**p)**(1/p),x)
```

output
```
Integral((b*tan(c + d*x)**p)**(1/p), x)
```

3.52.7 Maxima [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan{(dx+c)}^p)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="maxima")
```

output
```
integrate((b*tan(d*x + c)^p)^(1/p), x)
```

3.52.8 Giac [F]

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan{(dx+c)}^p)^{\left(\frac{1}{p}\right)}\,dx$$

input
```
integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")
```

output
```
integrate((b*tan(d*x + c)^p)^(1/p), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx = \int (b\tan(c+d\,x)^p)^{1/p}\,dx$$

input
```
int((b*tan(c + d*x)^p)^(1/p),x)
```

output
```
int((b*tan(c + d*x)^p)^(1/p), x)
```

3.53 $\int (a(b\tan(c+dx))^p)^n\,dx$

3.53.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}$$

output
```
hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(a*(b*t
an(d*x+c))^p)^n/d/(n*p+1)
```

3.53.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (a(b\tan(c+dx))^p)^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+np),\frac{1}{2}(3+np),-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}$$

input
```
Integrate[(a*(b*Tan[c + d*x])^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

3.53.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4142, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\tan(c+dx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a(b\tan(c+dx))^p)^n dx$$

$$\Big\downarrow \text{4142}$$

$$(b\tan(c+dx))^{-np} (a(b\tan(c+dx))^p)^n \int (b\tan(c+dx))^{np} dx$$

$$\Big\downarrow \text{3042}$$

$$(b\tan(c+dx))^{-np} (a(b\tan(c+dx))^p)^n \int (b\tan(c+dx))^{np} dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b(b\tan(c+dx))^{-np} (a(b\tan(c+dx))^p)^n \int \frac{(b\tan(c+dx))^{np}}{\tan^2(c+dx)b^2+b^2} d(b\tan(c+dx))}{d}$$

$$\Big\downarrow \text{278}$$

$$\frac{\tan(c+dx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(np+1), \frac{1}{2}(np+3), -\tan^2(c+dx)\right) (a(b\tan(c+dx))^p)^n}{d(np+1)}$$

input
```
Int[(a*(b*Tan[c + d*x])^p)^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d
*x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

3.53.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4142
```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.53.4 Maple [F]

$$\int (a(b\tan (dx+c))^p)^n \, dx$$

input
```
int((a*(b*tan(d*x+c))^p)^n,x)
```

output
```
int((a*(b*tan(d*x+c))^p)^n,x)
```

3.53.5 Fricas [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan(dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*tan(d*x + c))^p*a)^n, x)
```

3.53.6 Sympy [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int (a(b\tan(c+dx))^p)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*tan(c + d*x))**p)**n, x)
```

3.53.7 Maxima [F]

$$\int (a(b\tan(c+dx))^p)^n \, dx = \int ((b\tan(dx+c))^p a)^n \, dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*tan(d*x + c))^p*a)^n, x)
```

3.53.8 Giac [F]

$$\int (a(b\tan(c+dx))^p)^n\,dx = \int ((b\tan(dx+c))^p a)^n\,dx$$

input
```
integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*tan(d*x + c))^p*a)^n, x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int (a(b\tan(c+dx))^p)^n\,dx = \int \left(a\,(b\tan(c+d\,x))^p\right)^n\,dx$$

input
```
int((a*(b*tan(c + d*x))^p)^n,x)
```

output
```
int((a*(b*tan(c + d*x))^p)^n, x)
```

3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{21\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}+\frac{21\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b}\\ &\quad+\frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b}\\ &\quad-\frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b}\\ &\quad-\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}\end{aligned}$$

output
```
-21/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21
/64*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21/12
8*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^
(1/2)-21/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d
^(1/2)/b*2^(1/2)-7/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d-1/4*cos(b*x+a)
^4*(d*tan(b*x+a))^(7/2)/b/d^3
```

3.54.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{\left(21\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)\sqrt{\sin(2(a+bx))}+21\csc(a+bx)\log\left(\cos(a+bx)+\right.\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/64*((21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x]*Sqrt[Sin[2*(a
+ b*x)]] + 21*Csc[a + b*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
 + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 18*Sin[2*(a + b*x)] - 2*Sin[4*(a + b*x
)])*Sqrt[d*Tan[a + b*x]])/b
```

3.54.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3071, 252, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^4\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{7}{8}\int \frac{(d\tan(a+bx))^{5/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{4}\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{7/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{7/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 826

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\,t}{4(d^2\,t}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+b}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\,ta}{2(d^2\,ta}}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\text{arcta}\right.}{b}$$

$\downarrow$ 25

$$\frac{d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\text{arctan}\right.}{b}$$

$\downarrow$ 27

3.54. $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

$$d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.\right.$$
$$\overline{\hspace{20em}b}$$

$\big\downarrow$ 1103

$$d\left(\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\right.}{}\right.\right.\right.\right.$$
$$\overline{\hspace{20em}b}$$

input
```
Int[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (7*((3*((-(
ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + S
qrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2
]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/2 -
(d*Tan[a + b*x])^(3/2)/(2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.54. $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.54.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(197) = 394$.

Time = 13.87 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.41

method	result
default	$\frac{\left(16\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,(\cos^3(bx+a))\sin(bx+a)+16(\cos^2(bx+a))\sin(bx+a)\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-44\cos(bx+a)\sin(bx\right.}{}$

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*(16*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*cos(b*
x+a)^3*sin(b*x+a)+16*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)-44*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*s
in(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-44*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2)+21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2
*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+
a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc
(b*x+a)+2)/(-1+cos(b*x+a)))-21*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*s
in(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)
^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b
*x+a)+2)/(-1+cos(b*x+a)))-42*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+42*arctan((s
in(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+
a)-1)/(-1+cos(b*x+a))))*(d*tan(b*x+a))^(1/2)*cos(b*x+a)/(cos(b*x+a)+1)/(-c
os(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

3.54.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 947, normalized size of antiderivative = 3.68

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/256*(16*(4*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
 + a))*sin(b*x + a) + 21*b*(-d^2/b^4)^(1/4)*log(9261/2*d^2*cos(b*x + a)*si
n(b*x + a) + 9261/2*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 9261/
4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 21*b*(-d^2/b^4)^(1/4)
*log(9261/2*d^2*cos(b*x + a)*sin(b*x + a) - 9261/2*(b^3*(-d^2/b^4)^(3/4)*c
os(b*x + a)^2 - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a)) - 9261/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^
2/b^4)) + 21*I*b*(-d^2/b^4)^(1/4)*log(9261/2*d^2*cos(b*x + a)*sin(b*x + a)
 - 9261/2*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 + I*b*d*(-d^2/b^4)^(1/4)*
cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 9261/4*(2*b
^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 21*I*b*(-d^2/b^4)^(1/4)*log
(9261/2*d^2*cos(b*x + a)*sin(b*x + a) - 9261/2*(-I*b^3*(-d^2/b^4)^(3/4)*co
s(b*x + a)^2 - I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a)) + 9261/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d
^2/b^4)) + 21*b*(-d^2/b^4)^(1/4)*log(9261*d^2 + 18522*(b^3*(-d^2/b^4)^(3/4
)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*
sin(b*x + a)/cos(b*x + a))) - 21*b*(-d^2/b^4)^(1/4)*log(9261*d^2 - 18522*(
b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 21*I*b*(-d^2/b^4)^(1/4...
```

3.54.6 Sympy [F]

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\sin^4{(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**4, x)
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{128\,bd^5}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/128*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x +
a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(11*(d*
tan(b*x + a))^(7/2)*d^6 + 7*(d*tan(b*x + a))^(3/2)*d^8)/(d^4*tan(b*x + a)^
4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.95

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}-\frac{21\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d}\right.}{b}}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/128*(42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 42*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
b - 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
a))*sqrt(abs(d)) + abs(d))/b + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a)
- sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*(11*sqrt(d*tan
(b*x + a))*d^5*tan(b*x + a)^3 + 7*sqrt(d*tan(b*x + a))*d^5*tan(b*x + a))/(
(d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^4\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2), x)
```

3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\begin{aligned}\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{3\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{3\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}\\ &\quad+\frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}\\ &\quad-\frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b}\\ &\quad-\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}\end{aligned}$$

output
```
-3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/16*ln(d
^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(1/2)-
3/16*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b
*2^(1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.46

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$
$$-\frac{\left(3\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+3\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin\right.\right.}{8b}$$

input
```
Integrate[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/8*((3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + 3*Csc[a + b*x]
*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sqrt[Sin[2*
(a + b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/b
```

3.55.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^2\sqrt{d\tan(a+bx)}dx$$
$$\Big\downarrow \text{3071}$$
$$\frac{d\int \frac{(d\tan(a+bx))^{5/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \text{252}$$
$$\frac{d\left(\frac{3}{4}\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$
$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{3}{2}\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\Big\downarrow$ 826

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{3/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\Big\downarrow$ 1476

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)\right.\right.}{b}$$

$\Big\downarrow$ 1082

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan\ldots}\right.\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a\ldots}{2(d^2\tan^2(\ldots}\right.}{b}$$

$\Big\downarrow$ 1479

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\ldots}\right.}{}\right.\right.\right.}{b}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\ldots}\right.}{}\right.\right.\right.}{b}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2\ldots}\right.}{}\right.\right.\right.}{b}$$

3.55. $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

$$\Big\downarrow \text{1103}$$

$$\frac{d\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + A
rcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log
[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]
))/2))/2 - (d*Tan[a + b*x])^(3/2)/(2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.55. $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.55.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(171) = 342$.

Time = 0.92 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\left(4\cos(bx+a)\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+4\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-6\arctan\left(\frac{\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin}{(\cos(bx+}}}{-1+\cos(b}\right.\right.}{}$

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*(4*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)-6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((-sin(b*x+a)*2^(1/2)*
(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+
a)))-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3
+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+
a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))
)+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*
cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-
csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))*
(d*tan(b*x+a))^(1/2)*cos(b*x+a)/(cos(b*x+a)+1)/(-cos(b*x+a)*sin(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)*2^(1/2)
```

3.55.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 934, normalized size of antiderivative = 4.11

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - 3*
b*(-d^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b*x + a) + 27/2*(b^3*(-d^
2/b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 27/4*(2*b^2*d*cos(b*x + a)^2 - b^2
*d)*sqrt(-d^2/b^4)) + 3*b*(-d^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b
*x + a) - 27/2*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 - b*d*(-d^2/b^4)^(1/4)
*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 27/4*(2*b^
2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 3*I*b*(-d^2/b^4)^(1/4)*log(2
7/2*d^2*cos(b*x + a)*sin(b*x + a) - 27/2*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x +
 a)^2 + I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + 27/4*(2*b^2*d*cos(b*x + a)^2 - b^2*d)*sqrt(-d^2/b^4))
+ 3*I*b*(-d^2/b^4)^(1/4)*log(27/2*d^2*cos(b*x + a)*sin(b*x + a) - 27/2*(-I
*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)^2 - I*b*d*(-d^2/b^4)^(1/4)*cos(b*x + a)
*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 27/4*(2*b^2*d*cos(b*x +
 a)^2 - b^2*d)*sqrt(-d^2/b^4)) - 3*b*(-d^2/b^4)^(1/4)*log(27*d^2 + 54*(b^3
*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^(1/4)*cos(b*x
 + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 3*b*(-d^2/b^4)^(1/4)*log(27*
d^2 - 54*(b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-d^2/b^4)^
(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 3*I*b*(-d^2/b^4
)^(1/4)*log(27*d^2 - 54*(I*b^3*(-d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + ...
```

3.55.6 Sympy [F]

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\sin^2\left(a+bx\right)dx$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**2, x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{}\right.}{\sqrt{d}}\right.}{16\,bd^3}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
 + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(b*
x + a))^(3/2)*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{\frac{8\sqrt{d\tan(bx+a)}d^3\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+}{16\,d}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)
*b) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b +
 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*
sqrt(abs(d)) + abs(d))/b - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b)/d
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^2\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2), x)
```

3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

3.56.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{3/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

3.56.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.56.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2d}{b\sqrt{d\tan(bx+a)}}$	17
default	$-\frac{2d}{b\sqrt{d\tan(bx+a)}}$	17

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*sin(b*x + a))
```

3.56.6 Sympy [F]

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\csc^2{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**2, x)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt{d\tan{(bx+a)}}}{b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(d*tan(b*x + a))/(b*tan(b*x + a))
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\,d}{\sqrt{d\tan{(bx+a)}}b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*d/(sqrt(d*tan(b*x + a))*b)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{b\sin\left(a+b\,x\right)^2}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^2,x)
```

output
```
-(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b
*sin(a + b*x)^2)
```

3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)-2/5*d^3/b/(d*tan(b*x+a))^(5/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx=-\frac{2d(4+\csc^2(a+bx))}{5b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*(4 + Csc[a + b*x]^2))/(5*b*Sqrt[d*Tan[a + b*x]])
```

3.57.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{7/2}}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^2}{5(d\tan(a+bx))^{5/2}}-\frac{2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^2)/(5*(d*Tan[a + b*x])^(5/2)) - 2/Sqrt[d*Tan[a + b*x]]))/b
```

3.57.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.57.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{2\sqrt{d\tan(bx+a)}\,\left(4\left(\cot^3(bx+a)\right)-5\cot(bx+a)\left(\csc^2(bx+a)\right)\right)}{5b}$	43

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-5*cot(b*x+a)*csc(b*x+a)^2)
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^3-5\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(4*cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))
/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

3.57.6 Sympy [F]

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(a+bx)}\csc^4(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**4, x)
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^2\tan\left(bx+a\right)^2+d^2\right)d}{5\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(5*d^2*tan(b*x + a)^2 + d^2)*d/((d*tan(b*x + a))^(5/2)*b)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(5\,d^4\tan\left(bx+a\right)^2+d^4\right)}{5\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^2}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/5*(5*d^4*tan(b*x + a)^2 + d^4)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x + a)
^2)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.49

$$\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$$
$$= \frac{8\sqrt{-\frac{d\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}{5\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1)^3}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^4,x)
```

output
```
(8*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*(exp
(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*2i - exp(a*6i + b*x*6i)*1i - 1i))/
(5*b*(exp(a*2i + b*x*2i) - 1)^3)
```

3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}} - \frac{4d^3}{5b(d\tan(a+bx))^{5/2}} - \frac{2d}{b\sqrt{d\tan(a+bx)}}$$

output
```
-2*d/b/(d*tan(b*x+a))^(1/2)-2/9*d^5/b/(d*tan(b*x+a))^(9/2)-4/5*d^3/b/(d*ta
n(b*x+a))^(5/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2d(-21+20\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{45b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-21 + 20*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(45*
b*Sqrt[d*Tan[a + b*x]])
```

3.58.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{11/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{11/2}}+\frac{2d^2}{(d\tan(a+bx))^{7/2}}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{9(d\tan(a+bx))^{9/2}}-\frac{4d^2}{5(d\tan(a+bx))^{5/2}}-\frac{2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^4)/(9*(d*Tan[a + b*x])^(9/2)) - (4*d^2)/(5*(d*Tan[a + b*x])^(5/2
)) - 2/Sqrt[d*Tan[a + b*x]]))/b
```

3.58.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.58.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{2 \cot(bx+a) (\csc^4(bx+a)) \sqrt{d \tan(bx+a)}\,(32(\cos^4(bx+a))-72(\cos^2(bx+a))+45)}{45b}$	52

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/b*cot(b*x+a)*csc(b*x+a)^4*(d*tan(b*x+a))^(1/2)*(32*cos(b*x+a)^4-72*c
os(b*x+a)^2+45)
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(32\cos\left(bx+a\right)^5-72\cos\left(bx+a\right)^3+45\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(32*cos(b*x + a)^5 - 72*cos(b*x + a)^3 + 45*cos(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a))/((b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)*sin(b
*x + a))
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^4\tan\left(bx+a\right)^4+18\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)d}{45\,\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(45*d^4*tan(b*x + a)^4 + 18*d^4*tan(b*x + a)^2 + 5*d^4)*d/((d*tan(b*
x + a))^(9/2)*b)
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\left(45\,d^6\tan\left(bx+a\right)^4+18\,d^6\tan\left(bx+a\right)^2+5\,d^6\right)}{45\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^4}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/45*(45*d^6*tan(b*x + a)^4 + 18*d^6*tan(b*x + a)^2 + 5*d^6)/(sqrt(d*tan(
b*x + a))*b*d^5*tan(b*x + a)^4)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 8.13 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.65

$$\begin{aligned}\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx = &-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}\\ &+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2}\\ &-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3}\\ &-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^4}\\ &-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^5}\end{aligned}$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^6,x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i + b
*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)) - ((exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*
(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(9*b*(exp(a*2i + b*x
*2i) - 1)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(9*b*(exp(a*2i + b*x*2i) - 1)^5)
```

3.58. $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12b}$$

output
```
-5/6*d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/3*d*sin(b*x+a)^3/b/(d*tan(b*x+a
))^(1/2)-5/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ell
ipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2
)/b
```

3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.79 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.32

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\cos(2(a+bx))\sec(a+bx)\left(-5\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sec^2(a+bx)+(-\cdots\right.}{6b\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\left(-1+\tan^2(\cdots\right.}$$

input
```
Integrate[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/6*(Cos[2*(a + b*x)]*Sec[a + b*x]*(-5*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1
)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 + (-6 + Cos[2*(a + b*x)])*
Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sec
[a + b*x]^2]*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

3.59.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3042

$$\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3053

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3042

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3120

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

input

```
Int[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output

```
-1/3*(d*Sin[a + b*x]^3)/(b*Sqrt[d*Tan[a + b*x]]) + (5*(-((d*Sin[a + b*x])/
(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/6
```

3.59. $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.59.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.25 (sec) , antiderivative size = 1740, normalized size of antiderivative = 16.57

method	result	size
default	Expression too large to display	1740

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/b*csc(b*x+a)*(-6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot
(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*(cot
(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-c
ot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*
2^(1/2))+6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*
(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)-6*I*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+
csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*(cot(b*x+
a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*
x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*cos(b*x+a)-32*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))
^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))
^(1/2),1/2*2^(1/2))*cos(b*x+a)-8*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)+6*(cot(b*
x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(
b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(
1/2))+6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*...
```

3.59.5 Fricas [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.59.7 Maxima [F]

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^3, x)
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeDone
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2), x)
```

3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.60.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}$$

output
```
-d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^
(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a
)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Integrate[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt
[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

3.60.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
-((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

3.60.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 0.74 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.67

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\left(-\cot(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+\right.\right.}{}$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*(d*tan(b*x+a))^(1/2)*(-cot(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-csc(b*x+a)*(cot(b*x+a)-csc(b*x+
a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a)
)*2^(1/2)
```

3.60.5 Fricas [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

3.60.6 Sympy [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\sin{(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*sin(a + b*x), x)
```

3.60.7 Maxima [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\sin{(bx+a)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

3.60.8 Giac [F]

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\sin{(bx+a)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sin(a+b\,x)\ \sqrt{d\tan(a+b\,x)}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(1/2), x)
```

3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.61.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}$$

output
```
-csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+
1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2\sqrt[4]{-1}\cos(a+bx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}{b\sqrt{\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*(-1)^(1/4)*Cos[a + b*x]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*
x]]], -1]*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Tan[a + b*x]]
)
```

3.61.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*T
an[a + b*x]])/b
```

3.61.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.61.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.30

method	result
default	$\frac{\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)\sqrt{d\tan(bx-}{b}$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(
b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(
1/2))*(d*tan(b*x+a))^(1/2)*(cot(b*x+a)+csc(b*x+a))*2^(1/2)
```

3.61.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\sqrt{-i\,d}F(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)\,|-1}{b}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt(-
I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/b
```

3.61.6 Sympy [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\csc\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x), x)
```

3.61.7 Maxima [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

3.61.8 Giac [F]

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan(bx+a)}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x), x)
```

3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

output
```
-2/3*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x
+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{2\cos(2(a+bx))\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(\right.\right.\right.}{3bd\sqrt{\sec^2(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*Cos[2*(a + b*x)]*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2)*(Sqrt[Sec[a + b*
x]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -
1]*Tan[a + b*x]^(3/2)))/(3*b*d*Sqrt[Sec[a + b*x]^2]*(-1 + Tan[a + b*x]^2))
```

3.62.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF
[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.62.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(92) = 184$.

Time = 0.75 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.83

method	result
default	$\frac{\left(-2\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\mathrm{c}\right.\right.}{}$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(-2*sin(b*x+a)*cos(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a
)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/
2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Ellipt
icF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(b*x+a))*(d*ta
n(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*2^(1/2)
```

3.62.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.47

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{2\left(\left(\cos\left(bx+a\right)^2-1\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-i\,d}F(\cdots\right.}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b
*x + a) - I*sin(b*x + a)), -1) - sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x
 + a))/(b*cos(b*x + a)^2 - b)
```

3.62.6 Sympy [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(a+bx)}}\csc^3{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**3, x)
```

3.62.7 Maxima [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\csc{(bx+a)}^3\;dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

3.62.8 Giac [F]

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan{(bx+a)}}\csc{(bx+a)}^3\;dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{7b}$$

output
```
-4/7*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/7*d*csc(b*x+a)^3/b/(d*tan(b*x+a
))^(1/2)-4/7*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Elli
pticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)
/b
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.18

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\cos(2(a+bx))\csc^3(a+bx)\left((-2+\cos(2(a+bx)))\sec^2(a+bx)^{3/2}-4\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-\right.\right.}{7b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Cos[2*(a + b*x)]*Csc[a + b*x]^3*((-2 + Cos[2*(a + b*x)])*(Sec[a + b*
x]^2)^(3/2) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x
]]], -1]*Tan[a + b*x]^(7/2)))/(7*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x
]]*(-1 + Tan[a + b*x]^2))
```

3.63.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3079, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^5}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3079}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3053

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3120

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Csc[a + b*x]^3)/(7*b*Sqrt[d*Tan[a + b*x]]) + (6*((-2*d*Csc[a + b*x])
/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]
*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)))/7
```

3.63. $\int\csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(116) = 232.

Time = 0.70 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.29

method	result
default	$\frac{\sqrt{-\frac{d(\csc(bx+a)-\cot(bx+a))}{\left(\csc^2(bx+a)\right)(1-\cos(bx+a))^2-1}}\left(\left(\csc^2(bx+a)\right)(1-\cos(bx+a))^2-1\right)\left(\sin^3(bx+a)\right)\left(\left(\csc^8(bx+a)\right)(1-\cos(bx+a))^8+32(\csc^3(bx+a)\right.}{56b(1-\cos(bx+a))^3\sqrt{\csc(bx}}$

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/56/b*(-d/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)*(csc(b*x+a)-cot(b*x+a)))^(1/2
)*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(1-cos(b*x+a))^3*sin(b*x+a)^3*(csc(b*x
+a)^8*(1-cos(b*x+a))^8+32*csc(b*x+a)^3*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-
2*csc(b*x+a)+2*cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((
1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*(1-cos(b*x+a))^3+10*csc(b*x+a)
^6*(1-cos(b*x+a))^6-10*csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(csc(b*x+a)*(1-cos
(b*x+a))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))^(1/2)/(csc(b*x+a)^3*(1-cos(b*x
+a))^3-csc(b*x+a)+cot(b*x+a))^(1/2)*2^(1/2)
```

3.63.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx =$$

$$-\frac{2\left(2\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\sqrt{i\,d}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+2\left(\cos\left(bx\right.\right.\right.}{7\left(b\cos\right.}$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsi
n(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)
^2 + 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
 (2*cos(b*x + a)^3 - 3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b
*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

3.63.6 Sympy [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(a+bx\right)}\csc^5\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**5, x)
```

3.63.7 Maxima [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

3.63.8 Giac [F]

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \sqrt{d\tan\left(bx+a\right)}\csc\left(bx+a\right)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = \int \frac{\sqrt{d\tan\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5, x)
```

3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 277

$$\begin{aligned}\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{45d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} \\ &- \frac{45d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} \\ &+ \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} \\ &- \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{45d\sqrt{d\tan(a+bx)}}{16b} \\ &- \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}\end{aligned}$$

output
```
45/64*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-45/
64*d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+45/128
*d^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(
1/2)-45/128*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*
x+a))/b*2^(1/2)+45/16*d*(d*tan(b*x+a))^(1/2)/b-9/16*cos(b*x+a)^2*(d*tan(b*
x+a))^(5/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(9/2)/b/d^3
```

3.64.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.44

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{d\csc(a+bx)\left(-143\sin(a+bx)-45\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+45\log\left(\cos(a+\right.\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/64*(d*Csc[a + b*x]*(-143*Sin[a + b*x] - 45*ArcSin[Cos[a + b*x] - Sin[a
+ b*x]]*Sqrt[Sin[2*(a + b*x)]] + 45*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt
[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 14*Sin[3*(a + b*x)] + Sin[5*(
a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

3.64.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3071, 252, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^4(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{11/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{9}{8}\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\int \frac{(d\tan(a+bx))^{3/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 262

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-d^2\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+b\cdots}{4(d^2\tan^2(a+\cdots}\cdots}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{9/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)\right)-\frac{(d\tan(a+bx))^{5\cdots}}{2(d^2\tan^2(a+bx)\cdots}\cdots}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\,\mathrm{ta}\cdots}}{2d}\cdots}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^{2}\cdots}{d^4\tan^4(a\cdots}\cdots}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)\right)\cdots}{b}$$

$\downarrow$ 1479

3.64. $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

$$d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.\right.$$

$$b$$

$\downarrow$ 25

$$d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+b}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.\right.$$

$$b$$

$\downarrow$ 27

$$d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+b}}{2\sqrt{d}}}{2d}\right.\right.\right.\right.$$

$$b$$

$\downarrow$ 1103

$$d\left(\frac{9}{8}\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-}{2}\right.\right.\right.\right.$$

$$b$$

input
```
Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(9/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (9*(-1/2*(d
*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2) + (5*(-2*d^2*((-(ArcTan[1
- Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sq
rt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*
d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)) + 2
*Sqrt[d*Tan[a + b*x]]))/4))/8))/b
```

3.64. $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.64.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(213) = 426$.

Time = 3.46 (sec) , antiderivative size = 890, normalized size of antiderivative = 3.21

method	result	size
default	Expression too large to display	890

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*(d*tan(b*x+a))^(1/2)*(-16*2^(1/2)*cos(b*x+a)^4+68*cos(b*x+a)^2*2^(
1/2)+45*cot(b*x+a)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2
^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2
)*cot(b*x+a)-2*cot(b*x+a)+2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/
2)-45*cot(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x
+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-
90*cot(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(
b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-
1)/(-1+cos(b*x+a)))+90*cot(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+45*csc(b*x+a)*ln(-2*(-cos(b*x+a)*
sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)*(-cos(b*x
+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-45*csc(b*x+a)*(-cos(b*x+a)*sin(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)+128*2^(1/2)-90*csc(b*x+a)*(-cos(b*x+a)
*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a
)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+90*...
```

3.64.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.44

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/256*(45*I*(-d^6/b^4)^(1/4)*b*log(182250*d^5*cos(b*x + a)^2 + 182250*sqr
t(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 91125*d^5 - 182250*(I*(-d^
6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 45*I*(-d^6/b^4)^(1/4)*b*l
og(182250*d^5*cos(b*x + a)^2 + 182250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*
sin(b*x + a) - 91125*d^5 - 182250*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*
sin(b*x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/
cos(b*x + a))) - 45*(-d^6/b^4)^(1/4)*b*log(182250*d^5*cos(b*x + a)^2 - 182
250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 91125*d^5 + 182250*
((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + (-d^6/b^4)^(3/4)*b^3*c
os(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 45*(-d^6/b^4)^(1/4)*b*
log(182250*d^5*cos(b*x + a)^2 - 182250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)
*sin(b*x + a) - 91125*d^5 - 182250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*si
n(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(
b*x + a))) - 45*(-d^6/b^4)^(1/4)*b*log(-91125*d^5 + 182250*((-d^6/b^4)^(1/
4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*
sqrt(d*sin(b*x + a)/cos(b*x + a))) + 45*(-d^6/b^4)^(1/4)*b*log(-91125*d^5
- 182250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3
/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 45*I*(-d^6/b^
4)^(1/4)*b*log(-91125*d^5 - 182250*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + ...
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{90\,\sqrt{2}d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{d}}\right)+90\,\sqrt{2}d^{\frac{13}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{d}}\right)+45\,\sqrt{2}d^{\frac{13}{2}}\log}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/128*(90*sqrt(2)*d^(13/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d
*tan(b*x + a)))/sqrt(d)) + 90*sqrt(2)*d^(13/2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*sqrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 45*sqrt(2)*d^(13/2)*log(d*t
an(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 45*sqrt(2)*d^(13
/2)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 256*s
qrt(d*tan(b*x + a))*d^6 - 8*(17*(d*tan(b*x + a))^(5/2)*d^8 + 13*sqrt(d*tan
(b*x + a))*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5
)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.91

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{1}{128}\,d\left(\frac{90\,\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{|d|}}\right)}{b}+\frac{90\,\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\,\sqrt{d\tan(bx+a)}\right)}{2\,\sqrt{|d|}}\right)}{b}\right.$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/128*d*(90*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 90*sqrt(2)*sqrt(abs(d))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/b + 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x
 + a))*sqrt(abs(d)) + abs(d))/b - 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x +
a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 256*sqrt(d*ta
n(b*x + a))/b - 8*(17*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^2 + 13*sqrt(d*
tan(b*x + a))*d^4)/((d^2*tan(b*x + a)^2 + d^2)^2*b))
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \sin\left(a+b\,x\right)^4\,{\left(d\,\tan(a+b\,x)\right)}^{3/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2), x)
```

3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 247

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{5d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{5d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} + \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}$$

output
```
5/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-5/8*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+5/16*d^(3/
2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-5
/16*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*
2^(1/2)+5/2*d*(d*tan(b*x+a))^(1/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(5/2)
/b/d
```

3.65.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.46

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\csc(a+bx)\left(17\sin(a+bx)+5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-5\log\left(\cdots\right.\right.}{\cdots}$$

input
```
Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*Csc[a + b*x]*(17*Sin[a + b*x] + 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
+ b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])
/(8*b)
```

3.65.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^2(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{d\left(\frac{5}{4}\int \frac{(d\tan(a+bx))^{3/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-d^2\int\frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\int\frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int\frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)\right)-\frac{(d\tan(a+bx))^{5/2}}{2(d^2\tan^2(a+bx)+d^2)}}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\tan^2(a+}{d^4\tan^4(a+bx)-}\right.\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)\right)-\frac{}{2}}{b}$$

$\downarrow$ 1479

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 25

3.65. $\int\sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 27

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}\right.\right.\right.}{b}$$

$\downarrow$ 1103

$$\frac{d\left(\frac{5}{4}\left(2\sqrt{d\tan(a+bx)}-2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\cdots\right.}{}}{2d}\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/2*(d*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2) + (5*(-2*d^2*((
-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
 Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d
 + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))
/(2*d)) + 2*Sqrt[d*Tan[a + b*x]]))/4))/b
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.65. $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.65.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(187) = 374$.

Time = 2.33 (sec) , antiderivative size = 1083, normalized size of antiderivative = 4.38

method	result	size
default	Expression too large to display	1083

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*sin(b*x+a)*(4*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)+5*(-cos(b*x+a)*sin(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*si
n(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^
2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*
x+a)+2)/(-1+cos(b*x+a)))*cos(b*x+a)-5*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x
+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot
(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*
x+a)))*cos(b*x+a)+10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arcta
n((-sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos
(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)-10*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+16*sin(b*x+a)*2
^(1/2)+5*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*c
os(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+
a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*c
os(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-5*(-cos(b*x+a)*sin(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(
b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+
csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b...
```

3.65.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.81

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(5*I*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos(b*x + a)^2 + 250*sqrt(-d^6/b
^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d^5 - 250*(I*(-d^6/b^4)^(1/4)*
b*d^3*cos(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*s
qrt(d*sin(b*x + a)/cos(b*x + a))) - 5*I*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos
(b*x + a)^2 + 250*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d
^5 - 250*(-I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d^6/b^
4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 5*(-d^6/
b^4)^(1/4)*b*log(250*d^5*cos(b*x + a)^2 - 250*sqrt(-d^6/b^4)*b^2*d^2*cos(b
*x + a)*sin(b*x + a) - 125*d^5 + 250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*
sin(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/co
s(b*x + a))) + 5*(-d^6/b^4)^(1/4)*b*log(250*d^5*cos(b*x + a)^2 - 250*sqrt(
-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) - 125*d^5 - 250*((-d^6/b^4)^(1
/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)
*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 5*(-d^6/b^4)^(1/4)*b*log(-125*d^5 +
250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/4)*b
^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 5*(-d^6/b^4)^(1/4)
*b*log(-125*d^5 - 250*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) -
(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) +
5*I*(-d^6/b^4)^(1/4)*b*log(-125*d^5 - 250*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*
x + a)*sin(b*x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin...
```

3.65.6 Sympy [F]

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sin^2(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x)**2, x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.83

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{9}{2}}\log\left(d\right.}{}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*
x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan
(b*x + a))*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*sqrt(d*tan(b*x + a))*d^4)/(
b*d^3)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.91

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{1}{16}d\left(\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{10\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\right.$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/16*d*(10*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 10*sqrt(2)*sqrt(abs(d))*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d))
)/b + 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
 a))*sqrt(abs(d)) + abs(d))/b - 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a)
- sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x
 + a))*d^2/((d^2*tan(b*x + a)^2 + d^2)*b) - 32*sqrt(d*tan(b*x + a))/b)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin\left(a+b\,x\right)^2\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d*(d*tan(b*x+a))^(1/2)/b
```

3.66.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

input
```
Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sqrt[d*Tan[a + b*x]])/b
```

3.66.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{\sqrt{d\tan(a+bx)}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sqrt[d*Tan[a + b*x]])/b
```

3.66.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.66.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2d\sqrt{d\tan(bx+a)}}{b}$	17
default	$\frac{2d\sqrt{d\tan(bx+a)}}{b}$	17

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(d*tan(b*x+a))^(1/2)/b
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*d*sqrt(d*sin(b*x + a)/cos(b*x + a))/b
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan{(bx+a)})^{\frac{3}{2}}}{b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2*(d*tan(b*x + a))^(3/2)/(b*tan(b*x + a))
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan{(bx+a)}}d}{b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(d*tan(b*x + a))*d/b
```

3.66.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{b}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^2,x)
```

output
```
(2*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/b
```

3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d*(d*tan(b*x+a))^(1/2)/b-2/3*d^3/b/(d*tan(b*x+a))^(3/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d(-4+\csc^2(a+bx))\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d*(-4 + Csc[a + b*x]^2)*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.67.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{5/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{5/2}}+\frac{1}{\sqrt{d\tan(a+bx)}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(2\sqrt{d\tan(a+bx)}-\frac{2d^2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^2)/(3*(d*Tan[a + b*x])^(3/2)) + 2*Sqrt[d*Tan[a + b*x]]))/b
```

3.67.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.67.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{2\sqrt{d\tan(bx+a)}\, d\left(4\left(\cot^2(bx+a)\right)-3\left(\csc^2(bx+a)\right)\right)}{3b}$	38

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(d*tan(b*x+a))^(1/2)*d*(4*cot(b*x+a)^2-3*csc(b*x+a)^2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(4\,d\cos\left(bx+a\right)^2-3\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(4*d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*
x + a)^2 - b)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\,d^3\left(\frac{1}{(d\tan(bx+a))^{\frac{3}{2}}}-\frac{3\,\sqrt{d\tan(bx+a)}}{d^2}\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/3*d^3*(1/(d*tan(b*x + a))^(3/2) - 3*sqrt(d*tan(b*x + a))/d^2)/b
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2}{3}\,d\left(\frac{3\,\sqrt{d\tan\left(bx+a\right)}}{b}-\frac{d}{\sqrt{d\tan\left(bx+a\right)}b\tan\left(bx+a\right)}\right)$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/3*d*(3*sqrt(d*tan(b*x + a))/b - d/(sqrt(d*tan(b*x + a))*b*tan(b*x + a)))
```

3.67.9 Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.44

$$\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{8\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,(11\,\cos(2\,a+2\,b\,x)-5\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)-7)}{3\,b\,(15\,\cos(2\,a+2\,b\,x)-6\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)-10)}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^4,x)
```

output
```
(8*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(11*cos(2*a + 2*b
*x) - 5*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 7))/(3*b*(15*cos(2*a + 2*b*x
) - 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}} - \frac{4d^3}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d*(d*tan(b*x+a))^(1/2)/b-2/7*d^5/b/(d*tan(b*x+a))^(7/2)-4/3*d^3/b/(d*tan
(b*x+a))^(3/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d(-32+8\csc^2(a+bx)+3\csc^4(a+bx))\sqrt{d\tan(a+bx)}}{21b}$$

input
```
Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(2
1*b)
```

3.68.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^6}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 244$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{9/2}}+\frac{2d^2}{(d\tan(a+bx))^{5/2}}+\frac{1}{\sqrt{d\tan(a+bx)}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\left(-\frac{2d^4}{7(d\tan(a+bx))^{7/2}}-\frac{4d^2}{3(d\tan(a+bx))^{3/2}}+2\sqrt{d\tan(a+bx)}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^4)/(7*(d*Tan[a + b*x])^(7/2)) - (4*d^2)/(3*(d*Tan[a + b*x])^(3/2
)) + 2*Sqrt[d*Tan[a + b*x]]))/b
```

3.68.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.68.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\left(\csc^{4}\left(b x+a\right)\right) \sqrt{d \tan \left(b x+a\right)}\, d \left(32 \left(\cos^{4}\left(b x+a\right)\right)-56 \left(\cos^{2}\left(b x+a\right)\right)+21\right)}{21 b}$	47

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b*csc(b*x+a)^4*(d*tan(b*x+a))^(1/2)*d*(32*cos(b*x+a)^4-56*cos(b*x+a)^
2+21)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(32\,d\cos(bx+a)^4-56\,d\cos(bx+a)^2+21\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(32*d*cos(b*x + a)^4 - 56*d*cos(b*x + a)^2 + 21*d)*sqrt(d*sin(b*x + a
)/cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d^5\left(\frac{21\,\sqrt{d\tan(bx+a)}}{d^4}-\frac{14\,d^2\tan(bx+a)^2+3\,d^2}{(d\tan(bx+a))^{\frac{7}{2}}\,d^2}\right)}{21\,b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/21*d^5*(21*sqrt(d*tan(b*x + a))/d^4 - (14*d^2*tan(b*x + a)^2 + 3*d^2)/((
d*tan(b*x + a))^(7/2)*d^2))/b
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2}{21}d\left(\frac{21\sqrt{d\tan(bx+a)}}{b} - \frac{14\,d^4\tan(bx+a)^2+3\,d^4}{\sqrt{d\tan(bx+a)}bd^3\tan(bx+a)^3}\right)$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/21*d*(21*sqrt(d*tan(b*x + a))/b - (14*d^4*tan(b*x + a)^2 + 3*d^4)/(sqrt(
d*tan(b*x + a))*b*d^3*tan(b*x + a)^3))
```

3.68.9 Mupad [B] (verification not implemented)

Time = 7.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{\left(\frac{20\,d}{21\,b}-\frac{64\,d\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{21\,b}\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1} + \frac{20\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{21\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2} - \frac{24\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3} - \frac{16\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^4}$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^6,x)
```

output
```
(20*d*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2
i + b*x*2i) + 1))^(1/2))/(21*b*(exp(a*2i + b*x*2i) - 1)^2) - (((20*d)/(21*
b) - (64*d*exp(a*2i + b*x*2i))/(21*b))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) - (24*d*(exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2))/(7*b*(exp(a*2i + b*x*2i) - 1)^3) - (16*d*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*
b*(exp(a*2i + b*x*2i) - 1)^4)
```

3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}-\frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
7/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*
Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*
d*sin(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b+7/3*d^3*sin(b*x+a)^3/b/(d*tan(b*x+a)
)^(3/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{\left(-28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec(a+bx)+2\cos(a+bx)(13+\cos(2(a\right.}{12b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((-28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x] + 2*C
os[a + b*x]*(13 + Cos[2*(a + b*x)])*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])
^(3/2))/(12*b*Sqrt[Sec[a + b*x]^2])
```

3.69.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3074, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$\downarrow$ 3052

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

$\downarrow$ 3119

$$\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}-7d^2\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)$$

input
```
Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b - 7*d^2*(-1/3*(d*Sin[a + b*x]^
3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x]
)/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]))
```

3.69.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(123) = 246$.

Time = 0.82 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.72

method	result
default	$-\frac{\sin(bx+a)\left(42\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\right.\right.}{}$

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*sin(b*x+a)*(42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*
x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+c
sc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-2*2^(1/2)*cos(b*x+a)^4
+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(
b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(
1/2))-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/
2*2^(1/2))+11*cos(b*x+a)^2*2^(1/2)-21*2^(1/2)*cos(b*x+a)+12*2^(1/2))*(d*ta
n(b*x+a))^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

3.69.5 Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(d*cos(b*x + a)^2 - d)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(b*x
 + a), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.69.7 Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^3, x)
```

3.69.8 Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin(a+b\,x)^3\,(d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*
sin(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{b}$$

input
```
Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/b
```

3.70.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3074, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-3d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-3d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x
]]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

3.70.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(95) = 190.

Time = 0.77 (sec) , antiderivative size = 395, normalized size of antiderivative = 5.20

method	result
default	$-\frac{\sin(bx+a)\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\right.\right.}{}$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*sin(b*x+a)*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*
x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+
a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+6*(1+csc(b*x+a)-cot(b*x+a
))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2
)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1
/2)-3*2^(1/2)*cos(b*x+a)+2*2^(1/2))*(d*tan(b*x+a))^(1/2)*d/(cos(b*x+a)^2-1
)*2^(1/2)
```

3.70.5 Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*sin(b*x + a)*tan(b*x + a), x)
```

3.70.6 Sympy [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int(d\tan{(a+bx)})^{\frac{3}{2}}\sin{(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x), x)
```

3.70.7 Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int(d\tan{(bx+a)})^{\frac{3}{2}}\sin{(bx+a)}\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a), x)
```

3.70.8 Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \sin\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.71.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*
sin(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\cos(a+bx)\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]
*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.71.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3073, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-2d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-2d^2\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x
]]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

3.71.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(95) = 190.

Time = 0.62 (sec) , antiderivative size = 381, normalized size of antiderivative = 5.01

method	result
default	$-\frac{\sin(bx+a)\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\right.\right.}{}$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*sin(b*x+a)*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+
a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(1+csc(b*x+a)-cot(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellipt
icE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-(cot(b*x+a)-csc(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Elli
pticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(b*x+a)+2^(1
/2))*(d*tan(b*x+a))^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

3.71.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{-i\sqrt{i\,d}dE(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)+i\sqrt{-i\,d}dE(\arcsin(\cos(bx+a)-i\sin}{}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + I*
sqrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + I*sqr
t(I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - I*sqrt(-I
*d)*d*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) + 2*d*sqrt(d*s
in(b*x + a)/cos(b*x + a))*sin(b*x + a))/b
```

3.71.6 Sympy [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\csc(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*csc(a + b*x), x)
```

3.71.7 Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

3.71.8 Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x), x)
```

3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx=-\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
-4*d^2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)
/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2
*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*csc(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\cos(a+bx)\left(-6+3\csc^2(a+bx)+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\,\text{ta}}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]*(-6 + 3*Csc[a + b*x]^2 + 4*Hypergeometric2F1[3/4, 3/2, 7/
4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.72.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3073, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3073}$$

$$2d^2\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3050}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2d^2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d^2*Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]])
- (2*Sqrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(
b*Sqrt[Sin[2*a + 2*b*x]])))/(Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]]) + (2
*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

3.72. $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.72.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(119) = 238$.

Time = 0.67 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\csc(bx+a)\sqrt{d\tan(bx+a)}\,d\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx}\right.\right.}{}$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b*csc(b*x+a)*(d*tan(b*x+a))^(1/2)*d*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(cot(b*x+a)-csc(
b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(
1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-4*(
1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a
)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))
+2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b
*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1
/2))+2*2^(1/2)*cos(b*x+a)-2^(1/2))*2^(1/2)
```

3.72.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.73

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{2\left(i\sqrt{i}\,ddE(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\mid -1)\sin\left(bx+a\right)-i\sqrt{-i}\,ddE(\arcsin\left(\cos\left(bx+a\right)-i\right.\right.}{}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(I*d)*d*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*si
n(b*x + a) - I*sqrt(-I*d)*d*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a
)), -1)*sin(b*x + a) - I*sqrt(I*d)*d*elliptic_f(arcsin(cos(b*x + a) + I*si
n(b*x + a)), -1)*sin(b*x + a) + I*sqrt(-I*d)*d*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*d*cos(b*x + a)^2 - d)*sqrt(d*
sin(b*x + a)/cos(b*x + a)))/(b*sin(b*x + a))
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.72.7 Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

3.72.8 Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 277

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{77d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} + \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}$$

output
```
77/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/
64*d^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/128
*d^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(
1/2)+77/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*
x+a))/b*2^(1/2)+77/48*d*(d*tan(b*x+a))^(3/2)/b-11/16*cos(b*x+a)^2*(d*tan(b
*x+a))^(7/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(11/2)/b/d^3
```

3.73.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.51

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(128+204\cos^2(a+bx)+231\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin}$$

input
```
Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(128 + 204*Cos[a + b*x]^2 + 231*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot
[a + b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 231*Cot[a + b*x]*Csc[a + b
*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(
a + b*x)]] - 6*Cot[a + b*x]*Sin[4*(a + b*x)])*(d*Tan[a + b*x])^(3/2))/(192
*b)
```

3.73.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3071, 252, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{13/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{11}{8}\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\downarrow 252$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\int \frac{(d\tan(a+bx))^{5/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\downarrow 262$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-d^2\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\downarrow 266$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{11/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\downarrow 826$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)\right)\right.\right.}{b}$$

$$\downarrow 1476$$

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3}\ldots}\right.\right.\right.\right.\right.$$

$$\downarrow 1082$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.}{b}$$

$$\downarrow 217$$

$$\frac{d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+\ldots}\right.\right.\right.\right.}{b}$$

$$\downarrow 1479$$

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+\ldots}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.$$

3.73. $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

$$\Big\downarrow \ 25$$

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \ 27$$

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)}}{2\sqrt{d}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \ 1103$$

$$d\left(\frac{11}{8}\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan\right.}{}\right.\right.\right.\right.\right.$$

$$b$$

input
```
Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(11/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (11*(-1/2*
(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2) + (7*(-2*d^2*((-(ArcTan[
1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*
Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Tan[
a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/
2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2) + (2*(d*Tan[
a + b*x])^(3/2))/3))/4))/8))/b
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.73. $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.73.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(213) = 426$.

Time = 48.45 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\tan(bx+a)\sqrt{d\tan(bx+a)}\left(48(\cos^5(bx+a))\sqrt{2}-48\sqrt{2}\,(\cos^4(bx+a))-228(\cos^3(bx+a))\sqrt{2}+228(\cos^2(bx+a))\sqrt{2}-462\sqrt{-\frac{\cos(bx+}{(\cos(}}\right.}{}$

input
```
int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/b*tan(b*x+a)*(d*tan(b*x+a))^(1/2)*(48*cos(b*x+a)^5*2^(1/2)-48*2^(1/
2)*cos(b*x+a)^4-228*cos(b*x+a)^3*2^(1/2)+228*cos(b*x+a)^2*2^(1/2)-462*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b*x+a)*2^(1/2)*(-
cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)
))*cos(b*x+a)+462*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((
sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x
+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)-231*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/
2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^
(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)+231*cos(b*x+a)*(-cos(b*x+a)*sin(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)*ln(-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)*2^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(
1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-128*2^(1/2)*cos(b*x+a)+128*2^(1/2))/(-1+co
s(b*x+a))*d^2*2^(1/2)
```

3.73.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 1048, normalized size of antiderivative = 3.78

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/768*(231*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a
)*sin(b*x + a) + 456533/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/
b^4) + 456533/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^1
0/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*
(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x +
a) + 456533/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 45653
3/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4)
*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*I*(-d^10/b^4
)^(1/4)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x + a) - 45653
3/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 456533/2*(I*(-d
^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) + I*(-d^10/b^4)^(3/4)*b^3*c
os(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 231*I*(-d^10/b^4)^(1/4
)*b*cos(b*x + a)*log(-456533/2*d^8*cos(b*x + a)*sin(b*x + a) - 456533/4*(2
*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 456533/2*(-I*(-d^10/b
^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - I*(-d^10/b^4)^(3/4)*b^3*cos(b*
x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*(-d^10/b^4)^(1/4)*b*cos
(b*x + a)*log(456533*d^8 + 913066*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2
- (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos
(b*x + a))) + 231*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(456533*d^8 - 913066
*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^(3/4)*b^3*cos(b*...
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.87

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a}\right.}{\sqrt{d}}\right.}{}$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-1/384*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*
tan(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x
+ a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 256*(d*
tan(b*x + a))^(3/2)*d^6 - 24*(19*(d*tan(b*x + a))^(7/2)*d^8 + 15*(d*tan(b*
x + a))^(3/2)*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*
d^5)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{1}{384}d^2\left(\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}+\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}\right.$$

input
```
integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-1/384*d^2*(462*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
d)) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 462*sqrt(2)*abs(d)^(3/
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqr
t(abs(d)))/(b*d) - 231*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*s
qrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 231*sqrt(2)*abs(d)^(3/2
)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))
/(b*d) - 256*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 24*(19*sqrt(d*tan(b*x +
 a))*d^4*tan(b*x + a)^3 + 15*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a))/((d^2*
tan(b*x + a)^2 + d^2)^2*b))
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)^4\,(d\tan(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2), x)
```

3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 247

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{7d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

output
```
7/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/8*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/16*d^(5/
2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+7
/16*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*
2^(1/2)+7/6*d*(d*tan(b*x+a))^(3/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(7/2)
/b/d
```

3.74.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.51

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\Big(16+12\cos^2(a+bx)+21\arcsin(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin(2\ldots}}{}$$

input
```
Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(16 + 12*Cos[a + b*x]^2 + 21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[a
+ b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 21*Cot[a + b*x]*Csc[a + b*x]*
Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a +
b*x)]])*(d*Tan[a + b*x])^(3/2))/(24*b)
```

3.74.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3071, 252, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{9/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{7}{4}\int \frac{(d\tan(a+bx))^{5/2}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-d^2\int\frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\big\downarrow$ 266

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\int\frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{(d\tan(a+bx))^{7/2}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$\big\downarrow$ 826

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\int\frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)\right)-\frac{2}{}\right.}{b}$$

$\big\downarrow$ 1476

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 1082

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 217

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{}\right.\right.\right.}{b}$$

$\big\downarrow$ 1479

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 25

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d}{2\sqrt{2}\sqrt{d}}\right.\right.\right.\right.}{b}$$

$\big\downarrow$ 27

3.74. $\int\sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{}}{2\sqrt{d}}\right.\right.\right.\right.}{b}$$

$\Big\downarrow$ 1103

$$\frac{d\left(\frac{7}{4}\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+b\right.}{2\sqrt{2}}\right.\right.\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/2*(d*Tan[a + b*x])^(7/2)/(d^2 + d^2*Tan[a + b*x]^2) + (7*(-2*d^2*((
-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
 Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(
3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt
[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2) + (
2*(d*Tan[a + b*x])^(3/2))/3))/4))/b
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.74. $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.74.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(187) = 374$.

Time = 3.59 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.02

method	result
default	$\frac{\tan(bx+a)\sqrt{d\tan(bx+a)}\left(12(\cos^3(bx+a))\sqrt{2}+21\cos(bx+a)\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\ \ln\left(2\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\ \sqrt{2}\ \csc(bx+a)+2\right.\right.}{}$

input
```
int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*tan(b*x+a)*(d*tan(b*x+a))^(1/2)*(12*cos(b*x+a)^3*2^(1/2)+21*cos(b*x
+a)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(2*(-cos(b*x+a)*sin(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)+2*(-cos(b*x+a)*sin(b*x+a
)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-21*cos(b*x+a)
*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-2*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*csc(b*x+a)-2*(-cos(b*x+a)*sin(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)*cot(b*x+a)-2*cot(b*x+a)+2)-12*cos(b*x+a)^2
*2^(1/2)+42*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-sin(b
*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1
)/(-1+cos(b*x+a)))*cos(b*x+a)-42*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))*cos(b*x+a)+16*2^(1/2)*cos(b*x+a)-16
*2^(1/2))/(-1+cos(b*x+a))*d^2*2^(1/2)
```

3.74.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.19

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/96*(21*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-343/2*d^8*cos(b*x + a)*sin
(b*x + a) + 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) + 3
43/2*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4
)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*(-d^10/b^4)^
(1/4)*b*cos(b*x + a)*log(-343/2*d^8*cos(b*x + a)*sin(b*x + a) + 343/4*(2*b
^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-d^10/b^4) - 343/2*((-d^10/b^4)^(1/4
)*b*d^5*cos(b*x + a)*sin(b*x + a) - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*
sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)
*log(-343/2*d^8*cos(b*x + a)*sin(b*x + a) - 343/4*(2*b^2*d^3*cos(b*x + a)^
2 - b^2*d^3)*sqrt(-d^10/b^4) - 343/2*(I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x +
a)*sin(b*x + a) + I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a))) + 21*I*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(-343/2*d^8*
cos(b*x + a)*sin(b*x + a) - 343/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqr
t(-d^10/b^4) - 343/2*(-I*(-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a)
 - I*(-d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
))) - 21*(-d^10/b^4)^(1/4)*b*cos(b*x + a)*log(343*d^8 + 686*((-d^10/b^4)^(
1/4)*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^(3/4)*b^3*cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 21*(-d^10/b^4)^(1/4)*b*cos(b*x + a
)*log(343*d^8 - 686*((-d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)^2 - (-d^10/b^4)^
(3/4)*b^3*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))...
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.85

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\right.}{\sqrt{d}}\right.}{48\,bd^3}$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-1/48*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x +
a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 24*(d*tan
(b*x + a))^(3/2)*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*(d*tan(b*x + a))^(3/2
)*d^4)/(b*d^3)
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{1}{48}\left(\frac{24\sqrt{d\tan(bx+a)}d^2\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}-\frac{42\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}-\frac{42}{}\right.$$

input
```
integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
1/48*(24*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)
*b) - 42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) - 42*sqrt(2)*abs(d)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d) + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 21*sqrt(2)*abs(d)^(3/2)*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 3
2*sqrt(d*tan(b*x + a))*tan(b*x + a)/b)*d^2
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin\left(a+b\,x\right)^2\,\left(d\tan(a+b\,x)\right)^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2), x)
```

3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
2/3*d*(d*tan(b*x+a))^(3/2)/b
```

3.75.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx=\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.75.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{d\int \sqrt{d\tan(a+bx)}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.75.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.75.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$	17
default	$\frac{2d(d\tan(bx+a))^{\frac{3}{2}}}{3b}$	17

input
```
int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*(d*tan(b*x+a))^(3/2)/b
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\,\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a)/(b*cos(b*x + a))
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,(d\tan(bx+a))^{\frac{5}{2}}}{3\,b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(d*tan(b*x + a))^(5/2)/(b*tan(b*x + a))
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,\sqrt{d\tan(bx+a)}d^2\tan(bx+a)}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/b
```

3.75.9 Mupad [B] (verification not implemented)

Time = 3.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^2,x)
```

output
```
(2*d^2*sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2
))/(3*b*(cos(2*a + 2*b*x) + 1))
```

3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-2*d^3/b/(d*tan(b*x+a))^(1/2)+2/3*d*(d*tan(b*x+a))^(3/2)/b
```

3.76.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d(-1+3\cot^2(a+bx))\,(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*(-1 + 3*Cot[a + b*x]^2)*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.76.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \; 3071$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{3/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{3/2}}+\sqrt{d\tan(a+bx)}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(\frac{2}{3}(d\tan(a+bx))^{3/2}-\frac{2d^2}{\sqrt{d\tan(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^2)/Sqrt[d*Tan[a + b*x]] + (2*(d*Tan[a + b*x])^(3/2))/3))/b
```

3.76.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.76.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2\sqrt{d\tan(bx+a)}\, d^2(4\cot(bx+a)-\sec(bx+a)\csc(bx+a))}{3b}$	42

input
```
int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*(d*tan(b*x+a))^(1/2)*d^2*(4*cot(b*x+a)-sec(b*x+a)*csc(b*x+a))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(bx+a\right)^2-d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(4*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos
(b*x + a)*sin(b*x + a))
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\,d^3\left(\frac{3}{\sqrt{d\tan(bx+a)}} - \frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/3*d^3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/b
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{3}\,d^2\left(\frac{\sqrt{d\tan(bx+a)}\tan(bx+a)}{b} - \frac{3\,d}{\sqrt{d\tan(bx+a)}b}\right)$$

input
```
integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/3*d^2*(sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*d/(sqrt(d*tan(b*x + a))*b
))
```

3.76.9 Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

$$\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4\,d^2\,\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,{\sin\left(2\,a+2\,b\,x\right)}^2}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^4,x)
```

output
```
-(4*d^2*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2
*a + 2*b*x) + 1))^(1/2))/(3*b*sin(2*a + 2*b*x)^2)
```

3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d^5}{5b(d\tan(a+bx))^{5/2}} - \frac{4d^3}{b\sqrt{d\tan(a+bx)}} + \frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-4*d^3/b/(d*tan(b*x+a))^(1/2)-2/5*d^5/b/(d*tan(b*x+a))^(5/2)+2/3*d*(d*tan(
b*x+a))^(3/2)/b
```

3.77.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2d(-5+3\cot^2(a+bx)\,(9+\csc^2(a+bx)))\,(d\tan(a+bx))^{3/2}}{15b}$$

input
```
Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*(-5 + 3*Cot[a + b*x]^2*(9 + Csc[a + b*x]^2))*(d*Tan[a + b*x])^(3/2))
/(15*b)
```

3.77.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^6}dx$$

$$\Big\downarrow \text{ 3071}$$

$$\frac{d\int \frac{(\tan^2(a+bx)d^2+d^2)^2}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{7/2}}+\frac{2d^2}{(d\tan(a+bx))^{3/2}}+\sqrt{d\tan(a+bx)}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{2d^4}{5(d\tan(a+bx))^{5/2}}-\frac{4d^2}{\sqrt{d\tan(a+bx)}}+\frac{2}{3}(d\tan(a+bx))^{3/2}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^4)/(5*(d*Tan[a + b*x])^(5/2)) - (4*d^2)/Sqrt[d*Tan[a + b*x]] + (
2*(d*Tan[a + b*x])^(3/2))/3))/b
```

3.77.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.77.4 Maple [A] (verified)

Time = 73.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2\sec(bx+a)\left(\csc^3(bx+a)\right)\sqrt{d\tan(bx+a)}\,d^2\left(32\left(\cos^4(bx+a)\right)-40\left(\cos^2(bx+a)\right)+5\right)}{15b}$	55

input
```
int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/b*sec(b*x+a)*csc(b*x+a)^3*(d*tan(b*x+a))^(1/2)*d^2*(32*cos(b*x+a)^4-4
0*cos(b*x+a)^2+5)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{2\left(32\,d^2\cos\left(bx+a\right)^4-40\,d^2\cos\left(bx+a\right)^2+5\,d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{15\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(32*d^2*cos(b*x + a)^4 - 40*d^2*cos(b*x + a)^2 + 5*d^2)*sqrt(d*sin(b
*x + a)/cos(b*x + a))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2\,d^5\left(\frac{5\,(d\tan(bx+a))^{\frac{3}{2}}}{d^4}-\frac{3\left(10\,d^2\tan(bx+a)^2+d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}\,d^2}\right)}{15\,b}$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/15*d^5*(5*(d*tan(b*x + a))^(3/2)/d^4 - 3*(10*d^2*tan(b*x + a)^2 + d^2)/(
(d*tan(b*x + a))^(5/2)*d^2))/b
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2}{15}d^2\left(\frac{5\sqrt{d\tan(bx+a)}\tan(bx+a)}{b} - \frac{3\left(10\,d^3\tan(bx+a)^2+d^3\right)}{\sqrt{d\tan(bx+a)}bd^2\tan(bx+a)^2}\right)$$

input
```
integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
2/15*d^2*(5*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*(10*d^3*tan(b*x + a)^2
 + d^3)/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^2))
```

3.77.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.13

$$\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{32\,d^2\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\left(e^{a\,2i+b\,x\,2i}\,2i+e^{a\,4i+b\,x\,4i}\,3i+e^{a\,6i+b\,x\,6i}\,2i-e^{a\,8i+b\,x\,8i}\,2i-2i\right)}{15\,b\left(e^{a\,2i+b\,x\,2i}-1\right)^3\left(e^{a\,2i+b\,x\,2i}+1\right)}$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^6,x)
```

output
```
(32*d^2*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*2i -
exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i +
b*x*2i) + 1))
```

3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.78.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}+\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{4b} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
5/2*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+d^3*sin(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)+5/4*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*E
llipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1
/2)/b+2/3*d*sin(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b
```

3.78.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(120\sqrt[4]{-1}\cos(2(a+bx))\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(22\right.}{48b\tan^{\frac{3}{2}}(a+bx)\,(}$$

input
```
Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/48*(Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(120*(-1)^(1/4)*Cos[2*(a + b*x)]*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (22 + 77*Cos[2*(
a + b*x)] + 22*Cos[4*(a + b*x)] - Cos[6*(a + b*x)])*Sqrt[Sec[a + b*x]^2]*S
qrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Tan[a + b*x]^(3/2)*(-1 + Tan
[a + b*x]^2))
```

3.78.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3074, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\int \sin(a+bx)^3\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-3d^2\left(\frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\int\csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\int\frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$\big\downarrow$ 3081

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$\big\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)$$

$\big\downarrow$ 3053

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+}{3b\sqrt{d\tan(a}}\right.$$

$\big\downarrow$ 3042

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a+}{3b\sqrt{d\tan(a}}\right.$$

$\big\downarrow$ 3120

$$\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}-$$
$$3d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)-\frac{d\sin^3(a}{3b\sqrt{d\tan(}}\right.$$

3.78. $\int\sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

input
```
Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b) - 3*d^2*(-1/3*(d*Sin[a +
 b*x]^3)/(b*Sqrt[d*Tan[a + b*x]]) + (5*(-((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a
 + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b
*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/6)
```

3.78.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.78.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.73 (sec) , antiderivative size = 1840, normalized size of antiderivative = 13.43

method	result	size
default	Expression too large to display	1840

input
```
int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*tan(b*x+a)*(-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot
(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(
b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+6*I*EllipticPi((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1
/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x
+a)^2-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)
*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*EllipticPi((1+csc(b*x+a)-cot(b*x+a))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*cos(b*x+a)-6*EllipticPi
((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-cot(
b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/
2)*cos(b*x+a)^2+72*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+
a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc
(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+8*2^(1/2)*co
s(b*x+a)^4*sin(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(
b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+72*(cot(b*x+a)-csc(b*x+...
```

3.78.5 Fricas [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(d^2*cos(b*x + a)^2 - d^2)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan
(b*x + a)^2, x)
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)^3\,dx$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^3, x)
```

3.78.8 Giac [F(-2)]

Exception generated.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)^3\,(d\tan(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2), x)
```

3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{6b} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
5/3*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+5/6*d^2*csc(b*x+a)*(sin(a+1/4*Pi
+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(
2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*sin(b*x+a)*(d*tan(b*x+a))^(3
/2)/b
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)+(7+\right.}{6b\tan^{\frac{3}{2}}(a+bx)\left(-1+\tan^2(a+bx)\right)}$$

input
```
Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/6*(Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(10*(-1)^(1/4)*El
lipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (7 + 3*Cos[2*(a +
b*x)])*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b
*Tan[a + b*x]^(3/2)*(-1 + Tan[a + b*x]^2))
```

3.79.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 3074, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\Big\downarrow$ 3053

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

$\Big\downarrow$ 3120

$$\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{5}{3}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}\right)$$

input
```
Int[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b) - (5*d^2*(-((d*Sin[a + b*x
])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*
Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)))/3
```

3.79. $\int\sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.79.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.16

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(-5\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx\right.\right.}{}$

input
```
int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(-5*(cot(b*x+a)-csc(b*x+a))^(1/2)*(
-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF(
(1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+3*sin(b*x+a)*2^(1/
2)*cos(b*x+a)-5*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(
1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(
1/2),1/2*2^(1/2))+2*tan(b*x+a)*2^(1/2))*d^2/(cos(b*x+a)^2-1)*2^(1/2)
```

3.79.5 Fricas [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}} \sin(bx+a)\,dx$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a), x)
```

3.79.8 Giac [F(-2)]

Exception generated.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeDone
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \sin(a+b\,x)\,(d\tan(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)*(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)*(d*tan(a + b*x))^(5/2), x)
```

3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
1/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF
(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/
3*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Ta
n[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.80.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3074, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3074}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{1}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Int[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/3*(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]
*Sqrt[d*Tan[a + b*x]])/b + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

3.80.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.

Time = 1.63 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.69

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(-\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+}\right.\right.}{}$

input
```
int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+
a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+tan(b*x+a)*2^(1/2)
)*d^2/(cos(b*x+a)^2-1)*2^(1/2)
```

3.80.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{\sqrt{i}\,dd^2\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|\,-1)+\sqrt{-i}\,dd^2\cos(bx+a)\,F(\mathrm{ar}}{3\,b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
 + a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) -
 I*sin(b*x + a)), -1) + 2*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*
x + a))
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

3.80.8 Giac [F]

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

input
```
integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x), x)
```

3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-2/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Elliptic
F(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2
/3*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right.}{3b}$$

input
```
Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -
Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.81.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3073, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sq
rt[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b
)
```

3.81.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(95) = 190$.

Time = 2.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.69

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\,\sin(bx+a)\left(2\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a}\right.\right.}{}$

input
```
int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(d*tan(b*x+a))^(1/2)*sin(b*x+a)*(2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((
1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*(cot(b*x+a)-csc(b
*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1
/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+tan(b*x+a)*2^(1
/2))*d^2/(cos(b*x+a)^2-1)*2^(1/2)
```

3.81.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(\sqrt{i\,d}d^2\cos\left(bx+a\right)F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\sqrt{-i\,d}d^2\cos\left(bx+a\right)F(\arcsin\left(\cos\right.\right.}{3\,b\cos\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*d)*d^2*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x
 + a))
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

3.81.8 Giac [F]

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx=\int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{4d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-4/3*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-4/3*d^2*csc(b*x+a)*(sin(a+1/4*P
i+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin
(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)^3*(d*tan(b*x+a))
^(3/2)/b
```

3.82.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2d\csc^3(a+bx)\left(\cos(2(a+bx))\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\text{si}\right.}{3b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d*Csc[a + b*x]^3*(Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 2*(-1)^(1/4)
*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sin[2*(a + b*x)]*
Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(3*b*Sqrt[Sec[a + b*x]^2])
```

3.82.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3073, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^5}dx$$

$$\Big\downarrow \text{3073}$$

$$2d^2\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3079}$$

$$2d^2\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$2d^2\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3081}$$

$$2d^2\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3042

$$2d^2\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3053

$$2d^2\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3042

$$2d^2\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\downarrow$ 3120

$$2d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b) + 2*d^2*((-2*d*Csc[a + b
*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x
, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b))
```

3.82. $\int\csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(121) = 242.

Time = 20.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.35

method	result
default	$-\frac{\sqrt{d\tan(bx+a)}\, d^2\left(-4\cos(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,(\sin^3(bx+a)\right.}{}$

input
```
int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(d*tan(b*x+a))^(1/2)*d^2/(-1+cos(b*x+a))^2/(cos(b*x+a)+1)^2*(-4*cos
(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+
csc(b*x+a)-cot(b*x+a))^(1/2)*sin(b*x+a)^3*EllipticF((1+csc(b*x+a)-cot(b*x+
a))^(1/2),1/2*2^(1/2))-4*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*sin(b*x+a)^3*EllipticF((1+cs
c(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2)*sin(b*x+a)^2-
sin(b*x+a)*tan(b*x+a)*2^(1/2))*2^(1/2)
```

3.82.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{2\left(2\left(d^2\cos(bx+a)^3-d^2\cos(bx+a)\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+2\left(d^2\cos(bx\right.\right.}{3\left(b\cos(bx\right.}$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqrt(I*d)*elliptic_f(arcsi
n(cos(b*x + a) + I*sin(b*x + a)), -1) + 2*(d^2*cos(b*x + a)^3 - d^2*cos(b*
x + a))*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
 (2*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*
x + a)^3 - b*cos(b*x + a))
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.82.7 Maxima [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

3.82.8 Giac [F]

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^5\,dx$$

input
```
integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5, x)
```

3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

output
```
-40/21*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-20/21*d^3*csc(b*x+a)^3/b/(d*t
an(b*x+a))^(1/2)-40/21*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/
4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan
(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b
```

3.83.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$

$$-\frac{d^2\csc(a+bx)\left((1+10\cos(2(a+bx))-5\cos(4(a+bx)))\csc^3(a+bx)\sec(a+bx)\sqrt{\sec^2(a+bx)}+80\sqrt[4]{-\cdots}\right.}{21b\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/21*(d^2*Csc[a + b*x]*((1 + 10*Cos[2*(a + b*x)] - 5*Cos[4*(a + b*x)])*Cs
c[a + b*x]^3*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2] + 80*(-1)^(1/4)*EllipticF[I
*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[d*Ta
n[a + b*x]])/(b*Sqrt[Sec[a + b*x]^2])
```

3.83.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3073, 3042, 3079, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{5/2}}{\sin(a+bx)^7}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{10}{3}d^2\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}d^2\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^5}dx+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3079}$$

$$\frac{10}{3}d^2\left(\frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{3}d^2\left(\frac{6}{7}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3079

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right)+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3081

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right) \quad \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}\right) \quad \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3053

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a}{7b\sqrt{d\tan}} \quad \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc^3(a}{7b\sqrt{d\tan}} \quad \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

$\Big\downarrow$ 3120

3.83. $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

$$\frac{10}{3}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}\right)-\frac{2d\csc}{7b\sqrt{d}}\right.$$

$$\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

input
```
Int[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*d*Csc[a + b*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b) + (10*d^2*((-2*d*Csc[a +
 b*x]^3)/(7*b*Sqrt[d*Tan[a + b*x]]) + (6*((-2*d*Csc[a + b*x])/(3*b*Sqrt[d*
Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a
 + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)))/7))/3
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(147) = 294$.

Time = 151.87 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.26

method	result
default	$\sin(bx+a)\tan(bx+a)\left(40\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\text{co}\right.\right.$

input
```
int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/b*sin(b*x+a)*tan(b*x+a)*(40*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x
+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x
+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^4*sin(b*x+a)+40*(1+csc(b*x+a
)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)
^3*sin(b*x+a)-40*sin(b*x+a)*cos(b*x+a)^2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(
cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-40*sin(b*x+a)*cos(b*x+a)*(1+csc
(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot
(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-20*2
^(1/2)*cos(b*x+a)^4+30*cos(b*x+a)^2*2^(1/2)-7*2^(1/2))*(d*tan(b*x+a))^(1/2
)*d^2/(-1+cos(b*x+a))^3/(cos(b*x+a)+1)^3*2^(1/2)
```

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx =$$
$$-\frac{2\left(20\left(d^2\cos(bx+a)^5-2\,d^2\cos(bx+a)^3+d^2\cos(bx+a)\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))}{}$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/21*(20*(d^2*cos(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*s
qrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 20*(d^2*c
os(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sqrt(-I*d)*ellipt
ic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (20*d^2*cos(b*x + a)^4 -
 30*d^2*cos(b*x + a)^2 + 7*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(
b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**7*(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

3.83.8 Giac [F]

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int (d\tan(bx+a))^{\frac{5}{2}}\csc(bx+a)^7\,dx$$

input
```
integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \int \frac{(d\tan(a+b\,x))^{5/2}}{\sin(a+b\,x)^7}\,dx$$

input
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7,x)
```

output
```
int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7, x)
```

3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}}+\frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b\sqrt{d}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b\sqrt{d}} - \frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}$$

output
```
-5/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+5/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-5/128*l
n(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/
2)+5/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(
1/2)/d^(1/2)-5/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d-1/4*cos(b*x+a)^4*(
d*tan(b*x+a))^(5/2)/b/d^3
```

3.84.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.47

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$=\frac{\sec(a+bx)\left(-7\sin(a+bx)-5\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+5\log\left(\cos(a+bx)+\right.\right.}{64b\sqrt{d\tan(a+bx}}$$

input
```
Integrate[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sec[a + b*x]*(-7*Sin[a + b*x] - 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqr
t[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 6*Sin[3*(a + b*x)] + Sin[5*(a + b*x)]))/(
64*b*Sqrt[d*Tan[a + b*x]])
```

3.84.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{7/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{5}{8}\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \; 252$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \; 266$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \; 755$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{5/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\cdots}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}\cdots}{2d}\cdots\right.\right.\right.}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\cdots}{2(\cdots}\right.\right.}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\cdots}{4(\cdots}\right.}{b}$$

$$\Big\downarrow \; 1479$$

$$\frac{d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+b\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.\right.\right.}{b}$$

$$\Big\downarrow \; 25$$

3.84. $\displaystyle\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right.\right.\right.$$
$$b$$

$\downarrow$ 27

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right.\right.\right.$$
$$b$$

$\downarrow$ 1103

$$d\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.\right.$$
$$b$$

input
```
Int[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(5/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (5*(((-(Arc
Tan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt
[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]
*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sq
rt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d
))/2 - Sqrt[d*Tan[a + b*x]]/(2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.84. $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.84.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.

Time = 9.16 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.35

method	result
default	$\frac{\sin(bx+a)\left(16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\,(\cos^4(bx+a))+16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\,(\cos^3(bx+a))-36\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}}{}$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*sin(b*x+a)*(16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(
1/2)*cos(b*x+a)^4+16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/
2)*cos(b*x+a)^3-36*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1/2)
*cos(b*x+a)^2-36*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)-5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x
+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot
(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*
x+a)))+5*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)
^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*
x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a
)))-10*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+10*arctan((-sin(b*x+a)*2^(1/2)*(-co
s(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))
)/(cos(b*x+a)+1)/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*2^(1/2)
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.47

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/256*(5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x +
 a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*
x + a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*
x + a)/cos(b*x + a)) + 1) - 5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1
/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(b^3*d*(-1/(b
^4*d^2))^(3/4)*cos(b*x + a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*
x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 5*I*b*d*(-1/(b^4*d^2))^(1
/4)*log(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x
+ a)^2 - 2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - I*b*(-1/(b^4*d^2
))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)
 - 5*I*b*d*(-1/(b^4*d^2))^(1/4)*log(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x +
a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(
b*x + a)^2 + I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a)) + 1) + 5*b*d*(-1/(b^4*d^2))^(1/4)*log(2*(b^3*d*(-
1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*si
n(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - 5*b*d*(-1/(b^4*d^2))^
(1/4)*log(-2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))
^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) +
 5*I*b*d*(-1/(b^4*d^2))^(1/4)*log(-2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x
 + a)^2 + I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*si...
```

3.84.6 Sympy [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^4{(a+bx)}}{\sqrt{d\tan{(a+bx)}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.86

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{10\sqrt{2}d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{9}{2}}\log\left(d\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/128*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*
x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*(9*(d*tan(
b*x + a))^(5/2)*d^6 + 5*sqrt(d*tan(b*x + a))*d^8)/(d^4*tan(b*x + a)^4 + 2*
d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.96

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd} + \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd} - \frac{9\sqrt{d\tan\left(bx+a\right)}d^3\tan\left(bx+a\right)^2+5\sqrt{d\tan\left(bx+a\right)}d^3}{16\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2b}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
5/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 5/64*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d) + 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*ta
n(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 5/128*sqrt(2)*sqrt(abs(d))*log(
d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d)
 - 1/16*(9*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^2 + 5*sqrt(d*tan(b*x + a)
)*d^3)/((d^2*tan(b*x + a)^2 + d^2)^2*b)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2), x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b\sqrt{d}} - \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

output
```
-1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+1/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-1/16*ln(d
^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)+
1/16*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)
/d^(1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d
```

3.85.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx =$$
$$-\frac{\sec(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin\right.\right.}{8b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/8*(Sec[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqr
t[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)]))/(b*Sqrt[d*Tan[a + b*x]])
```

3.85.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{1}{4}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{}}{2d}\right.\right.}{b}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{\sqrt{}}{2(d^2\,\mathrm{ta}}\right.}{b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)-\frac{\sqrt{d\tan(a+bx)}}{2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{}\right.\right.}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\mathrm{ar}}{2d}\right.\right.}{b}$$

3.85. $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\ldots}{2d}\right.\right.}{b}$$

$\Big\downarrow$ 1103

$$\frac{d\left(\frac{1}{2}\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)\ldots\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*(((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[
d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) +
 Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqr
t[d]))/(2*d))/2 - Sqrt[d*Tan[a + b*x]]/(2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.85. $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.85.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(171) = 342$.

Time = 1.73 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.31

method	result
default	$-\frac{\sin(bx+a)\left(4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^2(bx+a))+4\cos(bx+a)\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+2\arctan\left(\frac{\sin(bx+a)\sqrt{2}\,\sqrt{-\frac{\cos(bx+ ...}{(\cos(...}}}{-1+c ...}\right.\right.}{}$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*sin(b*x+a)*(4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2)*cos(b*x+a)^2+4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+
a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)
*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x
+a)))-ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+
3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a
)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))
+ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot
(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc
(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))/(co
s(b*x+a)+1)/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)
^(1/2)*2^(1/2)
```

3.85.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 877, normalized size of antiderivative = 3.86

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
1/32*(b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)
*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x +
 a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + 1) - b*d*(-1/(b^4*d^2))^(1/4)*log(2*b^2*d*sqrt(-1/(b^4
*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(b^3*d*(-1/(b^4*d^
2))^(3/4)*cos(b*x + a)^2 + b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + I*b*d*(-1/(b^4*d^2))^(1/4)*log
(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2
- 2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - I*b*(-1/(b^4*d^2))^(1/4
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - I*b*
d*(-1/(b^4*d^2))^(1/4)*log(-2*b^2*d*sqrt(-1/(b^4*d^2))*cos(b*x + a)*sin(b*
x + a) - 2*cos(b*x + a)^2 - 2*(-I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^
2 + I*b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a
)/cos(b*x + a)) + 1) + b*d*(-1/(b^4*d^2))^(1/4)*log(2*(b^3*d*(-1/(b^4*d^2)
)^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))
*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - b*d*(-1/(b^4*d^2))^(1/4)*log(-2*
(b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 - b*(-1/(b^4*d^2))^(1/4)*cos(b*
x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) + I*b*d*(-1/(b
^4*d^2))^(1/4)*log(-2*(I*b^3*d*(-1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + I*b*(
-1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(...
```

3.85.6 Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd} + \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd} - \frac{\sqrt{d\tan(bx+a)}d}{2\left(d^2\tan(bx+a)^2+d^2\right)b}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 1/8*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(
b*d) + 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/16*sqrt(2)*sqrt(abs(d))*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1
/2*sqrt(d*tan(b*x + a))*d/((d^2*tan(b*x + a)^2 + d^2)*b)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2), x)
```

3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

3.86.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{5/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

3.86.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.86.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$	17
default	$-\frac{2d}{3b(d\tan(bx+a))^{\frac{3}{2}}}$	17

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

3.86.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^2}{3\left(bd\cos{(bx+a)}^2-bd\right)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2/(b*d*cos(b*x + a)^2 -
 b*d)
```

3.86.6 Sympy [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan(bx+a)}b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2}{3\sqrt{d\tan(bx+a)}b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$= -\frac{2\sqrt{\frac{d\sin(2a+2bx)}{\cos(2a+2bx)+1}}\,(\cos(2a+2bx)+2\cos(4a+4bx)-\cos(6a+6bx)-2)}{3bd\,(15\cos(2a+2bx)-6\cos(4a+4bx)+\cos(6a+6bx)-10)}$$

input
```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2)),x)
```

output
```
-(2*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
+ 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))/(3*b*d*(15*cos(2*a + 2*b*x)
- 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=-\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\frac{2d(-5+2\cos(2(a+bx)))\csc^2(a+bx)}{21b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-5 + 2*Cos[2*(a + b*x)])*Csc[a + b*x]^2)/(21*b*(d*Tan[a + b*x])^(3/2
))
```

3.87.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^4\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{9/2}}+\frac{1}{(d\tan(a+bx))^{5/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^2}{7(d\tan(a+bx))^{7/2}}-\frac{2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^2)/(7*(d*Tan[a + b*x])^(7/2)) - 2/(3*(d*Tan[a + b*x])^(3/2))))/b
```

3.87.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.87.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\frac{8\left(\cot^3(bx+a)\right)}{21}-\frac{2\cot(bx+a)\left(\csc^2(bx+a)\right)}{3}}{\sqrt{d\tan(bx+a)}\,b}$	43

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^3-7*cot(b*x+a)*csc(b*x+a)^2)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(4\cos\left(bx+a\right)^4-7\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\left(bd\cos\left(bx+a\right)^4-2\,bd\cos\left(bx+a\right)^2+bd\right)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(4*cos(b*x + a)^4 - 7*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
))/(b*d*cos(b*x + a)^4 - 2*b*d*cos(b*x + a)^2 + b*d)
```

3.87.6 Sympy [F]

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(7\,d^2\tan\left(bx+a\right)^2+3\,d^2\right)d}{21\,\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/21*(7*d^2*tan(b*x + a)^2 + 3*d^2)*d/((d*tan(b*x + a))^(7/2)*b)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=-\frac{2\left(7\,d^3\tan\left(bx+a\right)^2+3\,d^3\right)}{21\,\sqrt{d\tan\left(bx+a\right)}bd^3\tan\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/21*(7*d^3*tan(b*x + a)^2 + 3*d^3)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x +
 a)^3)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 530, normalized size of antiderivative = 12.33

$$\begin{aligned}\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=&\frac{344\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{105\,b\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}\\&+\frac{40\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{21\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}\\&+\frac{24\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{35\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}\\&-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,304\mathrm{i}}{105\,b\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}\\&+\frac{16\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}\\&+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,144\mathrm{i}}{35\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}\\&-\frac{16\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}\end{aligned}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2)),x)
```

output
```
(344*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(105*b*d*(exp(a*2i + b*x*2i) - 1)) + (40*(exp(a*2i
 + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1
))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35
*b*d*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*304i)/(105*b*d*(exp(
a*2i + b*x*2i)*1i - 1i)) + (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2
i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d*(exp(a*2i + b*x*2i)*1i -
 1i)^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

3.87. $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

output
```
-2/11*d^5/b/(d*tan(b*x+a))^(11/2)-4/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(
d*tan(b*x+a))^(3/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2d(-45+28\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{231b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*d*(-45 + 28*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(231
*b*(d*Tan[a + b*x])^(3/2))
```

3.88.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^6\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{13/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{13/2}}+\frac{2d^2}{(d\tan(a+bx))^{9/2}}+\frac{1}{(d\tan(a+bx))^{5/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{11(d\tan(a+bx))^{11/2}}-\frac{4d^2}{7(d\tan(a+bx))^{7/2}}-\frac{2}{3(d\tan(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(d*((-2*d^4)/(11*(d*Tan[a + b*x])^(11/2)) - (4*d^2)/(7*(d*Tan[a + b*x])^(7
/2)) - 2/(3*(d*Tan[a + b*x])^(3/2))))/b
```

3.88.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.88.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{2\cot(bx+a)\left(\csc^4(bx+a)\right)\left(32\left(\cos^4(bx+a)\right)-88\left(\cos^2(bx+a)\right)+77\right)}{231b\sqrt{d\tan(bx+a)}}$	52

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231/b*cot(b*x+a)*csc(b*x+a)^4*(32*cos(b*x+a)^4-88*cos(b*x+a)^2+77)/(d*t
an(b*x+a))^(1/2)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\left(32\cos\left(bx+a\right)^6-88\cos\left(bx+a\right)^4+77\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{231\left(bd\cos\left(bx+a\right)^6-3\,bd\cos\left(bx+a\right)^4+3\,bd\cos\left(bx+a\right)^2-bd\right)}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/231*(32*cos(b*x + a)^6 - 88*cos(b*x + a)^4 + 77*cos(b*x + a)^2)*sqrt(d*s
in(b*x + a)/cos(b*x + a))/(b*d*cos(b*x + a)^6 - 3*b*d*cos(b*x + a)^4 + 3*b
*d*cos(b*x + a)^2 - b*d)
```

3.88.6 Sympy [F]

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**6/sqrt(d*tan(a + b*x)), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^4\tan\left(bx+a\right)^4+66\,d^4\tan\left(bx+a\right)^2+21\,d^4\right)d}{231\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/231*(77*d^4*tan(b*x + a)^4 + 66*d^4*tan(b*x + a)^2 + 21*d^4)*d/((d*tan(
b*x + a))^(11/2)*b)
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(77\,d^5\tan\left(bx+a\right)^4+66\,d^5\tan\left(bx+a\right)^2+21\,d^5\right)}{231\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2/231*(77*d^5*tan(b*x + a)^4 + 66*d^5*tan(b*x + a)^2 + 21*d^5)/(sqrt(d*ta
n(b*x + a))*b*d^5*tan(b*x + a)^5)
```

3.88.9 Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 831, normalized size of antiderivative = 12.78

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(1/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*44864i)/(10395*b*d*(exp(a*2i + b*x*2i)*1i - 1i)) - (128
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(35*b*d*(exp(a*2i + b*x*2i) - 1)^2) - (7136*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1)
)^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i) - 1)^3) - (1216*(exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(231*b*d*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*
(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d*(ex
p(a*2i + b*x*2i) - 1)^5) - (41984*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d*(exp(a*2i
+ b*x*2i) - 1)) - (3904*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i)*1
i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2)*1088i)/(165*b*d*(exp(a*2i + b*x*2i)*1i - 1
i)^3) + (320*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4) +
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*1600i)/(99*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*(e
xp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b...
```

3.88. $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}} - \frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}} + \frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-7/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-7/30
*d*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)-1/5*d*sin(b*x+a)^5/b/(d*tan(b*x+a))
^(3/2)
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.94 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sin(a+bx)\left(-20\sin(2(a+bx))+3\sin(4(a+bx))+28\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{s}\right.}{120b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sin[a + b*x]*(-20*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + 28*Hypergeometr
ic2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]))/
(120*b*Sqrt[d*Tan[a + b*x]])
```

3.89.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{10}\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/5*(d*Sin[a + b*x]^5)/(b*(d*Tan[a + b*x])^(3/2)) + (7*(-1/3*(d*Sin[a + b
*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a +
b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/10
```

3.89.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(118) = 236$.

Time = 1.12 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\left(12\left(\cos^{5}(bx+a)\right)\sqrt{2}-38\left(\cos^{3}(bx+a)\right)\sqrt{2}+42\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\right.}{}$

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/b/(d*tan(b*x+a))^(1/2)*(12*cos(b*x+a)^5*2^(1/2)-38*cos(b*x+a)^3*2^(
1/2)+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(
cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
*2^(1/2))-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2
)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2
),1/2*2^(1/2))+42*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-
cot(b*x+a))^(1/2),1/2*2^(1/2))-21*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/
2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elliptic
F((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+47*2^(1/2)*cos(b*x+a)-21*2^
(1/2))*2^(1/2)
```

3.89.5 Fricas [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(
b*x + a)/(d*tan(b*x + a)), x)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^5}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

3.89.8 Giac [F]

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{\sin\left(bx+a\right)^5}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{\sin\left(a+b\,x\right)^5}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2), x)
```

3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/3*d
*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$
$$=\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(\right.\right.}{12bd\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b
*x)])) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*
Tan[a + b*x]))/(12*b*d*Sqrt[Sec[a + b*x]^2])
```

3.90.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
-1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 +
 b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.90.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.90.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(94) = 188.

Time = 0.99 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.77

method	result
default	$\frac{\left(2(\cos^3(bx+a))\sqrt{2}-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(b\right.\right.}{}$

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b/(d*tan(b*x+a))^(1/2)*(2*cos(b*x+a)^3*2^(1/2)-6*(1+csc(b*x+a)-cot(b*
x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)
*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(cot(b*x+a)-csc(
b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(
1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sec(b*x+a)*(
1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a
)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))
+3*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(
1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/
2),1/2*2^(1/2))-5*2^(1/2)*cos(b*x+a)+3*2^(1/2))*2^(1/2)
```

3.90.5 Fricas [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*
x + a)), x)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.90.8 Giac [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)^3}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2), x)
```

3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.91.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\sin(a+bx)\sqrt{d\tan(a+bx)}}{3bd}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*
Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)
```

3.91.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt
[d*Tan[a + b*x]])
```

3.91.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(69) = 138$.

Time = 0.83 (sec) , antiderivative size = 363, normalized size of antiderivative = 7.72

method	result
default	$-\frac{\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{\mathrm{co}}\cdots\right.}{\cdots}$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(d*tan(b*x+a))^(1/2)*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a
)-cot(b*x+a))^(1/2),1/2*2^(1/2))-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x
+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^
(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellip
ticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-sec(b*x+a)*(1+csc(b*x+a)
-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2)*cos
(b*x+a)-2^(1/2))*2^(1/2)
```

3.91.5 Fricas [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x + a)), x)
```

3.91.6 Sympy [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

3.91.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.91.8 Giac [F]

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(bx+a)}{\sqrt{d\tan(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\sin(a+b\,x)}{\sqrt{d\tan(a+b\,x)}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(1/2), x)
```

3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.92.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=-\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a
)^(1/2)/(d*tan(b*x+a))^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=$$
$$-\frac{2\cos(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*Cos[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*Sqrt[d*Tan[a + b*x]])
```

3.92.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3050}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3052}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*S
qrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt
[Sin[2*a + 2*b*x]])))/(Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.92.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.92.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(91) = 182$.

Time = 0.88 (sec) , antiderivative size = 349, normalized size of antiderivative = 4.85

method	result
default	$-\frac{\left(-2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sqrt{}\right.}{}$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(d*tan(b*x+a))^(1/2)*(-2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))+(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellipt
icE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a)-
cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2))*2^(
1/2)
```

3.92.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.35

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx =$$
$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^2+i\sqrt{i\,d}E(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|\,-1)\sin{(bx+a)}-i\sqrt{-i\,d}}{}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*ellipti
c_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)
*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sq
rt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
 + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(
b*x + a))/(b*d*sin(b*x + a))
```

3.92.6 Sympy [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

3.92.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.92.8 Giac [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc\left(bx+a\right)}{\sqrt{d\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)), x)
```

3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}} - \frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}} - \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-4/5*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin
(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x
+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*d*csc(b*x+a)/b/(d*tan(b*x+a))^(3/2)
```

3.93.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{6(-2+\cos(2(a+bx)))\cot(a+bx)\csc(a+bx)\sqrt{\sec^2(a+bx)} - 8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a\right.}{15b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(6*(-2 + Cos[2*(a + b*x)])*Cot[a + b*x]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]
- 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a +
 b*x]^2)/(15*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])
```

3.93.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3079, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3050}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

output
```
(-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) + (2*Sqrt[Sin[a + b*x]]*(
(-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*Ell
ipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))
/(5*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(113) = 226$.

Time = 1.03 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.63

method	result
default	$\frac{\left(4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-2\sqrt{\cot(}\right.}{}$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/b/(d*tan(b*x+a))^(1/2)*(4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)
-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+4*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))
^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x
+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x
+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*2^(1/2
)-cot(b*x+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

3.93.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.32

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = -\frac{2\left(\left(i\cos{(bx+a)}^2-i\right)\sqrt{i\,d}E(\arcsin{(\cos{(bx+a)}+i\sin{(bx+a)})}\,|-1)\sin{(bx+a)}+\left(-i\cos{(bx+a}\right.\right.}{}$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellip
tic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*
x + a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)),
-1)*sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos
(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d*cos(b*x + a)^2 - b*d)
*sin(b*x + a))
```

3.93.6 Sympy [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc^3{(a+bx)}}{\sqrt{d\tan{(a+bx)}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**3/sqrt(d*tan(a + b*x)), x)
```

3.93.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc{(bx+a)}^3}{\sqrt{d\tan{(bx+a)}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.93.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \int \frac{\csc{(bx+a)}^3}{\sqrt{d\tan{(bx+a)}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)^3\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

input

```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)),x)
```

output

```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)), x)
```

3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{3/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{3/2}} + \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}$$

output
```
-3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/128*l
n(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/
2)-3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(
3/2)*2^(1/2)+3/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3-1/4*cos(b*x+a)^4
*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.94.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(\cos(a+bx)-2\cos(3(a+bx))+\cos(5(a+bx))-3\arcsin(\cos(a+\right.}{}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]*(Cos[a + b*x] - 2*Cos[3*(a + b*x)] + Cos[5*(a + b*x)] - 3*Ar
cSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b
*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[
d*Tan[a + b*x]])/(64*b*d^2)
```

3.94.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{(d\tan(a+bx))^{5/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{3}{8}\int \frac{\sqrt{d\tan(a+bx)}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$$\Big\downarrow \text{253}$$

$$\frac{d\left(\frac{3}{8}\left(\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{4d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{3}{8}\left(\frac{\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 826

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{(d\tan(a+bx))^{3/2}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx}{d^4\tan^4(a+bx)+d}}{2d^2}}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{}{2d^2}\right.\right.}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{}{4(}\right.}{b}$$

$\downarrow$ 1479

3.94. $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+b\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 25

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 27

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 1103

$$d\left(\frac{3}{8}\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2\right.}{2\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.\right.}{b}$$

input
```
Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(-1/4*(d*Tan[a + b*x])^(3/2)/(d^2 + d^2*Tan[a + b*x]^2)^2 + (3*(((-(Arc
Tan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt
[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*
Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d
^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2)/(2*d^2)
+ (d*Tan[a + b*x])^(3/2)/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2))))/8))/b
```

3.94. $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.94. $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.94.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(197) = 394$.

Time = 10.06 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.42

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(16\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\left(\cos^3(bx+a)\right)\sin(bx+a)+16\left(\cos^2(bx+a)\right)\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(\ldots}{(\cos(bx+a)\ldots}}\right.}{}$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*csc(b*x+a)*(-1+cos(b*x+a))*(16*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)*cos(b*x+a)^3*sin(b*x+a)+16*cos(b*x+a)^2*sin(b*x+a)*2^
(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-12*cos(b*x+a)*sin(b*
x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-12*sin(b*x+a)
*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-6*arctan((sin(b*x
+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/
(-1+cos(b*x+a)))-6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))-3*ln(-(cot(b*x+a)*cos(b
*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3
*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b
*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+
a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*co
t(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+
a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a))))/(-cos(b*x+a)*sin(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))^(1/2)/d*2^(1/2)
```

3.94.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.84

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/256*(3*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/
2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*
cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*
d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) - 3*b*d^2*(-1/(b^4*d^6))
^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/(b^4*d^6))^(3/
4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)
*sqrt(-1/(b^4*d^6))) - 3*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)
*sin(b*x + a) + 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + I*b*d
*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b
*x + a)) + 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) +
3*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I
*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - I*b*d*(-1/(b^4*d^6))^(1/4)*
cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*
d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + 3*b*d^2*(-1/(b^4*d^6))
^(1/4)*log(2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d
*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) +
1) - 3*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos
(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*s
in(b*x + a)/cos(b*x + a)) + 1) + 3*I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*...
```

3.94.6 Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.88

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log(}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*ta
n(b*x + a))^(7/2)*d^4 - (d*tan(b*x + a))^(3/2)*d^6)/(d^4*tan(b*x + a)^4 +
2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}-\frac{3\sqrt{}}{}}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/128*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 6*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d
)))/(b*d^2) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*t
an(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 3*sqrt(2)*abs(d)^(3/2)*log(d
*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2
) + 8*(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 - sqrt(d*tan(b*x + a))*d^
3*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2), x)
```

3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

output
```
-1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/16*ln(d
^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)-
1/16*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)
*2^(1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.95.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.46

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\arcsin(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+\cdots\right.\right.}{8bd^2}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/8*((ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + Csc[a + b*x]*Log
[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sqrt[Sin[2*(a +
 b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(b*d^2)
```

3.95.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\sqrt{d\tan(a+bx)}}{(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{253}$$

$$\frac{d\left(\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{4d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{826}$$

$$\frac{d\left(\frac{\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{\cdots}}{2d^2}\right.}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan\cdots}{2d^2(d^2\tan\cdots}\right.}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{(d\tan(a+bx))^{3/2}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\cdots\right.}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

$$\Big\downarrow \text{25}$$

3.95. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

$$\frac{d\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{d\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+b\right.}{2\sqrt{2}\sqrt{d}}\right.}{2d^2}\right.}{b}$$

input
```
Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*(((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2)/(2*d^2) + (d*Tan[a + b*x])^(3/2)/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2))))/b
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.95. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.95.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(171) = 342$.

Time = 13.24 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\csc(bx+a)(-1+\cos(bx+a))\left(4\cos(bx+a)\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+4\sin(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+\ln\left(\frac{2\sin(\ldots}{\ldots}\right.\right.}{\ldots}$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*csc(b*x+a)*(-1+cos(b*x+a))*(4*cos(b*x+a)*sin(b*x+a)*2^(1/2)*(-cos(
b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+4*sin(b*x+a)*2^(1/2)*(-cos(b*x+a
)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot
(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc
(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-
csc(b*x+a)-2)/(-1+cos(b*x+a)))-ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*s
in(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)
^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b
*x+a)+2)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+2*arctan((sin(
b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-
1)/(-1+cos(b*x+a))))/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*ta
n(b*x+a))^(1/2)/d*2^(1/2)
```

3.95.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 971, normalized size of antiderivative = 4.28

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(
b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos
(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3
*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) - b*d^2*(-1/(b^4*d^6))^(1/4
)*log(1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*co
s(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*
sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt
(-1/(b^4*d^6))) - I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*
x + a) + 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + I*b*d*(-1/(b
^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)
) + 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + I*b*d^2
*(-1/(b^4*d^6))^(1/4)*log(1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^4*
(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x +
 a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3*cos(b
*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6))) + b*d^2*(-1/(b^4*d^6))^(1/4)*log(
2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d
^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - b*d^2*
(-1/(b^4*d^6))^(1/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin
(b*x + a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/c
os(b*x + a)) + 1) + I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(I*b^3*d^4*(-1/...
```

3.95.6 Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d \cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*
x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(b*x
+ a))^(3/2)*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\frac{8\sqrt{d\tan(bx+a)}d\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{\cdots}\right.}{\cdots}\right.}{bd\cdots}\cdots}{\cdots}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/16*(8*sqrt(d*tan(b*x + a))*d*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*b)
 + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 2*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
(b*d^2) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x
 + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2))/d
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

3.96.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(a+bx)^2(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{7/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 15$$

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

3.96.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.96.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$	17
default	$-\frac{2d}{5b(d\tan(bx+a))^{\frac{5}{2}}}$	17

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^3}{5\left(bd^2\cos{(bx+a)}^2-bd^2\right)\sin{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^3/((b*d^2*cos(b*x + a)^
2 - b*d^2)*sin(b*x + a))
```

3.96.6 Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,(d\tan{(bx+a)})^{\frac{3}{2}}\,b\tan{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/5/((d*tan(b*x + a))^(3/2)*b*tan(b*x + a))
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{5\,\sqrt{d\tan{(bx+a)}}bd\tan{(bx+a)}^2}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/5/(sqrt(d*tan(b*x + a))*b*d*tan(b*x + a)^2)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 381, normalized size of antiderivative = 19.05

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,14i}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,8i}{15\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^2} - \frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,32i}{15\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2} + \frac{8\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}$$

input

```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

output

```
(8*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*8i)/(15*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - (16*(exp(a*2i + b*x*
2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*
d^2*(exp(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*14i)/(5*b*d^2*(ex
p(a*2i + b*x*2i) - 1))
```

3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2(4+\csc^2(a+bx)-5\csc^4(a+bx))}{45bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(4 + Csc[a + b*x]^2 - 5*Csc[a + b*x]^4))/(45*b*d*Sqrt[d*Tan[a + b*x]])
```

3.97.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^4(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{11/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{11/2}}+\frac{1}{(d\tan(a+bx))^{7/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^2}{9(d\tan(a+bx))^{9/2}}-\frac{2}{5(d\tan(a+bx))^{5/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^2)/(9*(d*Tan[a + b*x])^(9/2)) - 2/(5*(d*Tan[a + b*x])^(5/2))))/b
```

3.97.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.97.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\frac{8\left(\cot^{4}(bx+a)\right)}{45}-\frac{2\left(\cot^{2}(bx+a)\right)\left(\csc^{2}(bx+a)\right)}{5}}{bd\sqrt{d\tan(bx+a)}}$	48

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b/(d*tan(b*x+a))^(1/2)/d*(4*cot(b*x+a)^4-9*cot(b*x+a)^2*csc(b*x+a)^2)
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.95

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\frac{2\left(4\cos\left(bx+a\right)^{5}-9\cos\left(bx+a\right)^{3}\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\left(bd^{2}\cos\left(bx+a\right)^{4}-2\,bd^{2}\cos\left(bx+a\right)^{2}+bd^{2}\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/45*(4*cos(b*x + a)^5 - 9*cos(b*x + a)^3)*sqrt(d*sin(b*x + a)/cos(b*x + a
))/((b*d^2*cos(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)*sin(b*x + a))
```

3.97.6 Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^4{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^2\tan{(bx+a)}^2+5\,d^2\right)d}{45\,(d\tan{(bx+a)})^{\frac{9}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/45*(9*d^2*tan(b*x + a)^2 + 5*d^2)*d/((d*tan(b*x + a))^(9/2)*b)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(9\,d^4\tan{(bx+a)}^2+5\,d^4\right)}{45\,\sqrt{d\tan{(bx+a)}}bd^5\tan{(bx+a)}^4}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/45*(9*d^4*tan(b*x + a)^2 + 5*d^4)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
 a)^4)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 684, normalized size of antiderivative = 15.91

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*6088i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*4024i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) + ((exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2)*200i)/(63*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) + ((exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(
63*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) + (1184*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(189*b*d^2
*(exp(a*2i + b*x*2i)*1i - 1i)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4192i)/(945*b*d^2*(exp
(a*2i + b*x*2i)*1i - 1i)^2) - (2176*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(315*b*d^2*(exp(a*2
i + b*x*2i)*1i - 1i)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2
i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*512i)/(63*b*d^2*(exp(a*2i + b
*x*2i)*1i - 1i)^4) + (32*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*d^2*(exp(a*2i + b*x*2i)*1
i - 1i)^5)
```

3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}} - \frac{4d^3}{9b(d\tan(a+bx))^{9/2}} - \frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

output
```
-2/13*d^5/b/(d*tan(b*x+a))^(13/2)-4/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(
d*tan(b*x+a))^(5/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{64+16\csc^2(a+bx)+10\csc^4(a+bx)-90\csc^6(a+bx)}{585bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(64 + 16*Csc[a + b*x]^2 + 10*Csc[a + b*x]^4 - 90*Csc[a + b*x]^6)/(585*b*d*
Sqrt[d*Tan[a + b*x]])
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^6(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{15/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{15/2}}+\frac{2d^2}{(d\tan(a+bx))^{11/2}}+\frac{1}{(d\tan(a+bx))^{7/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{13(d\tan(a+bx))^{13/2}}-\frac{4d^2}{9(d\tan(a+bx))^{9/2}}-\frac{2}{5(d\tan(a+bx))^{5/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*((-2*d^4)/(13*(d*Tan[a + b*x])^(13/2)) - (4*d^2)/(9*(d*Tan[a + b*x])^(9
/2)) - 2/(5*(d*Tan[a + b*x])^(5/2))))/b
```

3.98.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.98.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\left(\cot^{2}\left(bx+a\right)\right)\left(\csc^{4}\left(bx+a\right)\right)\left(32\left(\cos^{4}\left(bx+a\right)\right)-104\left(\cos^{2}\left(bx+a\right)\right)+117\right)}{585b\sqrt{d\tan\left(bx+a\right)}\,d}$	57

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585/b*cot(b*x+a)^2*csc(b*x+a)^4*(32*cos(b*x+a)^4-104*cos(b*x+a)^2+117)/
(d*tan(b*x+a))^(1/2)/d
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(32\cos(bx+a)^7 - 104\cos(bx+a)^5 + 117\cos(bx+a)^3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\left(bd^2\cos(bx+a)^6 - 3\,bd^2\cos(bx+a)^4 + 3\,bd^2\cos(bx+a)^2 - bd^2\right)\sin(bx+}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/585*(32*cos(b*x + a)^7 - 104*cos(b*x + a)^5 + 117*cos(b*x + a)^3)*sqrt(d
*sin(b*x + a)/cos(b*x + a))/((b*d^2*cos(b*x + a)^6 - 3*b*d^2*cos(b*x + a)^
4 + 3*b*d^2*cos(b*x + a)^2 - b*d^2)*sin(b*x + a))
```

3.98.6 Sympy [F]

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^4\tan(bx+a)^4 + 130\,d^4\tan(bx+a)^2 + 45\,d^4\right)d}{585\,(d\tan(bx+a))^{\frac{13}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/585*(117*d^4*tan(b*x + a)^4 + 130*d^4*tan(b*x + a)^2 + 45*d^4)*d/((d*ta
n(b*x + a))^(13/2)*b)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(117\,d^6\tan\left(bx+a\right)^4+130\,d^6\tan\left(bx+a\right)^2+45\,d^6\right)}{585\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^6}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/585*(117*d^6*tan(b*x + a)^4 + 130*d^6*tan(b*x + a)^2 + 45*d^6)/(sqrt(d*
tan(b*x + a))*b*d^7*tan(b*x + a)^6)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 14.66 (sec) , antiderivative size = 987, normalized size of antiderivative = 15.18

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

output
```
(128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(11*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(
a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
 + 1))^(1/2)*294464i)/(45045*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2
i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*24608i)/(2145*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b
*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*135104i)/(9009*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) - ((exp(a*2i + b*x*2
i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*13088i)/(1287*b*d^2*(exp(a*2i + b*x*2i) - 1)^5) - ((exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*384i
)/(143*b*d^2*(exp(a*2i + b*x*2i) - 1)^6) - (55808*(exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(6435
*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp
(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*7424i)/(1155*b*d
^2*(exp(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*18368i)/(2145*b*d^
2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*228736i)/(9009*b*d^2*(exp
(a*2i + b*x*2i)*1i - 1i)^4) - (17152*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp...
```

3.98. $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.99.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12bd^2}$$

output
```
-1/6*sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a
))^(1/2)-1/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ell
ipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2
)/b/d^2
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)\left(\sqrt{\sec^2(a+bx)}\sin(4(a+bx))+4\sqrt[4]{-1}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\tan}\right.}{24bd^2\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/24*(Csc[a + b*x]*(Sqrt[Sec[a + b*x]^2]*Sin[4*(a + b*x)] + 4*(-1)^(1/4)*
EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]]
)*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sec[a + b*x]^2])
```

3.99.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}dx}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}} - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\sin(a+bx)}} - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{2b} - \frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}}{6d^2} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*d*Sqrt[d*Tan[a + b*x]]) + (-((d*Sin[a + b*x])/(b*Sqrt[
d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*
a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b))/(6*d^2)
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 1048, normalized size of antiderivative = 9.36

method	result	size
default	Expression too large to display	1048

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/b*sec(b*x+a)*csc(b*x+a)*(6*I*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b
*x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)-6*I*(-csc(b*x+a
)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+
a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2)
)*sin(b*x+a)-8*2^(1/2)*cos(b*x+a)^4+8*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Elli
pticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sin(b*x+a)*EllipticPi
((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(cot(b*x+a)-csc(b*
x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/
2)-6*sin(b*x+a)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*(cot(b*x+a)-csc(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-cs
c(b*x+a)+1+cot(b*x+a))^(1/2)+8*cos(b*x+a)^3*2^(1/2)+4*cos(b*x+a)^2*2^(1/2)
+3*sin(b*x+a)*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b
*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+c
ot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(
b*x+a)))*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-3*sin(b*x+a)*ln((
2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x
+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*co
t(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))*(-cos(b...
```

3.99.5 Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(
b*x + a)^2), x)
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.99.8 Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^3}{(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.100.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}$$

output
```
sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)
^(1/2)/b/d/(d*tan(b*x+a))^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\cos(2(a+bx))\sec(a+bx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\right.}{b\sqrt{\sec^2(a+bx)}(d\tan(a+bx))^{3/2}\,(-1}$$

input
```
Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Cos[2*(a + b*x)]*Sec[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 - Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a
+ b*x]])*Tan[a + b*x]^(3/2))/(b*Sqrt[Sec[a + b*x]^2]*(d*Tan[a + b*x])^(3/2
)*(-1 + Tan[a + b*x]^2))
```

3.100.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.43, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3082, 3042, 3049, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sqrt{\sin(a+bx)}}dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{\cos(a+bx)^{3/2}}{\sqrt{\sin(a+bx)}}dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3049}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{1}{2}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{1}{2}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(a+bx)}\left(\frac{\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}{b}+\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}}\right)}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Sqrt[Sin[a + b*x]]*((Sqrt[Cos[a + b*x]]*Sqrt[Sin[a + b*x]])/b + (Elliptic
F[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[Cos[a + b*x]]*Sqrt[
Sin[a + b*x]])))/(d*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.100.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.100.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

method	result
default	$-\frac{\sec(bx+a)\csc(bx+a)\left(-\sin(bx+a)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\mathrm{cs}\right.\right.}{2b\sqrt{d\tan(bx+a)}\,d}$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*sec(b*x+a)*csc(b*x+a)*(-sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((
1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-2^(1/2)*c
os(b*x+a))*(cos(b*x+a)+1)/(d*tan(b*x+a))^(1/2)/d*2^(1/2)
```

3.100.5 Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

3.100.6 Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sin(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

3.100.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.100.8 Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.101.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}$$

output
```
-2/3*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x
+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\cos(2(a+bx))\sec(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(ia\right.\right.}{3b(d\tan(a+bx))^{3/2}\left(-1+\tan^2(a+\right.}$$

input
```
Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*Cos[2*(a + b*x)]*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2
] - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan
[a + b*x]^(3/2)))/(3*b*(d*Tan[a + b*x])^(3/2)*(-1 + Tan[a + b*x]^2))
```

3.101.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3077, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\sin(a+bx)}}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\sin(a+bx)}}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
 - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

3.101.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.101.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.38

method	result
default	$-\frac{\left(\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sec(bx}{3}$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(d*tan(b*x+a))^(1/2)/d*((cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a
)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2
)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF
((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+csc(b*x+a)*2^(1/2))*2^(1/2)
```

3.101.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{(\cos(bx+a)^2-1)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+(\cos(bx}{3\,(bd^2\cos}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*
x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*
x + a))/(b*d^2*cos(b*x + a)^2 - b*d^2)
```

3.101.6 Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

3.101.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.101.8 Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}} - \frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}} - \frac{2\csc(a+bx)\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{21bd^2}$$

output
```
2/21*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-2/7*csc(b*x+a)^3/b/d/(d*tan(b*x+a
))^(1/2)+2/21*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ell
ipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2
)/b/d^2
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc^3(a+bx)\left((1+10\cos(2(a+bx))+\cos(4(a+bx)))\sec^2(a+bx)^{3/2} - 8\sqrt[4]{-1}\,\text{c}\cdots\right.}{42bd\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a\cdots}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]^3*((1 + 10*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*(Sec[a + b*x
]^2)^(3/2) - 8*(-1)^(1/4)*Cos[2*(a + b*x)]*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(7/2)))/(42*b*d*Sqrt[Sec[a + b*x]^2]
*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

3.102.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}dx}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)^3}dx}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{3}\int \frac{\sqrt{d\tan(a+bx)}}{\sin(a+bx)}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\sin(a+bx)}}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3053

$$-\frac{\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2}{3}\sqrt{\sin(2a+2bx)}\csc(a+bx)\sqrt{d\tan(a+bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}}{7d^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Csc[a + b*x]^3)/(7*b*d*Sqrt[d*Tan[a + b*x]]) - ((-2*d*Csc[a + b*x])/(3
*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b))/(7*d^2)
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(123) = 246$.

Time = 0.86 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.08

method	result
default	$-\frac{\left(-3\left(\csc^{7}(bx+a)\right)(1-\cos(bx+a))^{8}+16\left(\csc^{2}(bx+a)\right)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{2-2\csc(bx+a)+2\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc}}{168b(1-\cos(bx+a))\sqrt{\left(\csc^{3}(bx+a)\right)(1-\cos(bx+a))^{3}-\csc(bx+a)+\cot(bx+a)}\,\sqrt{\csc(bx+a)(1-\cos(bx+}}$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/168/b/(1-cos(b*x+a))/(csc(b*x+a)^3*(1-cos(b*x+a))^3-csc(b*x+a)+cot(b*x+
a))^(1/2)/(csc(b*x+a)*(1-cos(b*x+a))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))^(1
/2)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(-d/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1
)*(csc(b*x+a)-cot(b*x+a)))^(3/2)*(-3*csc(b*x+a)^7*(1-cos(b*x+a))^8+16*csc(
b*x+a)^2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-2*csc(b*x+a)+2*cot(b*x+a))^(1/
2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2)
,1/2*2^(1/2))*(1-cos(b*x+a))^3-2*csc(b*x+a)^5*(1-cos(b*x+a))^6+2*csc(b*x+a
)*(1-cos(b*x+a))^2+3*sin(b*x+a))*2^(1/2)
```

3.102.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.46

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a) + i\sin(bx+$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*elliptic_f(arcsin(
cos(b*x + a) + I*sin(b*x + a)), -1) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 +
 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - (co
s(b*x + a)^3 + 2*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*c
os(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)
```

3.102.6 Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

3.102.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.102.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^3\,(d\tan(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 257

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3}$$

output
```
-3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/6
4*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/128*l
n(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/
2)+3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(
5/2)*2^(1/2)+1/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3-1/4*cos(b*x+a)^4
*(d*tan(b*x+a))^(1/2)/b/d^3
```

3.103.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$
$$-\frac{\csc(a+bx)\left(\sin(a+bx)+3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-3\log\left(\cos(a+bx)+\sin\right.\right.}{64bd^3}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
-1/64*(Csc[a + b*x]*(Sin[a + b*x] + 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*
Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a
 + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 2*Sin[3*(a + b*x)] + Sin[5*(a + b*x)])
*Sqrt[d*Tan[a + b*x]])/(b*d^3)
```

3.103.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3071, 252, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(a+bx)^4}{(d\tan(a+bx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3071}$$
$$\frac{d\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)d^2+d^2)^3}d(d\tan(a+bx))}{b}$$
$$\Big\downarrow \text{ 252}$$
$$\frac{d\left(\frac{1}{8}\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)^2}d(d\tan(a+bx))-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 253

$$\frac{d\left(\frac{1}{8}\left(\frac{3\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)}d(d\tan(a+bx))}{4d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 266

$$\frac{d\left(\frac{1}{8}\left(\frac{3\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{d\tan(a+bx)}}{4(d^2\tan^2(a+bx)+d^2)^2}\right)}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan}}{2d}\right.}{2d^2}\right.\right.}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d}}{2d^2(d^2}\right.\right.}{b}$$

$\downarrow$ 217

3.103. $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2(d^2\tan^2(a+bx)+d^2)}\right)-\frac{\sqrt{\cdots}}{4(d^2\,\mathrm{ta}\cdots}\cdots}{b}$$

$$\Big\downarrow \; 1479$$

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{2d^2}\cdots\right.\right.}{b}$$

$$\Big\downarrow \; 1103$$

3.103. $\displaystyle \int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

$$d\left(\frac{1}{8}\left(\frac{3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{2d^2}\right.\right.$$
$$\overline{\qquad b \qquad}$$

input
```
Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*(-1/4*Sqrt[d*Tan[a + b*x]]/(d^2 + d^2*Tan[a + b*x]^2)^2 + ((3*((-(ArcTa
n[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2
]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d
^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt
[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d))
)/(2*d^2) + Sqrt[d*Tan[a + b*x]]/(2*d^2*(d^2 + d^2*Tan[a + b*x]^2)))/8))/b
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.103. $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.103.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(197) = 394$.

Time = 15.09 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.35

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^4(bx+a))+16\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^3(bx+a))-4\sqrt{-\frac{\cos(bx+}{(\cos(b}}\right.}{}$

input
```
int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/b*csc(b*x+a)*(-1+cos(b*x+a))*(16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)*2^(1/2)*cos(b*x+a)^4+16*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*2^(1/2)*cos(b*x+a)^3-4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)*2^(1/2)*cos(b*x+a)^2-4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)+6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((sin(b*
x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)
/(-1+cos(b*x+a)))-3*ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x
+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-co
t(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1
+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot
(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3
+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+co
s(b*x+a))))/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)/(d*tan(b*x+a))
^(1/2)/d^2*2^(1/2)
```

3.103.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.72

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/256*(3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos
(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d^7*(-1/(b^4*d^10))^(3/
4)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))
*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*lo
g(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)
^2 - 2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10
))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)
 + 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(
b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b^3*d^7*(-1/(b^4*d^10))^(3
/4)*cos(b*x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/
4)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*
x + a)^2 - 2*(-I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + I*b*d^2*(-
1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
 + a)) + 1) + 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*(b^3*d^7*(-1/(b^4*d^10))
^(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4
)*log(-2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^
10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) -
1) + 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(I*b^3*d^7*(-1/(b^4*d^10))^...
```

3.103.6 Sympy [F]

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.85

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+}\right.}{2\sqrt{d}}\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
1/128*(6*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 6*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*d^(5/2)*log(d
*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*((d*tan(b*x
+ a))^(5/2)*d^4 - 3*sqrt(d*tan(b*x + a))*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*
tan(b*x + a)^2 + d^4))/(b*d^5)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3}\\
&+\frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3}\\
&+\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd^3}\\
&-\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{128\,bd^3}\\
&+\frac{\sqrt{d\tan\left(bx+a\right)}d^2\tan\left(bx+a\right)^2-3\sqrt{d\tan\left(bx+a\right)}d^2}{16\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2bd}\end{aligned}$$

input
```
integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
3/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/64*sqrt(2)*sqrt(abs(d))*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^3) + 3/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(
d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/128*sqrt(2)*sqrt(abs(d)
)*log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))
/(b*d^3) + 1/16*(sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 - 3*sqrt(d*tan(b*
x + a))*d^2)/((d^2*tan(b*x + a)^2 + d^2)^2*b*d)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2), x)
```

3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{5/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{5/2}} + \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

output
```
-3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/8*
arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/16*ln(d
^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)+
3/16*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)
*2^(1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3
```

3.104.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.50

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\csc(a+bx)\left(\sin(a+bx)-3\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}\right.}{}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Csc[a + b*x]*(Sin[a + b*x] - 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[S
in[2*(a + b*x)]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x
)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b
*d^3)
```

3.104.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3071, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(a+bx)^2}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{d\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \ 253$$

$$\frac{d\left(\frac{3\int \frac{1}{\sqrt{d\tan(a+bx)}\left(\tan^2(a+bx)d^2+d^2\right)}d(d\tan(a+bx))}{4d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2\left(d^2\tan^2(a+bx)+d^2\right)}\right)}{b}$$

$$\Big\downarrow \ 266$$

$$\frac{d\left(\frac{3\int \frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2\left(d^2\tan^2(a+bx)+d^2\right)}\right)}{b}$$

$\downarrow$ 755

$$\frac{d\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2\left(d^2\tan^2(a+bx)+d^2\right)}\right)}{b}$$

$\downarrow$ 1476

$$\frac{d\left(\frac{3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+b}}{2d}\right)}{2d^2}\right)}{b}$$

$\downarrow$ 1082

$$\frac{d\left(\frac{3\left(\frac{\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan}}{2d^2(d^2\tan^2}\right)}{b}$$

$\downarrow$ 217

$$\frac{d\left(\frac{3\left(\frac{\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{2d^2}+\frac{\sqrt{d\tan(a+bx)}}{2d^2\left(d^2\tan^2(a+bx)+d^2\right)}\right)}{b}$$

$\downarrow$ 1479

3.104. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

$$\frac{d\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

$\Big\downarrow$ 25

$$\frac{d\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\cdots}{\cdots}}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

$\Big\downarrow$ 27

$$\frac{d\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\cdots}{\cdots}}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

$\Big\downarrow$ 1103

$$\frac{d\left(\frac{3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\cdots\right)}{2d^2}\cdots\right)}{b}$$

input
```
Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

3.104. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

output
```
(d*((3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + A
rcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*L
og[d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d]
) + Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*
Sqrt[d]))/(2*d)))/(2*d^2) + Sqrt[d*Tan[a + b*x]]/(2*d^2*(d^2 + d^2*Tan[a +
 b*x]^2))))/b
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.104. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.104. $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.104.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(171) = 342$.

Time = 14.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.32

method	result
default	$\frac{\csc(bx+a)(-1+\cos(bx+a))\left(-4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\sqrt{2}\left(\cos^2(bx+a)\right)-4\cos(bx+a)\sqrt{2}\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}+6\arctan\left(\frac{\sin(bx\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/b*csc(b*x+a)*(-1+cos(b*x+a))*(-4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*2^(1/2)*cos(b*x+a)^2-4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x
+a)/(cos(b*x+a)+1)^2)^(1/2)+6*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+6*arctan((si
n(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a
)+1)/(-1+cos(b*x+a)))+3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+
a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(
b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2
)/(-1+cos(b*x+a)))-3*ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)-2*sin(b*x+a)*
(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x
+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(
-1+cos(b*x+a))))/(d*tan(b*x+a))^(1/2)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)/d^2*2^(1/2)
```

3.104.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 943, normalized size of antiderivative = 4.15

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(
b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 + 2*(b^3*d^7*(-1/(b^4*d^10))^(3/4
)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*
sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log
(2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x + a)^
2 - 2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + b*d^2*(-1/(b^4*d^10)
)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)
+ 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b
*x + a)*sin(b*x + a) - 2*cos(b*x + a)^2 - 2*(I*b^3*d^7*(-1/(b^4*d^10))^(3/
4)*cos(b*x + a)^2 - I*b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*I*b*d^3*(-1/(b^4*d^10))^(1/4
)*log(-2*b^2*d^5*sqrt(-1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*cos(b*x
 + a)^2 - 2*(-I*b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + I*b*d^2*(-1
/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a)) + 1) + 3*b*d^3*(-1/(b^4*d^10))^(1/4)*log(2*(b^3*d^7*(-1/(b^4*d^10))^
(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1) - 3*b*d^3*(-1/(b^4*d^10))^(1/4)
*log(-2*(b^3*d^7*(-1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 - b*d^2*(-1/(b^4*d^1
0))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1
) + 3*I*b*d^3*(-1/(b^4*d^10))^(1/4)*log(-2*(I*b^3*d^7*(-1/(b^4*d^10))^(...
```

3.104.6 Sympy [F]

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(b}{2\sqrt{d}}\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
1/16*(6*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 6*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*sqrt(d)*log(d*tan(b*x +
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*sqrt(d)*log(d*
tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*sqrt(d*tan(b*
x + a))*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} \\ &+ \frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3} \\ &+ \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd^3} \\ &- \frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{16\,bd^3} \\ &+ \frac{\sqrt{d\tan\left(bx+a\right)}}{2\left(d^2\tan\left(bx+a\right)^2+d^2\right)bd}\end{aligned}$$

input
```
integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
3/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/8*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d^3) + 3/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*t
an(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/16*sqrt(2)*sqrt(abs(d))*lo
g(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*
d^3) + 1/2*sqrt(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b*d)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2), x)
```

3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.105.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

3.105.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{1}{(d\tan(a+bx))^{9/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

3.105.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.105.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$	17
default	$-\frac{2d}{7b(d\tan(bx+a))^{\frac{7}{2}}}$	17

input
```
int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos{(bx+a)}^4}{7\left(bd^3\cos{(bx+a)}^4-2\,bd^3\cos{(bx+a)}^2+bd^3\right)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^4/(b*d^3*cos(b*x + a)^
4 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)
```

3.105.6 Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\,(d\tan(bx+a))^{\frac{5}{2}}\,b\tan(bx+a)}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/7/((d*tan(b*x + a))^(5/2)*b*tan(b*x + a))
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2}{7\,\sqrt{d\tan(bx+a)}bd^2\tan(bx+a)^3}$$

input
```
integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/7/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^3)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 530, normalized size of antiderivative = 26.50

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{46\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d^3\,\left(e^{a\,2i+b\,x\,2i}-1\right)} + \frac{12\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^3\left(e^{a\,2i+b\,x\,2i}-1\right)^2} + \frac{24\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}-1\right)^3} - \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,48i}{7\,b\,d^3\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)} + \frac{144\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2} + \frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,144i}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3} - \frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}$$

input
```
int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2)),x)
```

output
```
(46*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (12*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1)
)^(1/2))/(5*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) +
1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35
*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*48i)/(7*b*d^3*(exp
(a*2i + b*x*2i)*1i - 1i)) + (144*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^3*(exp(a*2i +
b*x*2i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d^3*(exp(a*2i + b*x*2
i)*1i - 1i)^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
- 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i)*1i -
1i)^4)
```

3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{2(-9+2\cos(2(a+bx)))\cot^4(a+bx)\csc^2(a+bx)\sqrt{d\tan(a+bx)}}{77bd^3}$$

input
```
Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(-9 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^4*Csc[a + b*x]^2*Sqrt[d*Tan[a +
b*x]])/(77*b*d^3)
```

3.106.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^4(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\tan^2(a+bx)d^2+d^2}{(d\tan(a+bx))^{13/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^2}{(d\tan(a+bx))^{13/2}}+\frac{1}{(d\tan(a+bx))^{9/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^2}{11(d\tan(a+bx))^{11/2}}-\frac{2}{7(d\tan(a+bx))^{7/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^2)/(11*(d*Tan[a + b*x])^(11/2)) - 2/(7*(d*Tan[a + b*x])^(7/2))))
/b
```

3.106.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.106.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\frac{8\left(\cot^{5}(bx+a)\right)}{77}-\frac{2\left(\cot^{3}(bx+a)\right)\left(\csc^{2}(bx+a)\right)}{7}}{d^{2}\sqrt{d\tan(bx+a)}\,b}$	48

input
```
int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/b/d^2/(d*tan(b*x+a))^(1/2)*(4*cot(b*x+a)^5-11*cot(b*x+a)^3*csc(b*x+a)
^2)
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^6-11\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{77\left(bd^3\cos\left(bx+a\right)^6-3\,bd^3\cos\left(bx+a\right)^4+3\,bd^3\cos\left(bx+a\right)^2-bd^3\right)}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/77*(4*cos(b*x + a)^6 - 11*cos(b*x + a)^4)*sqrt(d*sin(b*x + a)/cos(b*x +
 a))/(b*d^3*cos(b*x + a)^6 - 3*b*d^3*cos(b*x + a)^4 + 3*b*d^3*cos(b*x + a)
^2 - b*d^3)
```

3.106.6 Sympy [F]

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^2\tan\left(bx+a\right)^2+7\,d^2\right)d}{77\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/77*(11*d^2*tan(b*x + a)^2 + 7*d^2)*d/((d*tan(b*x + a))^(11/2)*b)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(11\,d^3\tan\left(bx+a\right)^2+7\,d^3\right)}{77\,\sqrt{d\tan\left(bx+a\right)}bd^5\tan\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/77*(11*d^3*tan(b*x + a)^2 + 7*d^3)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x
+ a)^5)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 831, normalized size of antiderivative = 19.33

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2)),x)
```

output
```
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*2048i)/(165*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)) - (7768
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(945*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) - (4232*(exp(a*
2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
 1))^(1/2))/(495*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - (1328*(exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2))/(231*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)
*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b
*d^3*(exp(a*2i + b*x*2i) - 1)^5) - (14456*(exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d^3*(
exp(a*2i + b*x*2i) - 1)) - (86528*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i
+ b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d^3*(exp(a*2
i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2
i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*3904i)/(315*b*d^3*(exp(a*2i +
 b*x*2i)*1i - 1i)^3) + (4160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x
*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*d^3*(exp(a*2i + b*x
*2i)*1i - 1i)^4) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i -
 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*1600i)/(99*b*d^3*(exp(a*2i + b*x*2i)
*1i - 1i)^5) - (64*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i...
```

3.106. $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2d^5}{15b(d\tan(a+bx))^{15/2}} - \frac{4d^3}{11b(d\tan(a+bx))^{11/2}} - \frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

output
```
-2/15*d^5/b/(d*tan(b*x+a))^(15/2)-4/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b
/(d*tan(b*x+a))^(7/2)
```

3.107.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2(-117+44\cos(2(a+bx))-4\cos(4(a+bx)))\cot^4(a+bx)\csc^4(a+bx)\sqrt{d\tan}}{1155bd^3}$$

input
```
Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(-117 + 44*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Cot[a + b*x]^4*Csc[a
+ b*x]^4*Sqrt[d*Tan[a + b*x]])/(1155*b*d^3)
```

3.107.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^6(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{d\int \frac{\left(\tan^2(a+bx)d^2+d^2\right)^2}{(d\tan(a+bx))^{17/2}}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{d\int \left(\frac{d^4}{(d\tan(a+bx))^{17/2}}+\frac{2d^2}{(d\tan(a+bx))^{13/2}}+\frac{1}{(d\tan(a+bx))^{9/2}}\right)d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{2d^4}{15(d\tan(a+bx))^{15/2}}-\frac{4d^2}{11(d\tan(a+bx))^{11/2}}-\frac{2}{7(d\tan(a+bx))^{7/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(d*((-2*d^4)/(15*(d*Tan[a + b*x])^(15/2)) - (4*d^2)/(11*(d*Tan[a + b*x])^(
11/2)) - 2/(7*(d*Tan[a + b*x])^(7/2))))/b
```

3.107.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.107.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\left(\cot^{3}\left(bx+a\right)\right)\left(\csc^{4}\left(bx+a\right)\right)\left(32\left(\cos^{4}\left(bx+a\right)\right)-120\left(\cos^{2}\left(bx+a\right)\right)+165\right)}{1155b\,d^{2}\sqrt{d\tan\left(bx+a\right)}}$	57

input
```
int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155/b*cot(b*x+a)^3*csc(b*x+a)^4*(32*cos(b*x+a)^4-120*cos(b*x+a)^2+165)
/d^2/(d*tan(b*x+a))^(1/2)
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(53) = 106$.

Time = 0.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.75

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(32\cos(bx+a)^8-120\cos(bx+a)^6+165\cos(bx+a)^4\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{1155\left(bd^3\cos(bx+a)^8-4\,bd^3\cos(bx+a)^6+6\,bd^3\cos(bx+a)^4-4\,bd^3\cos(bx+a)^2+bd^3\right)}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(32*cos(b*x + a)^8 - 120*cos(b*x + a)^6 + 165*cos(b*x + a)^4)*sqrt
(d*sin(b*x + a)/cos(b*x + a))/(b*d^3*cos(b*x + a)^8 - 4*b*d^3*cos(b*x + a)
^6 + 6*b*d^3*cos(b*x + a)^4 - 4*b*d^3*cos(b*x + a)^2 + b*d^3)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^4\tan(bx+a)^4+210\,d^4\tan(bx+a)^2+77\,d^4\right)d}{1155\,(d\tan(bx+a))^{\frac{15}{2}}\,b}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-2/1155*(165*d^4*tan(b*x + a)^4 + 210*d^4*tan(b*x + a)^2 + 77*d^4)*d/((d*t
an(b*x + a))^(15/2)*b)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\left(165\,d^5\tan\left(bx+a\right)^4+210\,d^5\tan\left(bx+a\right)^2+77\,d^5\right)}{1155\,\sqrt{d\tan\left(bx+a\right)}bd^7\tan\left(bx+a\right)^7}$$

input
```
integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
-2/1155*(165*d^5*tan(b*x + a)^4 + 210*d^5*tan(b*x + a)^2 + 77*d^5)/(sqrt(d
*tan(b*x + a))*b*d^7*tan(b*x + a)^7)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 1132, normalized size of antiderivative = 17.42

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(5/2)),x)
```

output
```
(199232*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2))/(12285*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (15813
76*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (4539104
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(225225*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) + (1152*(exp
(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i
) + 1))^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) + (74528*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(2145*b*d^3*(exp(a*2i + b*x*2i) - 1)^5) + (1088*(exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(55*b*d^3*(exp(a*2i + b*x*2i) - 1)^6) + (896*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195*b*d^3
*(exp(a*2i + b*x*2i) - 1)^7) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*439808i)/(27027*b*d^3*(e
xp(a*2i + b*x*2i)*1i - 1i)) + (1573888*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(135135*b*d^3*(e
xp(a*2i + b*x*2i)*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4557824i)/(225225*b*d^3
*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (7168*(exp(a*2i + b*x*2i) + 1)*(-(d*...
```

3.107. $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-3/40*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-
1/20*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)-3/70*sin(b*x+a)^5/b/d/(d*tan(b*
x+a))^(3/2)+1/7*sin(b*x+a)^7/b/d/(d*tan(b*x+a))^(3/2)
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.86 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(15\sin(a+bx)+29\sin(3(a+bx))+9\sin(5(a\right.}{}$$

input
```
Integrate[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(15*Sin[a + b*x] + 29*Sin[3*
(a + b*x)] + 9*Sin[5*(a + b*x)] - 5*Sin[7*(a + b*x)])) + 112*Hypergeometri
c2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(2240*b*d
^3*Sqrt[Sec[a + b*x]^2])
```

3.108.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^7}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{3\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sin(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{3\left(\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{7}{10}\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{3\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3081

$$\frac{3\left(\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3052

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

$\Big\downarrow$ 3119

$$\frac{3\left(\frac{7}{10}\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}\right)}{14d^2}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2)) + (3*(-1/5*(d*Sin[a + b*x]^5
)/(b*(d*Tan[a + b*x])^(3/2)) + (7*(-1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b
*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*
a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/10))/(14*d^2)
```

3.108. $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.108. $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(151) = 302$.

Time = 1.46 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.08

method	result
default	$\frac{\sec(bx+a)\left(\csc^2(bx+a)\right)(-1+\cos(bx+a))(\cos(bx+a)+1)\left(40\sqrt{2}\left(\cos^8(bx+a)\right)-108\sqrt{2}\left(\cos^6(bx+a)\right)+82\sqrt{2}\left(\cos^4(bx+a)\right)-21\sqrt{\cot(}\right.}{}$

input
```
int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/560/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(40*2^(1/2)
*cos(b*x+a)^8-108*2^(1/2)*cos(b*x+a)^6+82*2^(1/2)*cos(b*x+a)^4-21*(cot(b*x
+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b
*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*
x+a)+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(
cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
*2^(1/2))*cos(b*x+a)-21*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b
*x+a))^(1/2),1/2*2^(1/2))+42*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-
cot(b*x+a))^(1/2),1/2*2^(1/2))+7*cos(b*x+a)^2*2^(1/2)-21*2^(1/2)*cos(b*x+a
))/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2)
```

3.108.5 Fricas [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^7}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 1)*sqrt(
d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**7/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.108.7 Maxima [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

3.108.8 Giac [F]

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^7}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^7}{{\left(d\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2), x)
```

3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.109.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-3/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi
+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-
1/10*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)+1/5*sin(b*x+a)^5/b/d/(d*tan(b*x
+a))^(3/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}(\sin(3(a+bx))+\sin(5(a+bx)))+8\,\text{Hypergeo}\right.}{80bd^3\sqrt{\sec^2(a+bx}}$$

input
```
Integrate[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[3*(a + b*x)] + Sin[5*(a
 + b*x)])) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b
*x]*Tan[a + b*x]))/(80*b*d^3*Sqrt[Sec[a + b*x]^2])
```

3.109.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^5}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\sin(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{3\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{3\left(\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\sqrt{\sin(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{3\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\sin(a+bx)\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3\left(\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}\right)}{10d^2}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^5/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (3*(-1/3*(d*Sin[a + b*x]^3
)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])
/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])))/(10*d^2)
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(125) = 250$.

Time = 1.18 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.78

method	result
default	$-\frac{\sec(bx+a)\left(\csc^2(bx+a)\right)(-1+\cos(bx+a))(\cos(bx+a)+1)\left(4\sqrt{2}\left(\cos^6(bx+a)\right)-6\sqrt{2}\left(\cos^4(bx+a)\right)-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\right.}{}$

input
```
int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(4*2^(1/2)*
cos(b*x+a)^6-6*2^(1/2)*cos(b*x+a)^4-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-cs
c(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+cs
c(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+3*(cot(b*x+a)-csc(b*x+a
))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*
EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(1+csc
(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc
(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(c
ot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)
-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-
cos(b*x+a)^2*2^(1/2)+3*2^(1/2)*cos(b*x+a))/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2
)
```

3.109.5 Fricas [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^5}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(
b*x + a)/(d^3*tan(b*x + a)^3), x)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.109.7 Maxima [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

3.109.8 Giac [F]

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^5}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2), x)
```

3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}} + \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+1
/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}(\sin(a+bx)+\sin(3(a+bx)))+4\,\text{Hypergeomet}\right.}{12bd^3\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(Sqrt[d*Tan[a + b*x]]*(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*x
)]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan
[a + b*x]))/(12*b*d^3*Sqrt[Sec[a + b*x]^2])
```

3.110.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3076, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(a+bx)^3}{(d\tan(a+bx))^{5/2}}dx$$
$$\Big\downarrow \text{ 3076}$$
$$\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{2d^2}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{2d^2}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3081}$$
$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3052}$$
$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$
$$\Big\downarrow \text{ 3119}$$

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
 2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(99) = 198$.

Time = 0.94 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.96

method	result
default	$\frac{\sec(bx+a)\left(\csc^2(bx+a)\right)(-1+\cos(bx+a))(\cos(bx+a)+1)\left(2\sqrt{2}\left(\cos^4(bx+a)\right)-3\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+\right.}{}$

input
```
int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b*sec(b*x+a)*csc(b*x+a)^2*(-1+cos(b*x+a))*(cos(b*x+a)+1)*(2*2^(1/2)*c
os(b*x+a)^4-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/
2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/
2),1/2*2^(1/2))*cos(b*x+a)+6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+
1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-
cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*
(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF
((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*(1+csc(b*x+a)-cot(b*x+a))^
(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellip
ticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-3*2
^(1/2)*cos(b*x+a))/(d*tan(b*x+a))^(1/2)/d^2*2^(1/2)
```

3.110.5 Fricas [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(
b*x + a)^3), x)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.110.7 Maxima [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.110.8 Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(d\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2), x)
```

3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.111.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x
),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2*si
n(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=-\frac{2\cos(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{bd^2\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Cos[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*d^2*Sqrt[d*Tan[a + b*x]])
```

3.111.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3077, 3042, 3081, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{d^2}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{d^2}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sin(a+bx)\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b
*x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(97) = 194$.

Time = 0.95 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.69

method	result
default	$\frac{\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{\cot(}}{}$

input
```
int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(d*tan(b*x+a))^(1/2)/d^2*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+cs
c(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x
+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*
EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*sec(b*x+a)*(1+csc
(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc
(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1
/2)*cos(b*x+a)-3*2^(1/2))*2^(1/2)
```

3.111.5 Fricas [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

3.111.6 Sympy [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sin(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

3.111.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.111.8 Giac [F]

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)}{{\left(d\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(sin(a + b*x)/(d*tan(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)/(d*tan(a + b*x))^(5/2), x)
```

3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.112.1 Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
6/5*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/
sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/si
n(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*csc(b*x+a)/b/d/(d*tan(b*x+a))^
(3/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)-(3-4\csc^2(a+\right.}{5bd^3\sqrt{\sec^2(a+bx}}$$

input
```
Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 - (
3 - 4*Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*
Sqrt[d*Tan[a + b*x]])/(5*b*d^3*Sqrt[Sec[a + b*x]^2])
```

3.112.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {3042, 3077, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{3\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{5d^2}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx}{5d^2}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3050}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-2\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sqrt[Sin[a + b*x]]*(
(-2*Cos[a + b*x]^(3/2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*Ell
ipticE[a - Pi/4 + b*x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))
/(5*d^2*Sqrt[Cos[a + b*x]]*Sqrt[d*Tan[a + b*x]])
```

3.112.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(121) = 242$.

Time = 0.96 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.39

method	result
default	$\frac{\left(-6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+3\sqrt{\mathrm{co}}\right.}{}$

input
```
int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/b/(d*tan(b*x+a))^(1/2)/d^2*(-6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b
*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b
*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+c
sc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*
x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)
*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*sec(b*x+a)*(1+cs
c(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-cs
c(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+3*2
^(1/2)-cot(b*x+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

3.112.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.24

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$

$$-\frac{3\left(-i\cos\left(bx+a\right)^2+i\right)\sqrt{i\,d}E(\arcsin\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)|-1)\sin\left(bx+a\right)+3\left(i\cos\left(bx+a\right)^2\right.}{}$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(-I*cos(b*x + a)^2 + I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) +
 I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^2 - I)*sqrt(-I*d)*e
lliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*c
os(b*x + a)^2 - I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1)*sin(b*x + a) + 3*(-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_f(ar
csin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 2*(3*cos(b*x + a)^
4 - 2*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^3*cos(b*x +
 a)^2 - b*d^3)*sin(b*x + a))
```

3.112.6 Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.112.8 Giac [F]

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc{(bx+a)}}{(d\tan{(bx+a)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx=\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{5/2}} \,d x$$

input
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.113.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

output
```
4/15*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-4/15*(sin(a+1/4*Pi+b*x)^2)^(1/2
)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/
sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2/15*csc(b*x+a)/b/d/(d*tan(b*x+a
))^(3/2)-2/9*csc(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+\left(-6+3\csc^2(a\right.\right.}{45bd^3\sqrt{s}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + (
-6 + 3*Csc[a + b*x]^2 + 8*Csc[a + b*x]^4 - 5*Csc[a + b*x]^6)*Sqrt[Sec[a +
b*x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(45*b*d^3*Sqrt[Sec[a + b*x]^2]
)
```

3.113.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3050, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\sin(a+bx)^3\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{5}\int \frac{1}{\sin(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\int\frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\int\frac{\sqrt{\cos(a+bx)}}{\sin(a+bx)^{3/2}}dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3050

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-2\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3052

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

$\downarrow$ 3119

$$-\frac{\frac{2\sqrt{\sin(a+bx)}\left(-\frac{2\cos^{\frac{3}{2}}(a+bx)}{b\sqrt{\sin(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\cos(a+bx)}E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}}\right)}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}}{3d^2}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}$$

3.113. $\int\frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

input
```
Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Csc[a + b*x]^3)/(9*b*d*(d*Tan[a + b*x])^(3/2)) - ((-2*d*Csc[a + b*x])/
(5*b*(d*Tan[a + b*x])^(3/2)) + (2*Sqrt[Sin[a + b*x]]*((-2*Cos[a + b*x]^(3/
2))/(b*Sqrt[Sin[a + b*x]]) - (2*Sqrt[Cos[a + b*x]]*EllipticE[a - Pi/4 + b*
x, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]])))/(5*Sqrt[Cos[a + b*x
]]*Sqrt[d*Tan[a + b*x]]))/(3*d^2)
```

3.113.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(147) = 294$.

Time = 1.00 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.80

method	result
default	$\frac{\left(-12\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)}, \frac{\sqrt{2}}{2}\right)+6\sqrt{c}\right.}{}$

input
```
int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/45/b/(d*tan(b*x+a))^(1/2)/d^2*(-12*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc
(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc
(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-12*sec(b*x+a)*(1+csc(b*x+a)-cot
(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1
/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)
-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+
6*2^(1/2)+3*cot(b*x+a)*csc(b*x+a)*2^(1/2)-5*cot(b*x+a)*csc(b*x+a)^3*2^(1/2
))*2^(1/2)
```

3.113.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.21

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx =$$
$$-\frac{2\left(3\left(-i\cos(bx+a)^4+2i\cos(bx+a)^2-i\right)\sqrt{id}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a\right.}{}$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e
(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x +
a)^4 - 2*I*cos(b*x + a)^2 + I)*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) -
 I*sin(b*x + a)), -1)*sin(b*x + a) + 3*(I*cos(b*x + a)^4 - 2*I*cos(b*x + a
)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*s
in(b*x + a) + 3*(-I*cos(b*x + a)^4 + 2*I*cos(b*x + a)^2 - I)*sqrt(-I*d)*el
liptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - (6*cos(
b*x + a)^6 - 15*cos(b*x + a)^4 + 4*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos
(b*x + a)))/((b*d^3*cos(b*x + a)^4 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)*sin(b
*x + a))
```

3.113.6 Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(5/2), x)
```

3.113.7 Maxima [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.113.8 Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\,\tan\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)), x)
```

3.114 $\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

3.114.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

output
```
-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(1/2)-8/5*a^2*b*(a*sin(f*x+e)
)^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{a^2\sqrt{a\sin(e+fx)}(8\cot(e+fx)+\sin(2(e+fx)))\sqrt{b\tan(e+fx)}}{5f}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/5*(a^2*Sqrt[a*Sin[e + f*x]]*(8*Cot[e + f*x] + Sin[2*(e + f*x)])*Sqrt[b*
Tan[e + f*x]])/f
```

3.114.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3069}$$

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
 + f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

3.114.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(56) = 112$.

Time = 1.86 (sec) , antiderivative size = 444, normalized size of antiderivative = 6.53

method	result
default	$\frac{\csc(fx+e)a^2\sqrt{b\tan(fx+e)}\,\sqrt{\sin(fx+e)a}\left(4\left(\cos^3(fx+e)\right)-5\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e}}}{\cos(fx+e)+1}\right.\right.}{}$

input
```
int((sin(f*x+e)*a)^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/f*csc(f*x+e)*a^2*(b*tan(f*x+e))^(1/2)*(sin(f*x+e)*a)^(1/2)*(4*cos(f*x
+e)^3-5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
+1)/(cos(f*x+e)+1))*cos(f*x+e)+5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2
*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-5*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+5*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(c
os(f*x+e)+1))-20*cos(f*x+e))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\left(a^2\cos\left(fx+e\right)^3-5\,a^2\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{5\,f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/5*(a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))/(f*sin(f*x + e))
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

3.114.8 Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.114.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{a^2\sqrt{a\sin(e+fx)}\,(18\sin(2e+2fx)-\sin(4e+4fx))\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{10f\,(\cos(2e+2fx)-1)}$$

input
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
(a^2*(a*sin(e + f*x))^(1/2)*(18*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*s
in(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*f*(cos(2*e + 2*f*x) -
1))
```

3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

3.115.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}} + \frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

output
```
-2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(1/2)+4/3*a^2*(cos(1/2*f*x+1/
2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos
(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2ab\sqrt{a\sin(e+fx)}\left(-2\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+fx)\right)}{3f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*a*b*Sqrt[a*Sin[e + f*x]]*(-2*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (C
os[e + f*x]^2)^(1/4)*Sin[e + f*x]))/(3*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan
[e + f*x]])
```

3.115.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin
[e + f*x]])
```

3.115.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.115.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2\sqrt{\sin(fx+e)a}\,a\sqrt{b\tan(fx+e)}\left(2i\cot(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)+2i\csc(fx+e)\sqrt{\frac{\cos(}{\cos(f}}\right.}{3f}$

input
```
int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*(sin(f*x+e)*a)^(1/2)*a*(b*tan(f*x+e))^(1/2)*(2*I*cot(f*x+e)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e
)-cot(f*x+e)),I)+2*I*csc(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+cos(f*x+e))
```

3.115.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)-\sqrt{2}\sqrt{-ab}a\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(\right.}{3f}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
) - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f
*x + e)) - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e)))/f
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

3.115.8 Giac [F(-2)]

Exception generated.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2), x)
```

3.116 $\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$

3.116.1 Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx=-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

output
```
-2*b*(a*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx=-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3069}$$

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

3.116.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.37

method	result
risch	$-\frac{2i\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}\,\sqrt{\sin(fx+e)a}\,\left(e^{2i(fx+e)}+1\right)}{\left(e^{2i(fx+e)}-1\right)f}$
default	$-\frac{\cot(fx+e)\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos}{\cos(fx+e)+1}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{c}{(\cos}}}$

input
```
int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(-I*b*(exp(2*I*(f*x+e))-1)/(exp(2*I*(f*x+e))+1))^(1/2)/(exp(2*I*(f*x+
e))-1)*(sin(f*x+e)*a)^(1/2)*(exp(2*I*(f*x+e))+1)/f
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{f\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/(f*
sin(f*x + e))
```

3.116.6 Sympy [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin{(e+fx)}}\sqrt{b\tan{(e+fx)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x)), x)
```

3.116.7 Maxima [F]

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{a\sin{(fx+e)}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

3.116.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.116.9 Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.00

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\sin(2\,e+2\,f\,x)\,\sqrt{a\,\sin(e+f\,x)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\cos(e+f\,x)^2}}}{f\,\left(\cos(e+f\,x)^2-1\right)}$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2),x)
```

output
```
(sin(2*e + 2*f*x)*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(2*cos(e +
f*x)^2))^(1/2))/(f*(cos(e + f*x)^2 - 1))
```

3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

3.117.1 Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

output
```
2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos(e+fx)\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*Cos[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sqrt[b*Tan[e + f*x]])
/(f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]])
```

3.117.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*S
qrt[a*Sin[e + f*x]])
```

3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.117.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

method	result	size
default	$-\frac{2 i F\left(i(\csc(fx+e)-\cot(fx+e)), i\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{b\tan(fx+e)}}{f\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\sin(fx+e)a}}$	79

input
```
int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/f*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)/(1/(cos(f*x+e)+1))^(1/2)*(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)
```

3.117.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{-ab}\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2}\sqrt{-ab}\,\text{weierstrassPInverse}(-4,0,}{af}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x +
e)) + sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)))/(a*f)
```

3.117.6 Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan{(e+fx)}}}{\sqrt{a\sin{(e+fx)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

3.117.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

3.117.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)}}{\sqrt{a\,\sin(e+f\,x)}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

3.118.1 Optimal result

Integrand size = 25, antiderivative size = 107

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

output
```
-arctan(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/f/(a*sin
(f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(
1/2)/a/f/(a*sin(f*x+e))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right) + \operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sqrt{a\sin(e+fx)}}{a^2 f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
-((b*(ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sq
rt[a*Sin[e + f*x]])/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]]))
```

3.118.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{a\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{a\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*(ArcTan[Sqrt[Cos[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Co
s[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.118.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.53

method	result
default	$\frac{\cos(fx+e)\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\right)\sqrt{b\tan(fx+}}{2f(\cos(fx+e)+1)\sqrt{\sin(fx+e)a}\,\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,a}$

input
```
int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*cos(f*x+e)*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln(2*(2
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*(b*tan(f*x+e))^(1/2)/(cos(f*x
+e)+1)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(91) = 182$.

Time = 0.42 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \left[\frac{2\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)+\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(fx+e)^3+}{}\right.}{4\,af}\right.$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f
*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e))) + sq
rt(-b/a)*log(-(b*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5*b*cos(f*x + e)^2
 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x +
e) + 1)))/(a*f), 1/4*(2*sqrt(b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(b/a)*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f
*x + e))) + sqrt(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 +
6*b*cos(f*x + e) + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)
*sin(f*x + e))))/(a*f)]
```

3.118.6 Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan{(e+fx)}}}{(a\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

3.118.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

3.118.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2), x)
```

3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

3.119.1 Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

output
```
-b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^
(1/2)*(b*tan(f*x+e))^(1/2)/a^2/f/(a*sin(f*x+e))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b\left(-\sqrt[4]{\cos^2(e+fx)}+\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+fx)\right)}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(b*(-(Cos[e + f*x]^2)^(1/4) + EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e +
 f*x]))/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e +
f*x]])
```

3.119.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
-(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x
]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f
*x]])
```

3.119.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.119.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(f\right.}{f\,a^2\sqrt{\sin(fx+e)a}}$

input
```
int((b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*tan(f*x+e))^(1/2)/a^2/(sin(f*x+e)*a)^(1/2)*(I*(1/(cos(f*x+e)+1))^(1
/2)*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-c
sc(f*x+e)),I)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-cot(f*x+e))
```

3.119.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\sqrt{2}\cos\left(fx+e\right)^2-\sqrt{2}\right)\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(f\right.$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4,
 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sq
rt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*sqr
t(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a^3*f*c
os(f*x + e)^2 - a^3*f)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

3.119.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

3.120.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f} - \frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

output
```
-24/5*a^2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f
*x+e))^(1/2)-2/5*b*(a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2)/f+12/5*a^2*b*
(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

3.120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{a^2b\left(\cos^2(e+fx)^{3/4}(11+\cos(2(e+fx))) - 12\cos^2(e+fx)\,\text{Hypergeometri}}{5f\cos^2(e+fx)^{3/}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^2*b*((Cos[e + f*x]^2)^(3/4)*(11 + Cos[2*(e + f*x)]) - 12*Cos[e + f*x]^2
*Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sq
rt[b*Tan[e + f*x]])/(5*f*(Cos[e + f*x]^2)^(3/4))
```

3.120.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3078, 3042, 3074, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{6}{5}a^2\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}a^2\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3074}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3081}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6}{5}a^2\left(\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f) + (6*a^2*((-4*b^2
*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sqr
t[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/f))/5
```

3.120.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.120.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.94 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.71

method	result
default	$-\frac{2\sin(fx+e)\left(12i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\cot(fx+e)-\csc(fx+e)),i)\left(\cos^2(fx+e)\right)-12i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(\right.}{}$

input
```
int((sin(f*x+e)*a)^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f*sin(f*x+e)*(12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)^2-12*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)
-csc(f*x+e)),I)*cos(f*x+e)^2+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)-24*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(co
t(f*x+e)-csc(f*x+e)),I)*cos(f*x+e)+12*I*EllipticE(I*(cot(f*x+e)-csc(f*x+e)
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*Ellip
ticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)+cos(f*x+e)^3*sin(f*x+e)+sin(f*x+e)*cos(f*x+e)^2-7*sin(f
*x+e)*cos(f*x+e)+5*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*(sin(f*x+e)*a)^(1/2)*a
^2*b/(cos(f*x+e)-1)/(cos(f*x+e)+1)^2
```

3.120.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(6\sqrt{2}\sqrt{-ab}a^2b\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/5*(6*sqrt(2)*sqrt(-a*b)*a^2*b*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^2*b*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)
)) + (a^2*b*cos(f*x + e)^2 + 5*a^2*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
+ e)/cos(f*x + e)))/f
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

3.120.8 Giac [F]

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\,\sin(e+f\,x))^{5/2}\,(b\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2), x)
```

3.121 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

3.121.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}} - \frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

output
```
-2/3*b*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f+8/3*a^2*b*(b*tan(f*x+e)
)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{a^2b(7+\cos(2(e+fx)))\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^2*b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*
x]])
```

3.121.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{4}{3}a^2\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a^2\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

$$\Big\downarrow \text{3069}$$

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

3.121.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(56) = 112$.

Time = 1.79 (sec) , antiderivative size = 436, normalized size of antiderivative = 6.41

method	result
default	$\frac{\csc(fx+e)\sqrt{b\tan(fx+e)}\,ba\left(3\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ \ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e}{}$

input
```
int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/f*csc(f*x+e)*(b*tan(f*x+e))^(1/2)*b*a*(3*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-3*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f
*x+e)+1))*cos(f*x+e)+4*cos(f*x+e)^2+3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-3*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+12)*(sin
(f*x+e)*a)^(1/2)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(ab\cos\left(fx+e\right)^2+3\,ab\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{3\,f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(a*b*cos(f*x + e)^2 + 3*a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))/(f*sin(f*x + e))
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

3.121.8 Giac [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{a\,b\,(13\,\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\sqrt{a\,\sin(e+f\,x)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{6\,f\,{\sin(e+f\,x)}^2}$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
(a*b*(13*sin(e + f*x) + sin(3*e + 3*f*x))*(a*sin(e + f*x))^(1/2)*((b*sin(2
*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*f*sin(e + f*x)^2)
```

3.122 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx$

3.122.1 Optimal result

Integrand size = 25, antiderivative size = 84

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}$$

output
```
-4*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^
(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{2b\left(\cos^2(e+fx)^{3/4}-\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx}}{f\cos^2(e+fx)^{3/4}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*b*((Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^2*Hypergeometric2F1[1/4, 1/2,
3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(f*(Cos[e
 + f*x]^2)^(3/4))
```

3.122.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3074, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{ 3074}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-2b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{2b^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*
x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]
)/f
```

3.122.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.122.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.69

method	result
default	$-\frac{4\left(-\frac{b(\csc(fx+e)-\cot(fx+e))}{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1}\right)^{\frac{3}{2}}\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(i\sqrt{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e}\right.}{}$

input
```
int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/f*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)*
(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/
2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*
x+e)),I)-I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f
*x+e))^2+1)^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-c
os(f*x+e))^3)*(1/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e
)))^(1/2)/(1-cos(f*x+e))^2*sin(f*x+e)^2
```

3.122.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \frac{2\left(\sqrt{2}\sqrt{-ab}b\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))\right)}{}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2*(sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + sqrt(a*si
n(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/f
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.122.7 Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\sin\left(fx+e\right)}(b\tan\left(fx+e\right))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

3.122.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{a\,\sin\left(e+f\,x\right)}\,\left(b\tan(e+f\,x)\right)^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2), x)
```

3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$

3.123.1 Optimal result

Integrand size = 25, antiderivative size = 30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx=\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

output
```
2*b*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx=\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

3.123.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3069}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(26) = 52$.

Time = 2.07 (sec) , antiderivative size = 268, normalized size of antiderivative = 8.93

method	result
default	$\frac{\left(\cos(fx+e)\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)-\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}{\cos(}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{\sin(fx+e)a}\sqrt{-}}$

input
```
int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(cos(f*x+e)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-cos(f
*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*cos(f*x+e)*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b*
tan(f*x+e))^(1/2)*b/(cos(f*x+e)+1)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{af\sin(fx+e)}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e))/(a*f*sin(f*x +
e))
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

3.123.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\,b\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\cos(e+f\,x)^2}}}{f\,\sqrt{a\,\sin\left(e+f\,x\right)}}$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(1/2),x)
```

output
```
(2*b*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^(1/2))/(f*(a*sin(e + f*x))^
(1/2))
```

3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$

3.124.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}$$

output
```
-2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+
e))^(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/a^2/f
```

3.124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(2\cos(e+fx)\cos^2(e+fx)^{3/4} - \cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e\right.\right.}{af\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

output
```
((2*Cos[e + f*x]*(Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^3*Hypergeometric2F1
[1/4, 1/2, 3/2, Sin[e + f*x]^2])*(b*Tan[e + f*x])^(3/2))/(a*f*(Cos[e + f*x
]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

3.124.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3073, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{a^2}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e
+ f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(a^2*f)
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.124.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.66

method	result
default	$-\frac{2\left(i\sqrt{(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,F(i(\csc(fx+e)-\cot(fx+e)),i)-i\sqrt{(\csc^2(fx+e))(1-\right.}{}$

input
```
int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+
e))^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(csc(f*x+e)^2*(1-c
os(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticE(I
*(csc(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3+csc(f*x+e)-cot(f
*x+e))*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^
2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^2/(1
/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e)))^(3/2)
```

3.124.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{-ab}b\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin}{}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*sqrt(a*si
n(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^2*f)
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.124.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

3.124.8 Giac [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2), x)
```

3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

3.125.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b^2\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

output
```
b^2*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(
b*tan(f*x+e))^(1/2)-b^2*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3
/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(b*tan(f*x+e))^(1/2)/a^2/f/(a
*sin(f*x+e))^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.72

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{b\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+fx) - \text{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+f\right.}{a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(b*(ArcTan[(Cos[e + f*x]^2)^(1/4)]*Cos[e + f*x]^2 - ArcTanh[(Cos[e + f*x]^
2)^(1/4)]*Cos[e + f*x]^2 + 2*(Cos[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])
/(a^2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

3.125.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3073, 3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3073}$$

$$\frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{a^2}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{a^2}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b^2\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\int\frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{2b^2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(-2*b^2*(-1/2*ArcTan[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*
Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2
*b*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```

3.125. $\int\frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.125. $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

rule 3073
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Simp[b^2*((m + 2)/(a^2*(n - 1)))   Int[(a*Sin[e + f*
x])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2
*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.125.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

method	result
default	$\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\cos(fx+e)\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{}{(c}}}{\cos(fx+e)+}\right.\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{\sin(fx+e)a}\,a^2}$

input
```
int((b*tan(f*x+e))^(3/2)/(sin(f*x+e)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x+e)*arctan
(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+cos(f*x+e)*ln(2*(2*cos(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b*
tan(f*x+e))^(1/2)*b/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(s
in(f*x+e)*a)^(1/2)/a^2
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(125) = 250$.

Time = 0.48 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.61

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \left[\frac{2\,ab\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\,\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)\sin(fx+e)+ab\sqrt{-\frac{b}{a}}\,\mathrm{l}\ldots}{}\right.$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e)))*
sin(f*x + e) + a*b*sqrt(-b/a)*log(-(b*cos(f*x + e)^3 - 4*sqrt(a*sin(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e)
 - 5*b*cos(f*x + e)^2 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x
+ e)^2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*b*sqrt
(b*sin(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f*x + e)), -1/4*(2*a*b*sqrt(b/a)
*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a)
*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f*x + e)))*sin(f*x + e) - a*b*sqrt
(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f*x + e)
 + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*
sin(f*x + e) - 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))
/(a^3*f*sin(f*x + e))]
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.125.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

3.125.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
8/15*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e)
)^(1/2)-4/15*a^2*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)-2/9*b*(a*si
n(f*x+e))^(9/2)/f/(b*tan(f*x+e))^(3/2)
```

3.126.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{a^4\left(\cos^2(e+fx)^{3/4}(-17+5\cos(2(e+fx)))+12\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin\right.\right.}{90f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(a^4*((Cos[e + f*x]^2)^(3/4)*(-17 + 5*Cos[2*(e + f*x)]) + 12*Hypergeometri
c2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]
)/(90*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

3.126.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3078, 3042, 3078, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{3}a^2\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{3}a^2\left(\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\left(\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2}{3}a^2\left(\frac{2a^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a^2\left(\frac{2a^2\sqrt{a\sin(e+fx)}\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{3}a^2\left(\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (2*a^2*((-2*b
*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*EllipticE[(
e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
+ f*x]])))/3
```

3.126.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.126.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.77

method	result
default	$-\frac{2\sqrt{\sin(fx+e)a}\,a^4\left(12i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}-12i\cos(fx+e)E(i(\csc(fx+e)-\cot\right.}{}$

input
```
int((sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/f*(sin(f*x+e)*a)^(1/2)*a^4/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)*(12*I
*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-12*I*cos(f*x+e)*EllipticE(I*(csc(f*x+e
)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2
)-5*cos(f*x+e)^4*sin(f*x+e)+24*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-24*I*EllipticE(I*
(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e
)+1))^(1/2)-5*cos(f*x+e)^3*sin(f*x+e)+12*I*sec(f*x+e)*EllipticF(I*(csc(f*x
+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1
/2)-12*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+11*sin(f*x+e)*cos(f*x+e)^2+11*
sin(f*x+e)*cos(f*x+e)-12*sin(f*x+e))
```

3.126.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx =$$

$$-\frac{2\left(6\sqrt{2}\sqrt{-ab}a^4\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+6\sqrt{2}\sqrt{}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(6*sqrt(2)*sqrt(-a*b)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^4*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))
 - (5*a^4*cos(f*x + e)^4 - 11*a^4*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e)))/(b*f)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.126.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

3.126.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx=\int \frac{{\left(a\,\sin\left(e+f\,x\right)\right)}^{9/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2), x)
```

3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 68

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

output
```
-8/21*a^2*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)-2/7*b*(a*sin(f*x+e
))^(7/2)/f/(b*tan(f*x+e))^(3/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{a^3\cos(e+fx)(-11+3\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{21f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(a^3*Cos[e + f*x]*(-11 + 3*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(21*f*S
qrt[b*Tan[e + f*x]])
```

3.127.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3078}$$

$$\frac{4}{7}a^2\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4}{7}a^2\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3069}$$

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*
Sin[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

3.127.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

3.127.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{2a^{3}\sqrt{\sin(fx+e)a}\,(3(\cos^{3}(fx+e))-7\cos(fx+e))}{21f\sqrt{b\tan(fx+e)}}$	48

input
```
int((sin(f*x+e)*a)^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f*a^3*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)*(3*cos(f*x+e)^3-7*cos
(f*x+e))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(3\,a^{3}\cos\left(fx+e\right)^{4}-7\,a^{3}\cos\left(fx+e\right)^{2}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{21\,bf\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(3*a^3*cos(f*x + e)^4 - 7*a^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sq
rt(b*sin(f*x + e)/cos(f*x + e))/(b*f*sin(f*x + e))
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.127.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.127.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^3\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{-\frac{b\,\sin(2\,e+2\,f\,x)}{2\sin(e+f\,x)^2-2}}\,\left(22\,\sin\left(e+f\,x\right)+19\,\sin\left(3\,e+3\,f\,x\right)-3\,\sin\left(5\,e+5\,f\,x\right)\right)}{168\,b\,f\,\sin\left(e+f\,x\right)^2}$$

input
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
-(a^3*(a*sin(e + f*x))^(1/2)*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2)
)^(1/2)*(22*sin(e + f*x) + 19*sin(3*e + 3*f*x) - 3*sin(5*e + 5*f*x)))/(168
*b*f*sin(e + f*x)^2)
```

3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.128.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
4/5*a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))
^(1/2)-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.58 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a^2\left(\cos^2(e+fx)^{3/4}-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\sin(2(e+fx))}{5f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/5*(a^2*((Cos[e + f*x]^2)^(3/4) - Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e
 + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/(f*(Cos[e + f*x]^2)^(3/
4)*Sqrt[b*Tan[e + f*x]])
```

3.128.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3078, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3078}$$

$$\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}a^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{2a^2\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a^2\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipt
icE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*T
an[e + f*x]])
```

3.128.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.128.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.91

method	result
default	$\frac{2\sqrt{\sin(fx+e)a}\,a^2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\right.}{}$

input
```
int((sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*(sin(f*x+e)*a)^(1/2)*a^2/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)*(2*I*co
s(f*x+e)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-2*I*cos(f*x+e)*EllipticE(I*(cot(f*x+e)-cs
c(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)+4*
I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)-4*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(
I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+2*I*sec(f*x+e)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(co
s(f*x+e)+1))^(1/2)-2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*x+e)*cos(f
*x+e)^2-sin(f*x+e)*cos(f*x+e)+2*sin(f*x+e))
```

3.128.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx =$$
$$-\frac{2\left(\sqrt{a\sin\left(fx+e\right)}a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos\left(fx+e\right)^2+\sqrt{2}\sqrt{-ab}a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
 e)^2 + sqrt(2)*sqrt(-a*b)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*a^2*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(
b*f)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.128.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.129.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

output
```
-2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3069}$$

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

3.129.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{2\sqrt{\sin(fx+e)a}\,\cos(fx+e)a}{3f\sqrt{b\tan(fx+e)}}$	33

input
```
int((sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*(sin(f*x+e)*a)^(1/2)*cos(f*x+e)*a/(b*tan(f*x+e))^(1/2)
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2}{3\,bf\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
^2/(b*f*sin(f*x + e))
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.129.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.16

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{a\sqrt{a\sin(e+fx)}\,(\sin(e+fx)+\sin(3e+3fx))\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{6\,b\,f\sin(e+fx)^2}$$

input
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
-(a*(a*sin(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x))*((b*sin(2*e +
 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*b*f*sin(e + f*x)^2)
```

3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

3.130.1 Optimal result

Integrand size = 25, antiderivative size = 50

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)
```

3.130.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin
[2*(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

3.130.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])
```

3.130.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.130.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.12

method	result
default	$\frac{2\left(i\sqrt{(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,\sqrt{-(\csc^2(fx+e))(1-\cos(fx+e))^2+1}\,F(i(\csc(fx+e)-\cot(fx+e)),i)-i\sqrt{(\csc^2(fx+e))(1-\mathrm{co}}{f\sqrt{-\frac{b(\csc(}{\left(\csc^2(fx+}}}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(\mathrm{e}^{2i(fx+e)}-1\right)a\,\mathrm{e}^{-i(fx+e)}}}{f\sqrt{-\frac{ib\left(\mathrm{e}^{2i(fx+e)}-1\right)}{\mathrm{e}^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(ab\,\mathrm{e}^{2i(fx+e)}+ab\right)}{ab\sqrt{\mathrm{e}^{i(fx+e)}\left(ab\,\mathrm{e}^{2i(fx+e)}+ab\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(fx+e)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(fx+e)}-i\right)}\,\sqrt{i\mathrm{e}^{i(fx+e)}}\,\left(-2\right.}{\sqrt{ab\,\mathrm{e}^{3i(fx-}}}}{f\sqrt{}}$

input
```
int((sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*(I*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e
))^2+1)^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(csc(f*x+e)^2*(1-co
s(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticE(I*
(csc(f*x+e)-cot(f*x+e)),I)+csc(f*x+e)^3*(1-cos(f*x+e))^3+cot(f*x+e)-csc(f*
x+e))*(1/(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*a*(csc(f*x+e)-cot(f*x+e)))^(1/2
)/(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)/(cs
c(f*x+e)^2*(1-cos(f*x+e))^2-1)
```

3.130.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{\sqrt{2}\sqrt{-ab}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+\sqrt{2}\sqrt{-ab}}{bf}$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

3.130.6 Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

3.130.7 Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin(fx+e)}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.130.8 Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

3.131.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)/(b*tan(
f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e
)^(1/2)/(b*tan(f*x+e))^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
((ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sin[2*
(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e
+ f*x]])
```

3.131.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\sqrt{a\sin(e+fx)}\int \frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow 827$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow 216$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow 219$$

$$-\frac{2\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(-2*(-1/2*ArcTan[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt
[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.131.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\sin(fx+e)\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\right)}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{\sin(fx+e)a}\,\sqrt{b\tan(fx+e)}}$	158

input
```
int(1/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*sin(f*x+e)*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*c
os(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(cos(f*x+e)+1)/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(90) = 180$.

Time = 0.41 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.95

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$$

$$= \left[\frac{2\sqrt{-ab}\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right) - \sqrt{-ab}\log\left(-\frac{ab\cos(fx+e)^3-5\,ab\cos(fx+e)^2+4\sqrt{-ab}\sqrt{a\sin}}{\cos(fx+e)^3+}\right.}{4\,abf}\right.$$

$$-\frac{2\sqrt{ab}\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right) - \sqrt{ab}\log\left(\frac{4\sqrt{ab}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(}{\cos(f}}}{\left(\cos(fx+e)^2-2\cos(\right.}\right.}{4\,abf}$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*(2*sqrt(-a*b)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e)))
- sqrt(-a*b)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b
)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(
f*x + e) - 5*a*b*cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
3*cos(f*x + e) + 1)))/(a*b*f), -1/4*(2*sqrt(a*b)*arctan(2*sqrt(a*b)*sqrt(a
*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*
x + e) - a*b)*sin(f*x + e))) - sqrt(a*b)*log((4*sqrt(a*b)*(cos(f*x + e)^2
+ cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)) - (
a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e))))/(a*b*f)]
```

3.131.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(e+fx\right)}\sqrt{b\tan\left(e+fx\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x))), x)
```

3.131.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.131.8 Giac [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

3.132.1 Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1
/2)-b*(a*sin(f*x+e))^(1/2)/a^2/f/(b*tan(f*x+e))^(3/2)
```

3.132.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}\left(2\cos^2(e+fx)^{3/4}+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin^2(e+fx)\right)}{2a^2f\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-1/2*(b*Sqrt[a*Sin[e + f*x]]*(2*(Cos[e + f*x]^2)^(3/4) + Hypergeometric2F1
[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x]^2))/(a^2*f*(Cos[e + f*x]^2)^(
3/4)*(b*Tan[e + f*x])^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3079, 3042, 3081, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{2a^2}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{2a^2}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
 + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.132.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.16

method	result
default	$-\frac{i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,E(i(\csc(fx+e)-\cot(fx+e)),i)-i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+$

input
```
int(1/(sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/a*(I*sin(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-co
t(f*x+e)),I)-I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+I*tan(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-co
t(f*x+e)),I)-I*tan(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+1)
```

3.132.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2 + \left(\sqrt{2}\cos(fx+e)^2 -\right.}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
1/2*(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
^2 + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + (sqrt(2)*cos(
f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*b*f*cos(f*x + e)^2 - a^2*b*
f)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.132.8 Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

3.133.1 Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
1/4*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(
b*tan(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3
/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)-1/2*b/a^2/f/(a*sin(f*x+e))^(1/2)/
(b*tan(f*x+e))^(3/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{-4\cos^2(e+fx)^{3/4}\cot(e+fx) + \arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)\sin(2(\ldots}{8a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+\ldots}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(-4*(Cos[e + f*x]^2)^(3/4)*Cot[e + f*x] + ArcTan[(Cos[e + f*x]^2)^(1/4)]*S
in[2*(e + f*x)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)]*Sin[2*(e + f*x)])/(8*a^2
*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.133.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3079, 3042, 3081, 27, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3079}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{4a^2}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}dx}{4a^2}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}dx}{4a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)dx}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}}{\sin(e+fx)}dx}{4a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3045$$

$$-\frac{\sqrt{a\sin(e+fx)}\int\frac{\sqrt{\cos(e+fx)}}{1-\cos^2(e+fx)}d\cos(e+fx)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 266$$

$$-\frac{\sqrt{a\sin(e+fx)}\int\frac{\cos(e+fx)}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 827$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 216$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\sqrt{a\sin(e+fx)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}$$

input
```
Int[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
-1/2*b/(a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - ((-1/2*ArcTan
[Sqrt[Cos[e + f*x]]] + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[a*Sin[e + f*x]]
)/(2*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.133. $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

3.133. $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(120) = 240$.

Time = 1.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\csc(fx+e)\left(\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fa}{\cos(fx+e)+1}\right.\right.}{8f\sqrt{b\tan(fx+e)}}$

input
```
int(1/(sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/f*csc(f*x+e)*(cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2))+cos(f*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2))-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(b*tan(f
*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a^2
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(120) = 240$.

Time = 0.48 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.14

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \left[\frac{2\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\mathrm{c}}{(ab\cos(fx+e)+ab)\sin(fx+e}\right.}{}\right.$$

$$-\frac{2\sqrt{ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)\sin(fx+e)-\sqrt{ab}\left(\cos(fx+e)^2-\right.}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
+ a*b)*sin(f*x + e)))*sin(f*x + e) - sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-
(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x +
 e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*c
os(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1
))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*cos(f*x + e)^2)/((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e)), -1/16*
(2*sqrt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*s
in(f*x + e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log((4*sqrt(a*
b)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e
)/cos(f*x + e)) - (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x
+ e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) -
 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2)/
((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e))]
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.133.8 Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{5/2}\,\sqrt{b\tan(e+f\,x)}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.134.1 Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}} - \frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

output
```
-16/585*a^4*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)-2/117*a^2*(a*sin
(f*x+e))^(9/2)/b/f/(b*tan(f*x+e))^(1/2)+2/13*(a*sin(f*x+e))^(13/2)/b/f/(b*
tan(f*x+e))^(1/2)-64/585*a^6*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.46

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^6\cos^2(e+fx)(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))\sqrt{a\sin(e+fx)}}{2340bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^6*Cos[e + f*x]^2*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)])*Sq
rt[a*Sin[e + f*x]])/(2340*b*f*Sqrt[b*Tan[e + f*x]])
```

3.134.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}dx}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}dx}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{8}{9}a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{8}{9}a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{8}{9}a^2\left(\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{8}{9}a^2\left(\frac{4}{5}a^2\int\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{a^2\left(\frac{8}{9}a^2\left(-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{9/2}}{9f\sqrt{b\tan(e+fx)}}\right)}{13b^2}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(13/2))/(13*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a
*Sin[e + f*x])^(9/2))/(9*f*Sqrt[b*Tan[e + f*x]]) + (8*a^2*((-8*a^2*b*Sqrt[
a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e + f*x])^(5/2))
/(5*f*Sqrt[b*Tan[e + f*x]])))/9))/(13*b^2)
```

3.134.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

3.134.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

method	result	size
default	$-\frac{2a^{6}\sqrt{\sin(fx+e)a}\,(45(\cos^{6}(fx+e))-130(\cos^{4}(fx+e))+117(\cos^{2}(fx+e)))}{585f\sqrt{b\tan(fx+e)}\,b}$	63

input
```
int((sin(f*x+e)*a)^(13/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585/f*a^6*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/b*(45*cos(f*x+e)^6-
130*cos(f*x+e)^4+117*cos(f*x+e)^2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.58

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,a^{6}\cos\left(fx+e\right)^{7}-130\,a^{6}\cos\left(fx+e\right)^{5}+117\,a^{6}\cos\left(fx+e\right)^{3}\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{585\,b^{2}f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

output
```
-2/585*(45*a^6*cos(f*x + e)^7 - 130*a^6*cos(f*x + e)^5 + 117*a^6*cos(f*x +
 e)^3)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f
*x + e))
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(13/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx=\int \frac{(a\sin(fx+e))^{\frac{13}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

3.134.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.134.9 Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.03

$$\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx=\frac{(\cos(7e+7fx)-\sin(7e+7fx)\,1\mathrm{i})\sqrt{\frac{b(\sin(2e+2fx)-\cos(2e+2fx)\,1\mathrm{i}+1\mathrm{i})}{\cos(2e+2fx)+1+\sin(2e+2fx)\,1\mathrm{i}}}\left(\frac{a^6\cos(3e\right.}{}$$

input
```
int((a*sin(e + f*x))^(13/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
((cos(7*e + 7*f*x) - sin(7*e + 7*f*x)*1i)*((b*(sin(2*e + 2*f*x) - cos(2*e
+ 2*f*x)*1i + 1i))/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*((a
^6*cos(3*e + 3*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7
*f*x)*1i)*217i)/(9360*b^2*f) - (a^6*cos(5*e + 5*f*x)*(a*sin(e + f*x))^(1/2
)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*41i)/(1872*b^2*f) + (a^6*cos(7*
e + 7*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)
*1i)/(208*b^2*f) + (a^6*cos(e + f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f
*x) + sin(7*e + 7*f*x)*1i)*1991i)/(9360*b^2*f))*1i)/(2*sin(e + f*x))
```

3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.135.1 Optimal result

Integrand size = 25, antiderivative size = 109

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

output
```
-2/45*a^2*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)+2/9*(a*sin(f*x+e))
^(9/2)/b/f/(b*tan(f*x+e))^(1/2)-8/45*a^4*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f
*x+e))^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^4\cos^2(e+fx)(-13+5\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^4*Cos[e + f*x]^2*(-13 + 5*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(45*b
*f*Sqrt[b*Tan[e + f*x]])
```

3.135.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3076, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3076$$

$$\frac{a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3078$$

$$\frac{a^2\left(\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{a^2\left(\frac{4}{5}a^2\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}dx-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \ 3069$$

$$\frac{a^2\left(-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\right)}{9b^2}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(9/2))/(9*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-8*a^2*b*
Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e + f*x])^(
5/2))/(5*f*Sqrt[b*Tan[e + f*x]])))/(9*b^2)
```

3.135.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

3.135.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2a^4\sqrt{\sin(fx+e)a}\,(5(\cos^4(fx+e))-9(\cos^2(fx+e)))}{45f\sqrt{b\tan(fx+e)}\,b}$	53

input
```
int((sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f*a^4*(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/b*(5*cos(f*x+e)^4-9*c
os(f*x+e)^2)
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(5\,a^4\cos\left(fx+e\right)^5-9\,a^4\cos\left(fx+e\right)^3\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{45\,b^2f\sin\left(fx+e\right)}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/45*(5*a^4*cos(f*x + e)^5 - 9*a^4*cos(f*x + e)^3)*sqrt(a*sin(f*x + e))*sq
rt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x + e))
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.135.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

3.135.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.135.9 Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^4\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,\left(47\,\sin\left(2\,e+2\,f\,x\right)+16\,\sin\left(4\,e+4\,f\,x\right)-5\right.}{360\,b^2\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

input
```
int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
(a^4*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^
(1/2)*(47*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) - 5*sin(6*e + 6*f*x)))/(3
60*b^2*f*(cos(2*e + 2*f*x) - 1))
```

3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.136.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

output
```
-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(5/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2\cos^2(e+fx)\sqrt{a\sin(e+fx)}}{5bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*a^2*Cos[e + f*x]^2*Sqrt[a*Sin[e + f*x]])/(5*b*f*Sqrt[b*Tan[e + f*x]])
```

3.136.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3069}$$

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

3.136.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{2(\cos^2(fx+e))a^2\sqrt{\sin(fx+e)a}}{5fb\sqrt{b\tan(fx+e)}}$	40

input
```
int((sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f*cos(f*x+e)^2*a^2*(sin(f*x+e)*a)^(1/2)/b/(b*tan(f*x+e))^(1/2)
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{a\sin(fx+e)}a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^3}{5\,b^2f\sin(fx+e)}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
e)^3/(b^2*f*sin(f*x + e))
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

3.136.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.136.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^2\sqrt{a\sin(e+fx)}\,(2\sin(2e+2fx)+\sin(4e+4fx))\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{10b^2f\,(\cos(2e+2fx)-1)}$$

input
```
int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
(a^2*(a*sin(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*b^2*f*(cos(2*e + 2*f*x)
 - 1))
```

3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

3.137.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{a\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}} - \frac{a\,\mathrm{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

output
```
2*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-a*arctan(cos(f*x+e)^(1/2))
*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)-a*arctan
h(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x
+e))^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right)+2\sqrt[4]{\cos^2(e+fx)}\right)\sqrt{}}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
((-ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)] + 2*(C
os[e + f*x]^2)^(1/4))*Sqrt[a*Sin[e + f*x]])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sq
rt[b*Tan[e + f*x]])
```

3.137.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3075, 3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3075}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{b^2}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{b^2}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{b^2\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{b^2f\sqrt{a\sin(e+fx)}}$$

3.137. $\int\frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$

input
```
Int[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (2*a*(ArcTan[Sqrt[Co
s[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Cos[e + f*x]]*Sqrt[b*
Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3075
```
Int[Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]/((b_.)*tan[(e_.) + (f_.)*(x_)])^(3/
2), x_Symbol] :> Simp[2*(Sqrt[a*Sin[e + f*x]]/(b*f*Sqrt[b*Tan[e + f*x]])),
x] + Simp[a^2/b^2   Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2), x], x]
 /; FreeQ[{a, b, e, f}, x]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.137.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.47

method	result
default	$\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\cdots}}{\cos(fx+e)+\cdots}\right.\cdots}{2f(\cos(fx+e)+1)\sqrt{b\tan(fx+e)}\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,b}$

input
```
int((sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln(
(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*(sin(f*x+e)*a)^(1/2)/(cos(f
*x+e)+1)/(b*tan(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/b
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(121) = 242$.

Time = 0.48 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \left[\frac{2\,b\sqrt{-\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{a}{b}}\cos(fx+e)}{(a\cos(fx+e)+a)\sin(fx+e)}\right)\sin(fx+e)+b\sqrt{-\frac{a}{b}}\log\right.$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(-a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos
(f*x + e))*sqrt(-a/b)*cos(f*x + e)/((a*cos(f*x + e) + a)*sin(f*x + e)))*si
n(f*x + e) + b*sqrt(-a/b)*log(-(a*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-a/b)*cos(f*x + e)*sin(f*x + e) - 5
*a*cos(f*x + e)^2 - 5*a*cos(f*x + e) + a)/(cos(f*x + e)^3 + 3*cos(f*x + e)
^2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), 1/4*(2*b*sqrt(
a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(
a/b)*cos(f*x + e)/((a*cos(f*x + e) - a)*sin(f*x + e)))*sin(f*x + e) + b*sq
rt(a/b)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(a/b) - (a*cos(f*x + e)^2 + 6*a*cos(f*x +
e) + a)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e))
)*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
cos(f*x + e))/(b^2*f*sin(f*x + e))]
```

3.137.6 Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin{(e+fx)}}}{(b\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))/(b*tan(e + f*x))**(3/2), x)
```

3.137.7 Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.137.8 Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(b\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2), x)
```

3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

3.138.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\arctan\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/2/b/f/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)+1/4*arctan(cos(f*x+e)^(
1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a*sin(f*x+e))^(1/2)+1
/4*arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f
/(a*sin(f*x+e))^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(e+fx)}\right) + \operatorname{arctanh}\left(\sqrt[4]{\cos^2(e+fx)}\right) - 2\sqrt[4]{c\cdots}\right.}{4bf\sqrt[4]{\cos^2(e+fx)}(a\sin(e+fx))^{3/2}\sqrt{\cdots}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)] - 2*(Co
s[e + f*x]^2)^(1/4)*Csc[e + f*x]^2)*Sin[e + f*x]^2)/(4*b*f*(Cos[e + f*x]^2
)^(1/4)*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])
```

3.138.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3077, 3042, 3081, 27, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{4b^2}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{4b^2}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}dx}{4b^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}dx}{4ab^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}\sin(e+fx)}dx}{4ab^2\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}(1-\cos^2(e+fx))}d\cos(e+fx)}{4ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{1-\cos^2(e+fx)}d\sqrt{\cos(e+fx)}}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\int\frac{1}{\cos(e+fx)+1}d\sqrt{\cos(e+fx)}\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{1-\cos(e+fx)}d\sqrt{\cos(e+fx)}+\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(e+fx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(e+fx)}\right)\right)}{2ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-1/2*1/(b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + ((ArcTan[Sqrt[C
os[e + f*x]]]/2 + ArcTanh[Sqrt[Cos[e + f*x]]]/2)*Sqrt[Cos[e + f*x]]*Sqrt[b
*Tan[e + f*x]])/(2*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

3.138. $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.138. $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(125) = 250$.

Time = 1.10 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.91

method	result
default	$\frac{\csc(fx+e)\left(\cos(fx+e)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)-\cos(fx+e)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e}{\cos(fx+e)+1}\right.\right.}{8f\sqrt{b\tan(fx+e)}\sqrt{\sin(f}}$

input
```
int(1/(sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f*csc(f*x+e)*(cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2))-cos(f*x+e)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-arctan(1/2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(c
os(f*x+e)+1))-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))/(b*tan(f*x+e))^(1/2)
/(sin(f*x+e)*a)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/a/b
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(125) = 250$.

Time = 0.47 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.03

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \Bigg[-\frac{2\sqrt{-ab}\left(\cos\left(fx+e\right)^2-1\right)\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx}{\cos(fx}}}{(ab\cos(fx+e)+ab)\sin(}\right.}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
[-1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f
*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
 + a*b)*sin(f*x + e)))*sin(f*x + e) + sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(
-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x
+ e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*
cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) +
1))*sin(f*x + e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*cos(f*x + e))/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e)), -1/
16*(2*sqrt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e
))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b
)*sin(f*x + e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log(-(4*sqr
t(a*b)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
 + e)/cos(f*x + e)) + (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(
f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x +
e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
)/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))]
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.138.8 Giac [F]

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.139.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}} + \frac{8a^6\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-4/77*a^4*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)-2/77*a^2*(a*sin(f*
x+e))^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+2/11*(a*sin(f*x+e))^(11/2)/b/f/(b*tan
(f*x+e))^(1/2)+8/77*a^6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/
b^2/f/(a*sin(f*x+e))^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.71

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^5\left(\sqrt[4]{\cos^2(e+fx)}(-22\cos(e+fx)-17\cos(3(e+fx))+7\cos(5(e+fx)))+\cdots\right.}{616f\sqrt[4]{\cos^2(e+f\cdots}}$$

input
```
Integrate[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^5*((Cos[e + f*x]^2)^(1/4)*(-22*Cos[e + f*x] - 17*Cos[3*(e + f*x)] + 7*C
os[5*(e + f*x)]) + 64*Cot[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2])*S
qrt[a*Sin[e + f*x]]*Tan[e + f*x]^2)/(616*f*(Cos[e + f*x]^2)^(1/4)*(b*Tan[e
 + f*x])^(3/2))
```

3.139.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3076, 3042, 3078, 3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a \sin(e+fx))^{11/2}}{(b \tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a \sin(e+fx))^{11/2}}{(b \tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2 \int (a \sin(e+fx))^{7/2}\sqrt{b \tan(e+fx)}dx}{11b^2}+\frac{2(a \sin(e+fx))^{11/2}}{11bf\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \int (a \sin(e+fx))^{7/2}\sqrt{b \tan(e+fx)}dx}{11b^2}+\frac{2(a \sin(e+fx))^{11/2}}{11bf\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{6}{7}a^2 \int (a \sin(e+fx))^{3/2}\sqrt{b \tan(e+fx)}dx-\frac{2b(a \sin(e+fx))^{7/2}}{7f\sqrt{b \tan(e+fx)}}\right)}{11b^2}+\frac{2(a \sin(e+fx))^{11/2}}{11bf\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{6}{7}a^2 \int (a \sin(e+fx))^{3/2}\sqrt{b \tan(e+fx)}dx-\frac{2b(a \sin(e+fx))^{7/2}}{7f\sqrt{b \tan(e+fx)}}\right)}{11b^2}+\frac{2(a \sin(e+fx))^{11/2}}{11bf\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2}{3}a^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2}{3}a^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3081

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

$\Big\downarrow$ 3120

$$\frac{a^2\left(\frac{6}{7}a^2\left(\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)-\frac{2b(a\sin(e+fx))^{7/2}}{7f\sqrt{b\tan(e+fx)}}\right)}{11b^2}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(11/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a
*Sin[e + f*x])^(7/2))/(7*f*Sqrt[b*Tan[e + f*x]]) + (6*a^2*((-2*b*(a*Sin[e
+ f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]])))/7
))/(11*b^2)
```

3.139. $\int\frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}dx$

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.139.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07

method	result
default	$\frac{2i\sec(fx+e)\csc(fx+e)\left(7i(\cos^6(fx+e))-7i(\cos^5(fx+e))-13i(\cos^4(fx+e))+13i(\cos^3(fx+e))-4\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos}{\cos(}}\right.}{77f\sqrt{b\tan(fx+e)}\,b}$

input
```
int((sin(f*x+e)*a)^(11/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/77*I/f*sec(f*x+e)*csc(f*x+e)*(7*I*cos(f*x+e)^6-7*I*cos(f*x+e)^5-13*I*cos
(f*x+e)^4+13*I*cos(f*x+e)^3-4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)+4*I*cos(f
*x+e)^2-4*I*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a^5*(cos(f*x+e)+1)/(b*tan(f*x
+e))^(1/2)/b
```

3.139.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(2\sqrt{2}\sqrt{-ab}a^5\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\sqrt{2}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

output
```
2/77*(2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x + e) + I
*sin(f*x + e)) + 2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f
*x + e) - I*sin(f*x + e)) + (7*a^5*cos(f*x + e)^5 - 13*a^5*cos(f*x + e)^3
+ 4*a^5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e
)))/(b^2*f)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(11/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{11}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

3.139.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{11/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2), x)
```

3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.140.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/21*a^2*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/7*(a*sin(f*x+e))
^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+4/21*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*t
an(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{a^3\sqrt{a\sin(e+fx)}\Big(8\,\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}(5\sin(}{42bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(a^3*Sqrt[a*Sin[e + f*x]]*(8*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e
 + f*x]^2)^(1/4)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)])))/(42*b*f*(Cos[e +
f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

3.140.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3076, 3042, 3078, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3076}$$

$$\frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{a^2\left(\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{2}{3}a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\left(\frac{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\left(\frac{4a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}\right)}{7b^2}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (a^2*((-2*b*(a*S
in[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]
)))/(7*b^2)
```

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.140.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.97

method	result
default	$\frac{\left(\frac{3}{1985}-\frac{i}{41685}\right)\sec(fx+e)\csc(fx+e)\left(126i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+3i(\cos^4(fx+e))\cdot\right.}{}$

input
```
int((sin(f*x+e)*a)^(7/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(3/1985-1/41685*I)/f*sec(f*x+e)*csc(f*x+e)*(126*I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*
sin(f*x+e)+3*I*cos(f*x+e)^4-3*I*cos(f*x+e)^3-2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*sin
(f*x+e)+189*cos(f*x+e)^4-2*I*cos(f*x+e)^2-189*cos(f*x+e)^3+2*I*cos(f*x+e)-
126*cos(f*x+e)^2+126*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a^3*(cos(f*x+e)+1)/(
b*tan(f*x+e))^(1/2)/b
```

3.140.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\sqrt{2}\sqrt{-ab}a^3\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2}\sqrt{-}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e) + I*s
in(f*x + e)) + sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x +
 e) - I*sin(f*x + e)) - (3*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e))*sqrt(a
*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(b^2*f)
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.140.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

3.140.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{7/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2), x)
```

3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.141.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}$$

output
```
2/3*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/3*a^2*(cos(1/2*f*x+1/2
*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(
f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2a\sqrt{a\sin(e+fx)}\left(\text{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+\right.}{3bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*Sin[e + f*x]]*(EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e +
 f*x]^2)^(1/4)*Sin[e + f*x]))/(3*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e +
 f*x]])
```

3.141.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3076, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3076}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3b^2}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3b^2}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3081}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3b^2\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2a^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*
Sin[e + f*x]])
```

3.141.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3076
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Simp[a^2*((n + 1)/(b^2*m))   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
 f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
 && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.141.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

method	result
default	$\frac{2i\sec(fx+e)\csc(fx+e)\left(\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+i(\cos^2(fx+e))-i\cos(fx+e)\right)\sqrt{s}}{3f\sqrt{b\tan(fx+e)}\,b}$

input
```
int((sin(f*x+e)*a)^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/f*sec(f*x+e)*csc(f*x+e)*((1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*sin(f*x+e)+I*cos(f*
x+e)^2-I*cos(f*x+e))*(sin(f*x+e)*a)^(1/2)*a*(cos(f*x+e)+1)/(b*tan(f*x+e))^
(1/2)/b
```

3.141.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(fx+e)}a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)+\sqrt{2}\sqrt{-ab}a\text{weierstrassPInverse}(-4,$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
e) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(
f*x + e)) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) -
 I*sin(f*x + e)))/(b^2*f)
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.141.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

3.141.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(e+f\,x))^{3/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

3.142.1 Optimal result

Integrand size = 25, antiderivative size = 86

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \frac{-\sqrt[4]{\cos^2(e+fx)} - \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e+fx)),2\right)\sin(e+f}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-(Cos[e + f*x]^2)^(1/4) - EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*
x])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.142.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3077, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2b^2\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2b^2\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]])
```

3.142.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.142.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.77

method	result
default	$\frac{i\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)+i\tan(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)}{f\sqrt{b\tan(fx+e)}\,\sqrt{\sin(fx+e)a}\,b}$

input
```
int(1/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)/b*(I*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*sin(f*x+e)+I*tan(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-1)
```

3.142.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\left(\sqrt{2}\cos\left(fx+\right.\right.}{2\left(ab^{2}f\,c\right.}$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4
, 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*s
qrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sq
rt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a*b^2*
f*cos(f*x + e)^2 - a*b^2*f)
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.142.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.142.8 Giac [F]

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\sin\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(b\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

3.143.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/3/b/f/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/6/a^2/b/f/(a*sin(f*x+
e))^(1/2)/(b*tan(f*x+e))^(1/2)-1/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x
+e))^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(1-2\csc^2(e+fx)) - \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(\sin(e\right.}{6a^2bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((Cos[e + f*x]^2)^(1/4)*(1 - 2*Csc[e + f*x]^2) - EllipticF[ArcSin[Sin[e +
f*x]]/2, 2]*Sin[e + f*x])/(6*a^2*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.143.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3077, 3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{-\frac{\sqrt{\cos(e+fx)}\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}}{6b^2}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-1/3*1/(b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (-(b/(a^2*f*Sqr
t[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]]))/(6*b^2)
```

3.143.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.143.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.32

method	result
default	$-\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+i\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+}{6f\sqrt{b\tan(fx+e)}\,\sqrt{\sin(fx+e)a}\,a^2b}$

input
```
int(1/(sin(f*x+e)*a)^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)*a)^(1/2)/a^2/b*(I*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x
+e)),I)*sin(f*x+e)+I*tan(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+cot(f*x+e)^2+csc(f
*x+e)^2)
```

3.143.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{\left(\sqrt{2}\cos\left(fx+e\right)^4 - 2\sqrt{2}\cos\left(fx+e\right)^2 + \sqrt{2}\right)\sqrt{-ab}\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+\right.}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-1/12*((sqrt(2)*cos(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(
-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)
*cos(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(-a*b)*weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(cos(f*x + e)^3 + cos
(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*b^
2*f*cos(f*x + e)^4 - 2*a^3*b^2*f*cos(f*x + e)^2 + a^3*b^2*f)
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.143.7 Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.143.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{5/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$

3.144.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/5/b/f/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2)+1/30/a^2/b/f/(a*sin(f*x
+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/12/a^4/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f
*x+e))^(1/2)-1/12*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elliptic
F(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a^4/b^
2/f/(a*sin(f*x+e))^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(e+fx)}(5+2\csc^2(e+fx)-12\csc^4(e+fx)) - 5\,\text{Ellip}}{60a^4bf\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx}}$$

input
```
Integrate[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((Cos[e + f*x]^2)^(1/4)*(5 + 2*Csc[e + f*x]^2 - 12*Csc[e + f*x]^4) - 5*Ell
ipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x])/(60*a^4*b*f*(Cos[e + f*x]^
2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.144.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3077, 3042, 3079, 3042, 3079, 3042, 3081, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3077}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{5\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3079}$$

$$-\frac{\frac{5\left(\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\left(\frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2a^2}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3081}$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2a^2\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{5\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\right)}{6a^2}-\frac{b}{3a^2f(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}}{10b^2}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

input
```
Int[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

3.144. $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$

output
```
-1/5*1/(b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) - (-1/3*b/(a^2*f*
(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + (5*(-(b/(a^2*f*Sqrt[a*Sin[e
 + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])))/(6*a^2))/(10*b^
2)
```

3.144.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.144.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

method	result
default	$-\frac{5i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sin(fx+e)+5i\tan(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(f}{60f\sqrt{\sin(fx+e)a}\,\sqrt{b\tan(fx+e)}\,a^4b}$

input
```
int(1/(sin(f*x+e)*a)^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/60/f/(sin(f*x+e)*a)^(1/2)/(b*tan(f*x+e))^(1/2)/a^4/b*(5*I*EllipticF(I*(
cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)
+1))^(1/2)*sin(f*x+e)+5*I*tan(f*x+e)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)-5*cot(f*x+e)^
4+12*cot(f*x+e)^2*csc(f*x+e)^2+5*csc(f*x+e)^4)
```

3.144.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.58

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\left(\sqrt{2}\cos(fx+e)^6-3\sqrt{2}\cos(fx+e)^4+3\sqrt{2}\cos(fx+e)^2-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos}{}$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-1/120*(5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4 + 3*sqrt(2)*c
os(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e
) + I*sin(f*x + e)) + 5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4
 + 3*sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4,
0, cos(f*x + e) - I*sin(f*x + e)) + 2*(5*cos(f*x + e)^5 - 12*cos(f*x + e)^
3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
)/(a^5*b^2*f*cos(f*x + e)^6 - 3*a^5*b^2*f*cos(f*x + e)^4 + 3*a^5*b^2*f*cos
(f*x + e)^2 - a^5*b^2*f)
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.144.7 Maxima [F]

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(a\sin(fx+e))^{\frac{9}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.144.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(a\ \sin\left(e+f\,x\right)\right)^{9/2}\,\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

3.145.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

3.145.2 Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}\sin(2(e+fx}{17f\sqrt[4]{\cos^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
4/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(17*f*(Cos[e + f*x]^2)^(1/4))
```

3.145.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin (e+fx))^{4/3} \sqrt {d\tan (e+fx)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin (e+fx))^{4/3} \sqrt {d\tan (e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac {b\cos ^{\frac {3}{2}}(e+fx) (d\tan (e+fx))^{3/2} \int \frac {(b\sin (e+fx))^{11/6}}{\sqrt {\cos (e+fx)}}dx}{d (b\sin (e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac {b\cos ^{\frac {3}{2}}(e+fx) (d\tan (e+fx))^{3/2} \int \frac {(b\sin (e+fx))^{11/6}}{\sqrt {\cos (e+fx)}}dx}{d (b\sin (e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac {6 \cos ^2(e+fx)^{3/4} (b\sin (e+fx))^{4/3} (d\tan (e+fx))^{3/2} \text{Hypergeometric2F1}\left (\frac {3}{4},\frac {17}{12},\frac {29}{12},\sin ^2(e+fx)\right)}{17 d f}$$

input
```
Int[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```

3.145.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.145.4 Maple [F]

$$\int (b\sin (fx+e))^{\frac{4}{3}} \sqrt{d\tan (fx+e)}dx$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

3.145.5 Fricas [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin (fx+e))^{\frac{4}{3}} \sqrt{d\tan (fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sin(f*x + e), x)
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.145.7 Maxima [F]

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

3.145.8 Giac [F(-2)]

Exception generated.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\,\sin(e+f\,x))^{4/3}\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2), x)
```

3.146 $\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

3.146.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}}{11df}$$

output
```
6/11*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 11/12],[23/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

3.146.2 Mathematica [A] (verified)

Time = 10.62 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{11f\sqrt[4]{\cos^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
1/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(11*f*(Cos[e + f*x]^2)^(1/4))
```

3.146.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

input
```
Int[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.146.4 Maple [F]

$$\int (b\sin (fx+e))^{\frac{1}{3}} \sqrt{d\tan (fx+e)}dx$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

3.146.5 Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin (fx+e))^{\frac{1}{3}} \sqrt{d\tan (fx+e)}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.146.6 Sympy [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt[3]{b\sin{(e+fx)}}\sqrt{d\tan{(e+fx)}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

3.146.7 Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sin{(fx+e)})^{\frac{1}{3}}\sqrt{d\tan{(fx+e)}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.146.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\,\sin(e+f\,x))^{1/3}\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2), x)
```

3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

3.147.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

output
```
6/7*(cos(f*x+e)^2)^(3/4)*hypergeom([7/12, 3/4],[19/12],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(1/3)
```

3.147.2 Mathematica [A] (verified)

Time = 10.58 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

3.147.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

3.147.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.147.4 Maple [F]

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

3.147.5 Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sin(f*x + e)), x)
```

3.147.6 Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan{\left(e+fx \right)}}}{\sqrt[3]{b\sin{\left(e+fx \right)}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(1/3), x)
```

3.147.7 Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

3.147.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3), x)
```

3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$

3.148.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{6\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

output
```
6*(cos(f*x+e)^2)^(3/4)*hypergeom([1/12, 3/4],[13/12],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(4/3)
```

3.148.2 Mathematica [A] (verified)

Time = 10.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}(b\sin(e+fx))^{4/3}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(3*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*Sq
rt[d*Tan[e + f*x]])/(f*(Cos[e + f*x]^2)^(1/4)*(b*Sin[e + f*x])^(4/3))
```

3.148.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\int \frac{1}{\sqrt{\cos(e+fx)}(b\sin(e+fx))^{5/6}}dx}{d(b\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},\sin^2(e+fx)\right)}{df(b\sin(e+fx))^{4/3}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

3.148.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.148.4 Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

3.148.5 Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*cos(f*x + e)^2
- b^2), x)
```

3.148.6 Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(4/3), x)
```

3.148.7 Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

3.148.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3), x)
```

3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

3.149.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx)$$

output
```
6/23*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 23/12],[35/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

3.149.2 Mathematica [A] (verified)

Time = 10.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}{f}$$

input
```
Integrate[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 11/12, 23/12, Si
n[e + f*x]^2])*(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])/f
```

3.149.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{17/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)}{23df}$$

input
```
Int[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.149.4 Maple [F]

$$\int (b\sin(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

3.149.5 Fricas [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sin(f*x + e)*tan(
f*x + e), x)
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.149.7 Maxima [F]

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

3.149.8 Giac [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\,\sin(e+f\,x))^{4/3}\,(d\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2), x)
```

3.150 $\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

3.150.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

3.150.2 Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{12},\frac{17}{12},\sin^2(e+fx)\right)\right)\sqrt[3]{b\sin(e+fx)}\sqrt{d\tan(e+fx)}}{f}$$

input
```
Integrate[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/12, 17/12, Sin
[e + f*x]^2])*(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])/f
```

3.150.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \ 3082$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{11/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3057$$

$$\frac{6\cos^2(e+fx)^{5/4}\sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.150.4 Maple [F]

$$\int (b\sin (fx+e))^{\frac{1}{3}} (d\tan (fx+e))^{\frac{3}{2}}dx$$

input
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

3.150.5 Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin (fx+e))^{\frac{1}{3}} (d\tan (fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.150.7 Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

3.150.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\int \left(b\,\sin\left(e+f\,x\right)\right)^{1/3}\,\left(d\tan(e+f\,x)\right)^{3/2}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2), x)
```

3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

3.151.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

output
```
6/13*(cos(f*x+e)^2)^(5/4)*hypergeom([13/12, 5/4],[25/12],sin(f*x+e)^2)*(d*
tan(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(1/3)
```

3.151.2 Mathematica [A] (verified)

Time = 10.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = -\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},\sin^2(e+fx)\right)\right)\sqrt{d\tan(e+fx)}}{f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, Sin
[e + f*x]^2])*Sqrt[d*Tan[e + f*x]])/(f*(b*Sin[e + f*x])^(1/3))
```

3.151.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{7/6}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{(b\sin(e+fx))^{7/6}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{25}{12},\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x
]^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

3.151.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.151.4 Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

3.151.5 Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sin
(f*x + e)), x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(1/3),x)
```

output
```
Timed out
```

3.151.7 Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

3.151.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin(e+f\,x))^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3), x)
```

3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$

3.152.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx=\frac{6\cos^2(e+fx)^{5/4}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

output
```
6/7*(cos(f*x+e)^2)^(5/4)*hypergeom([7/12, 5/4],[19/12],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(4/3)
```

3.152.2 Mathematica [A] (verified)

Time = 10.83 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = -\frac{2d\left(-7+4\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{2/3}\sqrt{d\tan(e+fx)}}{7b^2f}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(-2*d*(-7 + 4*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/12, 19/12, S
in[e + f*x]^2])*(b*Sin[e + f*x])^(2/3)*Sqrt[d*Tan[e + f*x]])/(7*b^2*f)
```

3.152.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}dx$$

$$\Big\downarrow \; 3082$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos^{\frac{3}{2}}(e+fx)}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos(e+fx)^{3/2}}dx}{d(b\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3057$$

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

3.152.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.152.4 Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

3.152.5 Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*
cos(f*x + e)^2 - b^2), x)
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.152.7 Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

3.152.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx=\int \frac{{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{4/3}} \,d x$$

input
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3), x)
```

3.153 $\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

3.153.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

3.153.2 Mathematica [A] (verified)

Time = 11.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = -\frac{3d\left(-1+\text{Hypergeometric2F1}\left(\frac{5}{12},\frac{5}{4},\frac{17}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}}{f}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*d*(-1 + Hypergeometric2F1[5/12, 5/4, 17/12, -Tan[e + f*x]^2]*(Sec[e +
f*x]^2)^(1/4))*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3))/f
```

3.153.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \ 3082$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{4}{3}}(e+fx)}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \ 3042$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{11/6}}{\cos(e+fx)^{4/3}}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \ 3057$$

$$\frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{7/3}\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x
]^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

3.153.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.153.4 Maple [F]

$$\int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

3.153.5 Fricas [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.153.7 Maxima [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

3.153.8 Giac [F]

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sin(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,dx=\int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{4/3}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3), x)
```

3.154 $\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$

3.154.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

output
```
6/11*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 11/12],[23/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

3.154.2 Mathematica [A] (verified)

Time = 10.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(\frac{11}{12},\frac{5}{4},\frac{23}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}}{11df}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Hypergeometric2F1[11/12, 5/4, 23/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^
(1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

3.154.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{23}{12},\sin^2(e+fx)\right)}{11df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x
]^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

3.154.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.154.4 Maple [F]

$$\int \sqrt{b \sin \left(f x +e \right)}\, \left(d \tan \left(f x +e \right)\right)^{\frac{1}{3}}d x$$

input
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

3.154.5 Fricas [F]

$$\int \sqrt{b \sin(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx = \int \sqrt{b \sin \left(f x + e\right)} \left(d \tan \left(f x + e\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.154.6 Sympy [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(e+fx\right)}\sqrt[3]{d\tan\left(e+fx\right)}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sin(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

3.154.7 Maxima [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.154.8 Giac [F]

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sin\left(fx+e\right)}\left(d\tan\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sin(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{1/3}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3), x)
```

3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

3.155.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

output
```
6/7*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 7/12],[19/12],sin(f*x+e)^2)*(b*si
n(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

3.155.2 Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}}{7df}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Hypergeometric2F1[7/12, 5/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(
1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

3.155.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}\sqrt[6]{b\sin(e+fx)}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

3.155.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.155.4 Maple [F]

$$\int \frac{\sqrt{b \sin (fx+e)}}{(d \tan (fx+e))^{\frac{1}{3}}} dx$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

3.155.5 Fricas [F]

$$\int \frac{\sqrt{b \sin(e+fx)}}{\sqrt[3]{d \tan(e+fx)}} dx = \int \frac{\sqrt{b \sin (fx+e)}}{(d \tan (fx+e))^{\frac{1}{3}}} dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

3.155.6 Sympy [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin{(e+fx)}}}{\sqrt[3]{d\tan{(e+fx)}}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

3.155.7 Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.155.8 Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{{\left(d\tan\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

3.156.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

output
```
6*hypergeom([-1/6, 1/12],[13/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(1/2)/d/f/(c
os(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

3.156.2 Mathematica [A] (verified)

Time = 31.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{3\sqrt[4]{\sec^2(e+fx)}\sqrt{b\sin(e+fx)}\left(13\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{1}{4},\frac{13}{12},-\tan^2(e+fx\right.\right.}{}$$

input
```
Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Sec[e + f*x]^2)^(1/4)*Sqrt[b*Sin[e + f*x]]*(13*Hypergeometric2F1[1/12,
 1/4, 13/12, -Tan[e + f*x]^2] + 13*Hypergeometric2F1[1/12, 5/4, 13/12, -Ta
n[e + f*x]^2] - Hypergeometric2F1[13/12, 5/4, 25/12, -Tan[e + f*x]^2]*Tan[
e + f*x]^2))/(13*d*f*(d*Tan[e + f*x])^(1/3))
```

3.156.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b \sin(e+fx)}}{(d \tan(e+fx))^{4/3}} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{b \sin(e+fx)}}{(d \tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \ 3082$$

$$\frac{b \sqrt[3]{b \sin(e+fx)} \int \frac{\cos^{\frac{4}{3}}(e+fx)}{(b \sin(e+fx))^{5/6}}dx}{d \sqrt[3]{\cos(e+fx)} \sqrt[3]{d \tan(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{b \sqrt[3]{b \sin(e+fx)} \int \frac{\cos(e+fx)^{4/3}}{(b \sin(e+fx))^{5/6}}dx}{d \sqrt[3]{\cos(e+fx)} \sqrt[3]{d \tan(e+fx)}}$$

$$\Big\downarrow \ 3057$$

$$\frac{6 \sqrt{b \sin(e+fx)} \text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{13}{12},\sin^2(e+fx)\right)}{df \sqrt[6]{\cos^2(e+fx)} \sqrt[3]{d \tan(e+fx)}}$$

input
```
Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x
]])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

3.156.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.156.4 Maple [F]

$$\int \frac{\sqrt{b \sin \left(f x + e\right)}}{\left(d \tan \left(f x + e\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

3.156.5 Fricas [F]

$$\int \frac{\sqrt{b \sin(e+fx)}}{(d \tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b \sin \left(f x + e\right)}}{\left(d \tan \left(f x + e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
 x)
```

3.156.6 Sympy [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin{(e+fx)}}}{(d\tan{(e+fx)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(4/3), x)
```

3.156.7 Maxima [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.156.8 Giac [F]

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sin{(fx+e)}}}{(d\tan{(fx+e)})^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{{\left(d\tan\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

3.157.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{6\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)(b\sin(e+fx)}{23df}$$

output
```
6/23*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 23/12],[35/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

3.157.2 Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3d\left(-\text{Hypergeometric2F1}\left(\frac{11}{12},\frac{7}{4},\frac{23}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)+\sqrt[4]{\sec^2}\right.}{f\sqrt[4]{\sec^2(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*d*(-(Hypergeometric2F1[11/12, 7/4, 23/12, -Tan[e + f*x]^2]*Sec[e + f*x]
^2) + (Sec[e + f*x]^2)^(1/4))*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3
))/(f*(Sec[e + f*x]^2)^(1/4))
```

3.157.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{4}{3}}(e+fx)}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\int \frac{(b\sin(e+fx))^{17/6}}{\cos(e+fx)^{4/3}}dx}{d(b\sin(e+fx))^{7/3}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{35}{12},\sin^2(e+fx)\right)}{23df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

3.157.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.157.4 Maple [F]

$$\int (b\sin (fx+e))^{\frac{3}{2}} (d\tan (fx+e))^{\frac{4}{3}}\, dx$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

3.157.5 Fricas [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\, dx = \int (b\sin (fx+e))^{\frac{3}{2}} (d\tan (fx+e))^{\frac{4}{3}}\, dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sin(f*x + e)*tan(
f*x + e), x)
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.157.7 Maxima [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

3.157.8 Giac [F]

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx=\int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\,\sin(e+f\,x))^{3/2}\,(d\tan(e+f\,x))^{4/3}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3), x)
```

3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

3.158.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos^2(e+fx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)(b\sin(e+fx))}{17df}$$

output
```
6/17*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 17/12],[29/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

3.158.2 Mathematica [A] (verified)

Time = 10.69 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{6\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{17}{12},\frac{7}{4},\frac{29}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{7/4}(}{17bf}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*Cos[e + f*x]*Hypergeometric2F1[17/12, 7/4, 29/12, -Tan[e + f*x]^2]*(Sec
[e + f*x]^2)^(7/4)*(b*Sin[e + f*x])^(5/2)*(d*Tan[e + f*x])^(1/3))/(17*b*f)
```

3.158.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \; 3082$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}dx}{d(b\sin(e+fx))^{4/3}}$$

$$\Big\downarrow \; 3057$$

$$\frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{29}{12},\sin^2(e+fx)\right)}{17df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

3.158.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.158.4 Maple [F]

$$\int (b\sin(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}} dx$$

input
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

3.158.5 Fricas [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sin(f*x + e), x)
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

3.158.8 Giac [F]

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
sage0*x
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\,\sin(e+f\,x))^{3/2}\,(d\tan(e+f\,x))^{1/3}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3), x)
```

3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

3.159.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\frac{6\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d}{13df}$$

output
```
6/13*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 13/12],[25/12],sin(f*x+e)^2)*(b*
sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

3.159.2 Mathematica [A] (verified)

Time = 10.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\frac{2d\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{13}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e}{3f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*(-1 + Hypergeometric2F1[1/12, 3/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f
*x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(3*f*(d*Tan[e + f*x])^(4/3))
```

3.159.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\int \sqrt[3]{\cos(e+fx)}(b\sin(e+fx))^{7/6}dx}{d(b\sin(e+fx))^{2/3}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6\sqrt[3]{\cos^2(e+fx)}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{25}{12},\sin^2(e+fx)\right)}{13df}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x
]^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

3.159.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.159.4 Maple [F]

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

3.159.5 Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d*tan
(f*x + e)), x)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Timed out
```

3.159.7 Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.159.8 Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx=\int \frac{{\left(b\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(d\,\tan\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

3.160.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{6\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

output
```
6/7*hypergeom([-1/6, 7/12],[19/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(3/2)/d/f/
(cos(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

3.160.2 Mathematica [A] (verified)

Time = 10.70 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2\left(7+2\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{19}{12},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e+fx))^{3/2}}{21df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*(7 + 2*Hypergeometric2F1[7/12, 3/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(21*d*f*(d*Tan[e + f*x])^(1/3))
```

3.160.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \cos^{\frac{4}{3}}(e+fx)\sqrt[6]{b\sin(e+fx)}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt[3]{b\sin(e+fx)}\int \cos(e+fx)^{4/3}\sqrt[6]{b\sin(e+fx)}dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{6(b\sin(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{19}{12},\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(
3/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

3.160.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.160.4 Maple [F]

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

3.160.5 Fricas [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d^2*t
an(f*x + e)^2), x)
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.160.7 Maxima [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.160.8 Giac [F]

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\,\sin\left(e+f\,x\right))^{3/2}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$

3.161.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},\frac{6+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4f(4+m)}$$

output
```
hypergeom([2, 2+1/2*m],[3+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(4+m)/a^4/f/
(4+m)
```

3.161.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{4+m}{2},1+\frac{4+m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m}{f(4+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(Hypergeometric2F1[2, (4 + m)/2, 1 + (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*
x]^4*(a*Sin[e + f*x])^m)/(f*(4 + m))
```

3.161.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3072, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+3}}{\left(a^2-a^2\sin^2(e+fx)\right)^2}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\sin(e+fx))^{m+4}\,\text{Hypergeometric2F1}\left(2,\frac{m+4}{2},\frac{m+6}{2},\sin^2(e+fx)\right)}{a^4f(m+4)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x
])^(4 + m))/(a^4*f*(4 + m))
```

3.161.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.161.4 Maple [F]

$$\int (\sin(fx+e)\,a)^m \left(\tan^3(fx+e)\right) dx$$

input
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^3,x)
```

output
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^3,x)
```

3.161.5 Fricas [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

3.161.6 Sympy [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**3, x)
```

3.161.7 Maxima [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

3.161.8 Giac [F]

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^3*(a*sin(e + f*x))^m, x)
```

3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$

3.162.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2f(2+m)}$$

output
```
hypergeom([1, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(2+m)/a^2/f/
(2+m)
```

3.162.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},1+\frac{2+m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m}{f(2+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, Sin[e + f*x]^2]*Sin[e + f*
x]^2*(a*Sin[e + f*x])^m)/(f*(2 + m))
```

3.162.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3072, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+1}}{a^2-a^2\sin^2(e+fx)}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\sin(e+fx))^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},\sin^2(e+fx)\right)}{a^2f(m+2)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x
])^(2 + m))/(a^2*f*(2 + m))
```

3.162.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.162.4 Maple [F]

$$\int (\sin (fx+e)\, a)^{m} \tan (fx+e)\, dx$$

input
```
int((sin(f*x+e)*a)^m*tan(f*x+e),x)
```

output
```
int((sin(f*x+e)*a)^m*tan(f*x+e),x)
```

3.162.5 Fricas [F]

$$\int (a\sin(e+fx))^m \tan(e+fx)\, dx = \int (a\sin (fx+e))^{m} \tan (fx+e)\; dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e), x)
```

3.162.6 Sympy [F]

$$\int (a\sin(e+fx))^m \tan(e+fx)\, dx = \int (a\sin (e+fx))^{m} \tan (e+fx)\, dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e),x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x), x)
```

3.162.7 Maxima [F]

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

3.162.8 Giac [F]

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx = \int \tan(e+f\,x)\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)*(a*sin(e + f*x))^m, x)
```

3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$

3.163.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx=\frac{(a\sin(e+fx))^m}{fm}$$

output
```
(a*sin(f*x+e))^m/f/m
```

3.163.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx=\frac{(a\sin(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sin[e + f*x])^m/(f*m)
```

3.163.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3072, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int (a\sin(e+fx))^{m-1}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a\sin(e+fx))^m}{fm}$$

input
```
Int[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sin[e + f*x])^m/(f*m)
```

3.163.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.163.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{(\sin(fx+e)a)^m}{fm}$
default	$\frac{(\sin(fx+e)a)^m}{fm}$
risch	$\left(e^{i(fx+e)}\right)^{-m}\left(e^{2i(fx+e)}-1\right)^{m}\left(\frac{1}{2}\right)^{m}a^{m}e^{-\frac{i\pi m\left(\operatorname{csgn}(i\sin(fx+e)a)^3+\operatorname{csgn}(i\sin(fx+e)a)^2\operatorname{csgn}(\sin(fx+e)a)-\operatorname{csgn}(\sin(fx+e)a)^3\right.}{}}$

input
```
int(cot(f*x+e)*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
```

output
```
(sin(f*x+e)*a)^m/f/m
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin(fx+e))^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
(a*sin(f*x + e))^m/(f*m)
```

3.163.6 Sympy [F]

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin{(e+fx)})^m \cot{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x), x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{a^m \sin{(fx+e)}^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
a^m*sin(f*x + e)^m/(f*m)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\sin{(fx+e)})^m}{fm}$$

input
```
integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
(a*sin(f*x + e))^m/(f*m)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a\sin(e+fx))^m\,dx = \frac{(a\,\sin\left(e+f\,x\right))^m}{f\,m}$$

input
```
int(cot(e + f*x)*(a*sin(e + f*x))^m,x)
```

output
```
(a*sin(e + f*x))^m/(f*m)
```

3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$

3.164.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)} - \frac{(a\sin(e+fx))^m}{fm}$$

output
```
-a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)-(a*sin(f*x+e))^m/f/m
```

3.164.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{(2-m+m\csc^2(e+fx))\,(a\sin(e+fx))^m}{f(-2+m)m}$$

input
```
Integrate[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

output
```
((2 - m + m*Csc[e + f*x]^2)*(a*Sin[e + f*x])^m)/(f*(-2 + m)*m)
```

3.164.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3072, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int (a\sin(e+fx))^{m-3}\left(a^2-a^2\sin^2(e+fx)\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(a^2(a\sin(e+fx))^{m-3}-(a\sin(e+fx))^{m-1}\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2(a\sin(e+fx))^{m-2}}{2-m}-\frac{(a\sin(e+fx))^m}{m}}{f}$$

input
```
Int[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

output
```
(-((a^2*(a*Sin[e + f*x])^(-2 + m))/(2 - m)) - (a*Sin[e + f*x])^m/m)/f
```

3.164.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.164.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.19 (sec) , antiderivative size = 2751, normalized size of antiderivative = 59.80

method	result	size
risch	Expression too large to display	2751

input
```
int(cot(f*x+e)^3*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
```

output
```
-1/(-2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*exp(I*(f*x+e))^(-m)*(exp(2*I*(f*x+e))
-1)^m*(1/2)^m*a^m*(m*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*si
n(e))^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2*c
sgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*c
os(e)+a*cos(f*x)*sin(e))^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*
x)*sin(e))^2*csgn(I*a)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*
sin(e))^2*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(-1/2*I*m*csgn(a*si
n(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(I*a)*Pi*csgn(sin(f*x)*cos(e)+cos(f*x
)*sin(e)))*exp(1/2*I*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2
*I*Pi*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(-I*(f*x+e))))*e
xp(1/2*I*Pi*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2*csgn(I*exp(2*I*(f*x+
e))-I))*exp(1/2*I*Pi*m*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))*csgn(I*exp(-I
*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))-I))*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e
)+I*a*cos(f*x)*sin(e))^2*Pi)*exp(1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(
f*x)*sin(e))*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(-1/2*I*Pi*m
)*exp(4*I*f*x)*exp(4*I*e)-2*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(
f*x)*sin(e))^3*Pi)*exp(-1/2*I*m*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(
e))^2*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*Pi)*exp(1/2*I*m*csgn(a*sin
(f*x)*cos(e)+a*cos(f*x)*sin(e))^3*Pi)*exp(1/2*I*m*csgn(a*sin(f*x)*cos(e)+a
*cos(f*x)*sin(e))^2*csgn(I*a)*Pi)*exp(-1/2*I*m*csgn(a*sin(f*x)*cos(e)+a...
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m-2)\cos\left(fx+e\right)^2+2\right)\left(a\sin\left(fx+e\right)\right)^m}{fm^2-\left(fm^2-2\,fm\right)\cos\left(fx+e\right)^2-2\,fm}$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
((m - 2)*cos(f*x + e)^2 + 2)*(a*sin(f*x + e))^m/(f*m^2 - (f*m^2 - 2*f*m)*c
os(f*x + e)^2 - 2*f*m)
```

3.164.6 Sympy [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int \left(a\sin\left(e+fx\right)\right)^m\cot^3\left(e+fx\right)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**3, x)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{\frac{a^m\sin(fx+e)^m}{m}-\frac{a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}}{f}$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
-(a^m*sin(f*x + e)^m/m - a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2))/f
```

3.164.8 Giac [F]

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^3, x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx = -\frac{(a\,\sin(e+f\,x))^m\,\left(m-4\sin(2\,e+2\,f\,x)^2+m\,\left(2\sin(2\,e+2\,f\,x)^2-1\right)+16\sin(e+f\,x)^2\right)}{f\,m\,\left(2\sin(2\,e+2\,f\,x)^2-8\sin(e+f\,x)^2\right)\,(m-2)}$$

input
```
int(cot(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

output
```
-((a*sin(e + f*x))^m*(m - 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
 1) + 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(
m - 2))
```

3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$

3.165.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)} + \frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)} + \frac{(a\sin(e+fx))^m}{fm}$$

output
```
-a^4*(a*sin(f*x+e))^(-4+m)/f/(4-m)+2*a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)+(a*
sin(f*x+e))^m/f/m
```

3.165.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{(8-6m+m^2-2(-4+m)m\csc^2(e+fx)+(-2+m)m\csc^4(e+fx))\,(a\sin(e+fx))^m}{f(-4+m)(-2+m)m}$$

input
```
Integrate[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

output
```
((8 - 6*m + m^2 - 2*(-4 + m)*m*Csc[e + f*x]^2 + (-2 + m)*m*Csc[e + f*x]^4)
*(a*Sin[e + f*x])^m)/(f*(-4 + m)*(-2 + m)*m)
```

3.165.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3072, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int (a\sin(e+fx))^{m-5}\left(a^2-a^2\sin^2(e+fx)\right)^2d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(a^4(a\sin(e+fx))^{m-5}-2a^2(a\sin(e+fx))^{m-3}+(a\sin(e+fx))^{m-1}\right)d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^4(a\sin(e+fx))^{m-4}}{4-m}+\frac{2a^2(a\sin(e+fx))^{m-2}}{2-m}+\frac{(a\sin(e+fx))^m}{m}}{f}$$

input
```
Int[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

output
```
(-((a^4*(a*Sin[e + f*x])^(-4 + m))/(4 - m)) + (2*a^2*(a*Sin[e + f*x])^(-2
+ m))/(2 - m) + (a*Sin[e + f*x])^m/m)/f
```

3.165.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.165.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.02 (sec) , antiderivative size = 6931, normalized size of antiderivative = 96.26

method	result	size
risch	Expression too large to display	6931

input
```
int(cot(f*x+e)^5*(sin(f*x+e)*a)^m,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.56

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\left((m^2-6\,m+8)\cos\left(fx+e\right)^4+4\,(m-4)\cos\left(fx+e\right)^2+8\right)\left(a\sin\left(fx+e\right)\right)^m}{\left(fm^3-6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4+fm^3-6\,fm^2-2\left(fm^3-6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^2+8\,fm}$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
((m^2 - 6*m + 8)*cos(f*x + e)^4 + 4*(m - 4)*cos(f*x + e)^2 + 8)*(a*sin(f*x
 + e))^m/((f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 - 6*f*m^2 - 2*(
f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

3.165.6 Sympy [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \int \left(a\sin\left(e+fx\right)\right)^m\cot^5\left(e+fx\right)\,dx$$

input
```
integrate(cot(f*x+e)**5*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**5, x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \frac{\frac{a^m\sin(fx+e)^m}{m} - \frac{2\,a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2} + \frac{a^m\sin(fx+e)^m}{(m-4)\sin(fx+e)^4}}{f}$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
(a^m*sin(f*x + e)^m/m - 2*a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2) + a^
m*sin(f*x + e)^m/((m - 4)*sin(f*x + e)^4))/f
```

3.165.8 Giac [F]

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^5, x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.04

$$\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx =$$

$$-\frac{(a\sin(e+fx))^m\left(2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}-1\right)\left(-\frac{2\left(2\sin(2e+2fx)^2-1\right)\left(-2\sin(2e+2fx)^2+\mathrm{s}\right.}{fm}\right.}{}$$

input
```
int(cot(e + f*x)^5*(a*sin(e + f*x))^m,x)
```

output
```
-((a*sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*(((s
in(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(6*m^2 - 4*m + 48))/(f*m*(m
^2 - 6*m + 8)) - (2*(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*si
n(2*e + 2*f*x)^2 + 1))/(f*m) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)
*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(8*m - 4*m^2 + 32))/(f*m*(m^2 - 6*m + 8)))
)/(16*sin(e + f*x)^4)
```

3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$

3.166.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5 f(5+m)}$$

output
```
hypergeom([5/2, 5/2+1/2*m],[7/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x
+e))^(5+m)*(cos(f*x+e)^2)^(1/2)/a^5/f/(5+m)
```

3.166.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7+m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(5+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
 f*x]^2]*Sin[e + f*x]^4*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(5 + m))
```

3.166.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3080}$$

$$\frac{\int \sec^4(e+fx)(a\sin(e+fx))^{m+4}dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+4}}{\cos(e+fx)^4}dx}{a^4}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{m+7}{2},\sin^2(e+fx)\right)}{a^5f(m+5)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
 f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

3.166.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

3.166.4 Maple [F]

$$\int (\sin (fx+e)\, a)^{m} \left(\tan^{4}(fx+e)\right) dx$$

input
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^4,x)
```

output
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^4,x)
```

3.166.5 Fricas [F]

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin(fx+e))^m \tan(fx+e)^4 \, dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

3.166.6 Sympy [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx = \int (a\sin{(e+fx)})^m\tan^4{(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**4, x)
```

3.166.7 Maxima [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx = \int (a\sin{(fx+e)})^m\tan{(fx+e)}^4\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

3.166.8 Giac [F]

$$\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx = \int (a\sin{(fx+e)})^m\tan{(fx+e)}^4\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.167 $\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx$

3.167.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

output
```
hypergeom([3/2, 3/2+1/2*m],[5/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x
+e))^(3+m)*(cos(f*x+e)^2)^(1/2)/a^3/f/(3+m)
```

3.167.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5+m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(3+m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
 f*x]^2]*Sin[e + f*x]^2*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(3 + m))
```

3.167.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3080}$$

$$\frac{\int \sec^2(e+fx)(a\sin(e+fx))^{m+2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a\sin(e+fx))^{m+2}}{\cos(e+fx)^2}dx}{a^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{m+5}{2},\sin^2(e+fx)\right)}{a^3f(m+3)}$$

input
```
Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
 f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

3.167.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

3.167.4 Maple [F]

$$\int (\sin (fx+e) a)^m (\tan^2 (fx+e)) dx$$

input
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^2,x)
```

output
```
int((sin(f*x+e)*a)^m*tan(f*x+e)^2,x)
```

3.167.5 Fricas [F]

$$\int (a\sin(e+fx))^m \tan^2(e+fx) \, dx = \int (a \sin (fx+e))^m \tan (fx+e)^2 \, dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

3.167.6 Sympy [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx = \int (a\sin{(e+fx)})^m\tan^2{(e+fx)}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((a*sin(e + f*x))**m*tan(e + f*x)**2, x)
```

3.167.7 Maxima [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx = \int (a\sin{(fx+e)})^m\tan{(fx+e)}^2\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

3.167.8 Giac [F]

$$\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx = \int (a\sin{(fx+e)})^m\tan{(fx+e)}^2\,dx$$

input
```
integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$

3.168.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

output
```
-a*cos(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2+1/2*m],sin(f*x+e)^2)*(a*si
n(f*x+e))^(-1+m)/f/(1-m)/(cos(f*x+e)^2)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \frac{a\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{-1+m}}{f(-1+m)}$$

input
```
Integrate[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

output
```
(a*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin
[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(-1 + m))/(f*(-1 + m))
```

3.168.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \; 3080$$

$$a^2\int \cos^2(e+fx)(a\sin(e+fx))^{m-2}dx$$

$$\Big\downarrow \; 3042$$

$$a^2\int \cos(e+fx)^2(a\sin(e+fx))^{m-2}dx$$

$$\Big\downarrow \; 3057$$

$$-\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

output
```
-((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f
*x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

3.168.4 Maple [F]

$$\int \left(\cot^{2}\left(fx+e\right)\right) \left(\sin \left(fx+e\right) a\right)^{m} dx$$

input
```
int(cot(f*x+e)^2*(sin(f*x+e)*a)^m,x)
```

output
```
int(cot(f*x+e)^2*(sin(f*x+e)*a)^m,x)
```

3.168.5 Fricas [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int \left(a\sin\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

3.168.6 Sympy [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**2, x)
```

3.168.7 Maxima [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

3.168.8 Giac [F]

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^2\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$

3.169.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a^3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)(a\sin(e+fx))^{-3+m}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

output
```
-a^3*cos(f*x+e)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(f*x+e)^2)*(a
*sin(f*x+e))^(-3+m)/f/(3-m)/(cos(f*x+e)^2)^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),\frac{1}{2}(-1+m),\sin^2(e+fx)\right)\sec(e+fx)(}{f(-3+m)}$$

input
```
Integrate[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Hypergeometric2F1[-3/2, (-3 + m)/2, (
-1 + m)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^m)/(f*(-3 + m))
```

3.169.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3080, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3080}$$

$$a^4\int \cos^4(e+fx)(a\sin(e+fx))^{m-4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \cos(e+fx)^4(a\sin(e+fx))^{m-4}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

output
```
-((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e
+ f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

3.169.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3080
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Simp[1/a^n   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /
; FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

3.169.4 Maple [F]

$$\int \left(\cot^{4}\left(f x +e \right)\right) \left(\sin \left(f x +e \right) a \right)^{m}d x$$

input
```
int(cot(f*x+e)^4*(sin(f*x+e)*a)^m,x)
```

output
```
int(cot(f*x+e)^4*(sin(f*x+e)*a)^m,x)
```

3.169.5 Fricas [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx=\int \left(a \sin \left(f x + e\right)\right)^{m} \cot \left(f x + e\right)^{4}\,{d x}$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

3.169.6 Sympy [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m \cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*cot(e + f*x)**4, x)
```

3.169.7 Maxima [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

3.169.8 Giac [F]

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx=\int \cot(e+f\,x)^4\,(a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.170 $\int (a \sin(e+fx))^m (b \tan(e+fx))^{3/2} \, dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a \sin(e+fx))^m (b \tan(e+fx))^{3/2} \, dx = \frac{2 \cos^2(e+fx)^{5/4} \text{Hypergeometric2F1}\left(\frac{5}{4}, \frac{1}{4}(5+2m), \frac{1}{4}(9+2m), \sin^2(e+fx)\right) (a \sin(e+fx))}{bf(5+2m)}$$

output
```
2*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 5/4+1/2*m],[9/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(5/2)/b/f/(5+2*m)
```

3.170.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (a \sin(e+fx))^m (b \tan(e+fx))^{3/2} \, dx = \frac{2 \text{Hypergeometric2F1}\left(\frac{2+m}{2}, \frac{1}{4}(5+2m), \frac{1}{4}(9+2m), -\tan^2(e+fx)\right) \sec^2(e+fx)^{m/2} (a \sin(e}{bf(5+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5
 + 2*m))
```

3.170.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(e+fx))^{3/2}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \; 3082$$

$$\frac{a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\int \frac{(a\sin(e+fx))^{m+\frac{3}{2}}}{\cos^{\frac{3}{2}}(e+fx)}dx}{b(a\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\int \frac{(a\sin(e+fx))^{m+\frac{3}{2}}}{\cos(e+fx)^{3/2}}dx}{b(a\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3057$$

$$\frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(2m+5),\frac{1}{4}(2m+9),\sin^2(e+fx)\right)}{bf(2m+5)}$$

input
```
Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m)
)
```

3.170.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.170.4 Maple [F]

$$\int (\sin (fx+e)\, a)^{m} (b\tan (fx+e))^{\frac{3}{2}}\, dx$$

input
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(3/2),x)
```

output
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(3/2),x)
```

3.170.5 Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\, dx = \int (b\tan (fx+e))^{\frac{3}{2}} (a\sin (fx+e))^{m}\, dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m*b*tan(f*x + e), x)
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

3.170.8 Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \int (a\,\sin(e+f\,x))^m\,(b\tan(e+f\,x))^{3/2}\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2), x)
```

3.171 $\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx$

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e+}{bf(3+2m)}$$

output
```
2*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 3/4+1/2*m],[7/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2)/b/f/(3+2*m)
```

3.171.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(3+2m),\frac{1}{4}(7+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m(b\,\text{ta}}{bf(3+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3
 + 2*m))
```

3.171.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\int \frac{(a\sin(e+fx))^{m+\frac{1}{2}}}{\sqrt{\cos(e+fx)}}dx}{b(a\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\int \frac{(a\sin(e+fx))^{m+\frac{1}{2}}}{\sqrt{\cos(e+fx)}}dx}{b(a\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(2m+3),\frac{1}{4}(2m+7),\sin^2(e+fx)\right)}{bf(2m+3)}$$

input
```
Int[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m)
)
```

3.171.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.171.4 Maple [F]

$$\int (\sin (fx+e) a)^m \sqrt{b \tan (fx+e)}dx$$

input
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(1/2),x)
```

output
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^(1/2),x)
```

3.171.5 Fricas [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan(fx+e)}(a\sin(fx+e))^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

3.171.6 Sympy [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int (a\sin{(e+fx)})^m\sqrt{b\tan{(e+fx)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((a*sin(e + f*x))**m*sqrt(b*tan(e + f*x)), x)
```

3.171.7 Maxima [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan{(fx+e)}}(a\sin{(fx+e)})^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

3.171.8 Giac [F]

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan{(fx+e)}}(a\sin{(fx+e)})^m\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,\sqrt{b\tan\left(e+f\,x\right)}\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2), x)
```

3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b\tan(e-}{bf(1+2m)}$$

output
```
2*(cos(f*x+e)^2)^(1/4)*hypergeom([1/4, 1/4+1/2*m],[5/4+1/2*m],sin(f*x+e)^2
)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2)/b/f/(1+2*m)
```

3.172.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(1+2m),\frac{1}{4}(5+2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\sqrt{b}}{bf(1+2m)}$$

input
```
Integrate[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[e + f*x]^2]
*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 +
 2*m))
```

3.172.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \sqrt{\cos(e+fx)}(a\sin(e+fx))^{m-\frac{1}{2}}dx}{b\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\int \sqrt{\cos(e+fx)}(a\sin(e+fx))^{m-\frac{1}{2}}dx}{b\sqrt{a\sin(e+fx)}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(2m+1),\frac{1}{4}(2m+5),\sin^2(e+fx)\right)}{bf(2m+1)}$$

input
```
Int[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

3.172.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.172.4 Maple [F]

$$\int \frac{(\sin(fx+e)\,a)^m}{\sqrt{b\tan(fx+e)}}dx$$

input
```
int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(1/2),x)
```

output
```
int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(1/2),x)
```

3.172.5 Fricas [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b*tan(f*x + e)), x)
```

3.172.6 Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$$

input
```
integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((a*sin(e + f*x))**m/sqrt(b*tan(e + f*x)), x)
```

3.172.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

3.172.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^m}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2), x)
```

3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

output
```
-2*hypergeom([-1/4, -1/4+1/2*m],[3/4+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^m
/b/f/(1-2*m)/(cos(f*x+e)^2)^(1/4)/(b*tan(f*x+e))^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1}{4}(-1+2m),\frac{1}{4}(3+2m),-\tan^2(e+fx)\right)\sec^2(e+f}{bf(-1+2m)\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[(2 + m)/2, (-1 + 2*m)/4, (3 + 2*m)/4, -Tan[e + f*x]^2
]*(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m)/(b*f*(-1 + 2*m)*Sqrt[b*Tan[e
+ f*x]])
```

3.173.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\sqrt{a\sin(e+fx)}\int \cos^{\frac{3}{2}}(e+fx)(a\sin(e+fx))^{m-\frac{3}{2}}dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\sin(e+fx)}\int \cos(e+fx)^{3/2}(a\sin(e+fx))^{m-\frac{3}{2}}dx}{b\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3057}$$

$$-\frac{2(a\sin(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(2m-1),\frac{1}{4}(2m+3),\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

input

```
Int[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

output

```
(-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*
Sin[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]
])
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.173.4 Maple [F]

$$\int \frac{(\sin (fx+e) a)^m}{(b \tan (fx+e))^{\frac{3}{2}}} dx$$

input
```
int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(3/2),x)
```

output
```
int((sin(f*x+e)*a)^m/(b*tan(f*x+e))^(3/2),x)
```

3.173.5 Fricas [F]

$$\int \frac{(a \sin(e+fx))^m}{(b \tan(e+fx))^{3/2}} dx = \int \frac{(a \sin (fx+e))^m}{(b \tan (fx+e))^{\frac{3}{2}}} dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b^2*tan(f*x + e)^2), x)
```

3.173.6 Sympy [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((a*sin(e + f*x))**m/(b*tan(e + f*x))**(3/2), x)
```

3.173.7 Maxima [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

3.173.8 Giac [F]

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin\left(e+f\,x\right))^m}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2), x)
```

3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$

3.174.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b}{bf(1+m+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/2+1/2
*m+1/2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+m+n)
```

3.174.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.82 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.13

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$
$$= \frac{(3+m+}{f(1+m+n)\left((3+m+n)\,\text{AppellF1}\left(\frac{1}{2}(1+m+n),n,1+m,\frac{1}{2}(3+m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\right.\right.\right.}$$

input
```
Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m + n)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(a*Sin[e + f*x])^m*(b*Tan[e + f*
x])^n)/(f*(1 + m + n)*((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 +
m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*AppellF1[(
3 + m + n)/2, n, 2 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] - n*AppellF1[(3 + m + n)/2, 1 + n, 1 + m, (5 + m + n)/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

3.174.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{m+n}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{m+n}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\sin(e+fx))^m(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(e+fx)\right)}{bf(m+n+1)}$$

input
```
Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2,
(3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + m + n))
```

3.174.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.174.4 Maple [F]

$$\int (\sin (fx+e)\, a)^m\, (b\tan (fx+e))^n\, dx$$

input
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^n,x)
```

output
```
int((sin(f*x+e)*a)^m*(b*tan(f*x+e))^n,x)
```

3.174.5 Fricas [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.174.6 Sympy [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(e+fx))^m\,(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*sin(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.174.7 Maxima [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.174.8 Giac [F]

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sin{(fx+e)})^m\,(b\tan{(fx+e)})^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\,\sin{(e+f\,x)})^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$

3.175.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}$$

output
```
hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(5+n)/b
^5/f/(5+n)
```

3.175.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{5+n}{2},\frac{7+n}{2},-\tan^2(e+fx)\right)\tan^5(e+fx)(b\tan(e+fx))^n}{f(5+n)}$$

input
```
Integrate[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^
5*(b*Tan[e + f*x])^n)/(f*(5 + n))
```

3.175.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3071, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^4(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int \frac{(b\tan(e+fx))^{n+4}}{(\tan^2(e+fx)b^2+b^2)^3}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\tan(e+fx))^{n+5}\,\text{Hypergeometric2F1}\left(3,\frac{n+5}{2},\frac{n+7}{2},-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

input
```
Int[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*
x])^(5 + n))/(b^5*f*(5 + n))
```

3.175.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.175.4 Maple [F]

$$\int \left(\sin^4(fx+e)\right)(b\tan(fx+e))^n\,dx$$

input
```
int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

3.175.5 Fricas [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e))^n, x)
```

3.175.6 Sympy [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n\sin^4(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x)**4, x)
```

3.175.7 Maxima [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

3.175.8 Giac [F]

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^4\,dx$$

input
```
integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx = \int \sin(e+f\,x)^4\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)^4*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^4*(b*tan(e + f*x))^n, x)
```

3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$

3.176.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}$$

output
```
hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(3+n)/b
^3/f/(3+n)
```

3.176.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},-\tan^2(e+fx)\right)\tan^3(e+fx)(b\tan(e+fx))^n}{f(3+n)}$$

input
```
Integrate[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*Tan[e + f*x]^
3*(b*Tan[e + f*x])^n)/(f*(3 + n))
```

3.176.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3071, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^2(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int \frac{(b\tan(e+fx))^{n+2}}{(\tan^2(e+fx)b^2+b^2)^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\tan(e+fx))^{n+3}\,\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

input
```
Int[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*
x])^(3 + n))/(b^3*f*(3 + n))
```

3.176.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.176.4 Maple [F]

$$\int \left(\sin^2(fx+e)\right)(b\tan(fx+e))^n\,dx$$

input
```
int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

3.176.5 Fricas [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n, x)
```

3.176.6 Sympy [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \sin^2(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x)**2, x)
```

3.176.7 Maxima [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

3.176.8 Giac [F]

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)^2\,dx$$

input
```
integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx=\int \sin(e+f\,x)^2\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)^2*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^2*(b*tan(e + f*x))^n, x)
```

3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$

3.177.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

3.177.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(b\tan(e+fx))^{-1+n}}{f(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(b*Tan[e + f*x])^(-1 + n))/(f*(-1 + n))
```

3.177.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3071, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int (b\tan(e+fx))^{n-2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

output
```
-((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

3.177.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.177.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{e^{n \ln(b \tan(fx+e))}}{f(-1+n) \tan(fx+e)}$	30
default	$\frac{e^{n \ln(b \tan(fx+e))}}{f(-1+n) \tan(fx+e)}$	30
risch	Expression too large to display	1750

input
```
int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
1/f/(-1+n)*exp(n*ln(b*tan(f*x+e)))/tan(f*x+e)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n \cos(fx+e)}{(fn-f)\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(b*sin(f*x + e)/cos(f*x + e))^n*cos(f*x + e)/((f*n - f)*sin(f*x + e))
```

3.177.6 Sympy [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(e+fx)})^n \csc^2{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**2, x)
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \frac{b^n\tan{(fx+e)}^n}{f(n-1)\tan{(fx+e)}}$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
b^n*tan(f*x + e)^n/(f*(n - 1)*tan(f*x + e))
```

3.177.8 Giac [F]

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan{(fx+e)})^n \csc{(fx+e)}^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^2, x)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)}^n}{2\,f\,\left({\cos\left(e+f\,x\right)}^2-1\right)\,\left(n-1\right)}$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^2,x)
```

output
```
-(sin(2*e + 2*f*x)*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^n)/(2*f*(cos(
e + f*x)^2 - 1)*(n - 1))
```

3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$

3.178.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx=-\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

3.178.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx =\frac{b(-2+n+\cos(2(e+fx)))\csc^2(e+fx)(b\tan(e+fx))^{-1+n}}{f(-3+n)(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-2 + n + Cos[2*(e + f*x)])*Csc[e + f*x]^2*(b*Tan[e + f*x])^(-1 + n))/(
f*(-3 + n)*(-1 + n))
```

3.178.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \ 3071$$

$$\frac{b\int (b\tan(e+fx))^{n-4}\left(\tan^2(e+fx)b^2+b^2\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \ 244$$

$$\frac{b\int \left(b^2(b\tan(e+fx))^{n-4}+(b\tan(e+fx))^{n-2}\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\left(-\frac{b^2(b\tan(e+fx))^{n-3}}{3-n}-\frac{(b\tan(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-((b^2*(b*Tan[e + f*x])^(-3 + n))/(3 - n)) - (b*Tan[e + f*x])^(-1 + n)
/(1 - n)))/f
```

3.178.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.178.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 20.03 (sec) , antiderivative size = 5281, normalized size of antiderivative = 99.64

method	result	size
risch	Expression too large to display	5281

input
```
int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(2\cos\left(fx+e\right)^3+(n-3)\cos\left(fx+e\right)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left(fn^2-\left(fn^2-4\,fn+3\,f\right)\cos\left(fx+e\right)^2-4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(2*cos(f*x + e)^3 + (n - 3)*cos(f*x + e))*(b*sin(f*x + e)/cos(f*x + e))^n/
((f*n^2 - (f*n^2 - 4*f*n + 3*f)*cos(f*x + e)^2 - 4*f*n + 3*f)*sin(f*x + e)
)
```

3.178.6 Sympy [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan\left(e+fx\right))^n\csc^4\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**4, x)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}}{f}$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
(b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + b^n*tan(f*x + e)^n/((n - 3)*t
an(f*x + e)^3))/f
```

3.178.8 Giac [F]

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^4, x)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx = -\frac{2\left(-\frac{b\sin(2e+2fx)}{2\sin(e+fx)^2-2}\right)^n(9\sin(2e+2fx)-6\sin(4e+4fx)+\sin(6e+6fx)-4n\sin(2e+2fx)+}{f\left(30\sin(e+fx)^2-12\sin(2e+2fx)^2+2\sin(3e+3fx)^2\right)(n^2-4n+3)}$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^4,x)
```

output
```
-(2*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))^n*(9*sin(2*e + 2*f*x) -
 6*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - 4*n*sin(2*e + 2*f*x) + 2*n*sin(4*
e + 4*f*x)))/(f*(2*sin(3*e + 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e +
 f*x)^2)*(n^2 - 4*n + 3))
```

3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$

3.179.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)} - \frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^5*(b*tan(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(
b*tan(f*x+e))^(-1+n)/f/(1-n)
```

3.179.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{b(8-6n+n^2+2(-3+n)\cos(2(e+fx))+\cos(4(e+fx)))\csc^4(e+fx)(b\tan(e+fx))^{-1+n}}{f(-5+n)(-3+n)(-1+n)}$$

input
```
Integrate[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(8 - 6*n + n^2 + 2*(-3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Csc[e
+ f*x]^4*(b*Tan[e + f*x])^(-1 + n))/(f*(-5 + n)*(-3 + n)*(-1 + n))
```

3.179.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3071, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^6}dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{b\int (b\tan(e+fx))^{n-6}\left(\tan^2(e+fx)b^2+b^2\right)^2d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(b^4(b\tan(e+fx))^{n-6}+2b^2(b\tan(e+fx))^{n-4}+(b\tan(e+fx))^{n-2}\right)d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{b^4(b\tan(e+fx))^{n-5}}{5-n}-\frac{2b^2(b\tan(e+fx))^{n-3}}{3-n}-\frac{(b\tan(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

output
```
(b*(-((b^4*(b*Tan[e + f*x])^(-5 + n))/(5 - n)) - (2*b^2*(b*Tan[e + f*x])^(
-3 + n))/(3 - n) - (b*Tan[e + f*x])^(-1 + n)/(1 - n)))/f
```

3.179.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.179.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 151.82 (sec) , antiderivative size = 10580, normalized size of antiderivative = 132.25

method	result	size
risch	Expression too large to display	10580

input
```
int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.80

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$$
$$= \frac{\left(8\cos\left(fx+e\right)^5+4\left(n-5\right)\cos\left(fx+e\right)^3+\left(n^2-8n+15\right)\cos\left(fx+e\right)\right)\left(\frac{b\sin(f}{\cos(fx}\right.}{\left(\left(fn^3-9fn^2+23fn-15f\right)\cos\left(fx+e\right)^4+fn^3-9fn^2-2\left(fn^3-9fn^2+23fn-15f\right)\cos\left(fx+e\right.\right.}$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
(8*cos(f*x + e)^5 + 4*(n - 5)*cos(f*x + e)^3 + (n^2 - 8*n + 15)*cos(f*x +
e))*(b*sin(f*x + e)/cos(f*x + e))^n/(((f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*co
s(f*x + e)^4 + f*n^3 - 9*f*n^2 - 2*(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(f
*x + e)^2 + 23*f*n - 15*f)*sin(f*x + e))
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{2\,b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}+\frac{b^n\tan(fx+e)^n}{(n-5)\tan(fx+e)^5}}{f}$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
(b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + 2*b^n*tan(f*x + e)^n/((n - 3)
*tan(f*x + e)^3) + b^n*tan(f*x + e)^n/((n - 5)*tan(f*x + e)^5))/f
```

3.179.8 Giac [F]

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^6, x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^6}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^6,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^6, x)
```

3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$

3.180.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{6+n}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3+1/2*n],sin(f*
x+e)^2)*sin(f*x+e)^3*(b*tan(f*x+e))^(1+n)/b/f/(4+n)
```

3.180.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.30 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.85

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{}{f(2+n)\left(-2(4+n)\,\text{AppellF1}\left(1+\frac{n}{2},n,4,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx\right)\right.}$$

input
```
Integrate[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
(4*(4 + n)*(AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[e + f*x]^3*(b*Tan[e
 + f*x])^n)/(f*(2 + n)*(-2*(4 + n)*AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 +
n/2, n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[
2 + n/2, n, 5, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-App
ellF1[2 + n/2, 1 + n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + AppellF1[2 + n/2, 1 + n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]))*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 3, 2 + n/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

3.180.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^3(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)\sin^{n+3}(e+fx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}\sin(e+fx)^{n+3}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+6}{2},\sin^2(e+fx)\right)}{bf(n+4)}$$

input
```
Int[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
 n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 +
n))
```

3.180.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

3.180.4 Maple [F]

$$\int \left(\sin^{3}\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)\right)^{n}dx$$

input
```
int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

3.180.5 Fricas [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n*sin(f*x + e), x)
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

3.180.7 Maxima [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

3.180.8 Giac [F]

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \sin(fx+e)^3\,dx$$

input
```
integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \int \sin(e+f\,x)^3\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)^3*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^3*(b*tan(e + f*x))^n, x)
```

3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$

3.181.1 Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[2+1/2*n],sin(f*
x+e)^2)*sin(f*x+e)*(b*tan(f*x+e))^(1+n)/b/f/(2+n)
```

3.181.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.32

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{8(4+n)\,\text{AppellF1}\left(1+\frac{n}{2},n,2\right.}{f(2+n)\left(2\left(2\,\text{AppellF1}\left(2+\frac{n}{2},n,3,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2},\right.\right.\right.}$$

input
```
Integrate[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
(8*(4 + n)*AppellF1[1 + n/2, n, 2, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(2
 + n)*(2*(2*AppellF1[2 + n/2, n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 2,
 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

3.181.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)\sin^{n+1}(e+fx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{-n-1}(e+fx)\cos^{n+1}(e+fx)(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}\sin(e+fx)^{n+1}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+4}{2},\sin^2(e+fx)\right)}{bf(n+2)}$$

input
```
Int[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
 n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n)
)
```

3.181.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.181.4 Maple [F]

$$\int \sin (fx+e) (b\tan (fx+e))^{n} dx$$

input
```
int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

3.181.5 Fricas [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n dx = \int (b\tan (fx+e))^{n} \sin (fx+e) dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*sin(f*x + e), x)
```

3.181.6 Sympy [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(e+fx))^n\sin(e+fx)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*sin(e + f*x), x)
```

3.181.7 Maxima [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

3.181.8 Giac [F]

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan(fx+e))^n\sin(fx+e)\,dx$$

input
```
integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \int \sin\left(e+f\,x\right)\,\left(b\tan(e+f\,x)\right)^n\,dx$$

input
```
int(sin(e + f*x)*(b*tan(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)*(b*tan(e + f*x))^n, x)
```

3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$

3.182.1 Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([1-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

3.182.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b\tan(e+fx))^n}{fn}$$

input
```
Integrate[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[
(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*n)
```

3.182.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-1}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos(e+fx)^{-n}\sin(e+fx)^{n-1}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{2-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

3.182.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.182.4 Maple [F]

$$\int \csc(fx+e)\,(b\tan(fx+e))^n\,dx$$

input
```
int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

3.182.5 Fricas [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e), x)
```

3.182.6 Sympy [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x), x)
```

3.182.7 Maxima [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

3.182.8 Giac [F]

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x),x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x), x)
```

3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$

3.183.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

3.183.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 13.22 (sec) , antiderivative size = 743, normalized size of antiderivative = 9.53

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$$
$$= \frac{\cot^2\left(\frac{1}{2}(e+fx)\right)\text{Hypergeometric2F1}\left(-1+\frac{n}{2},n,\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b}{f(-8+4n)}$$
$$+\frac{(4+n)\,\text{AppellF1}}{4f(2+n)\left(2\left(\text{AppellF1}\left(2+\frac{n}{2},n,2,3+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-n\,\text{AppellF1}\left(2+\frac{n}{2}\right.\right.\right.}$$
$$+\frac{\text{Hypergeometric2F1}\left(1+\frac{n}{2},n,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n\tan^2\left(\frac{1}{2}(e+fx\right.}{f(8+4n)}$$
$$+\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\left((2+n)\,\text{Hypergeometric2F1}\left(\frac{n}{2},n,1+\frac{n}{2}\right.\right.}{fn(2+n)\left(-8\,\text{AppellF1}\left(1+\frac{n}{2},n,1,2+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)+\right.}$$

input
```
Integrate[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2
]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*(-8 + 4*n)) +
 ((4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f*x])^n)/(4*f*(2 + n)*(2*(AppellF1[2 +
 n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1
[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1
 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))) + (Hypergeometric2F1[1 + n
/2, n, 2 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Ta
n[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(8 + 4*n)) + (4*Cos[(e + f*x)/2]^2
*Cot[(e + f*x)/2]*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x
)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Tan[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(f*n*(2 + n)*(-8*Appel
lF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(
e + f*x)/2] + (8*(2*AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] - 2*n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + ((4 + n)*Cot[(e + f*x)/2]^4)/(Cos[e + f*x]
*Sec[(e + f*x)/2]^2)^n)*Tan[(e + f*x)/2]^3)/(4 + n)))
```

3.183.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^3}dx$$

$$\Big\downarrow \text{3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-3}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos(e+fx)^{-n}\sin(e+fx)^{n-3}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{4-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

3.183.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.183.4 Maple [F]

$$\int \left(\csc^{3}\left(f x +e \right)\right) \left(b \tan \left(f x +e \right)\right)^{n}d x$$

input
```
int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

3.183.5 Fricas [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int \left(b \tan \left(f x + e\right)\right)^{n} \csc \left(f x + e\right)^{3}\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

3.183.6 Sympy [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan{(e+fx)})^n\csc^3{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**3, x)
```

3.183.7 Maxima [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan{(fx+e)})^n\csc{(fx+e)}^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

3.183.8 Giac [F]

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx=\int (b\tan{(fx+e)})^n\csc{(fx+e)}^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

3.183.9 Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin{(e+f\,x)}^3}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^3,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^3, x)
```

3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$

3.184.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = -\frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

output
```
-cos(f*x+e)*hypergeom([3-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*ta
n(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

3.184.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.17 (sec) , antiderivative size = 1017, normalized size of antiderivative = 13.04

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

output
```
(3*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]
^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(16*f*(-2 + n)
) + (Cot[(e + f*x)/2]^2*((-2 + n)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-2
+ n/2, n, -1 + n/2, Tan[(e + f*x)/2]^2] + (-4 + n)*Hypergeometric2F1[-1 +
n/2, n, n/2, Tan[(e + f*x)/2]^2])*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*T
an[e + f*x])^n)/(16*f*(-4 + n)*(-2 + n)) + (3*(4 + n)*AppellF1[1 + n/2, n,
 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*Ta
n[e + f*x])^n)/(16*f*(2 + n)*(2*(AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*Ap
pellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1
 + Cos[e + f*x]))) + (3*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(b*Tan[e +
 f*x])^n)/(16*f*(2 + n)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f
*x)/2]^2*((4 + n)*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^
2] + (2 + n)*Hypergeometric2F1[2 + n/2, n, 3 + n/2, Tan[(e + f*x)/2]^2]*Ta
n[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(16*f*(2 + n)*(4 + n)) + (3*Cos[(e +
 f*x)/2]^2*Cot[(e + f*x)/2]*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Ta
n[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*(b*Tan[e + f*x])^n)/(f*n*(2 +...
```

3.184.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3081, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sin(e+fx)^5}dx$$

$$\Big\downarrow \text{ 3081}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int \cos^{-n}(e+fx)\sin^{n-5}(e+fx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin^{-n}(e+fx)\cos^n(e+fx)(b\tan(e+fx))^n\int\cos(e+fx)^{-n}\sin(e+fx)^{n-5}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{6-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
 f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

3.184.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

3.184.4 Maple [F]

$$\int \left(\csc^{5}\left(fx+e\right)\right) \left(b\tan\left(fx+e\right)\right)^{n} dx$$

input
```
int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

3.184.5 Fricas [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{5}\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

3.184.6 Sympy [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int \left(b\tan\left(e+fx\right)\right)^{n}\csc^{5}\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**5*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*csc(e + f*x)**5, x)
```

3.184.7 Maxima [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

3.184.8 Giac [F]

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n\csc(fx+e)^5\,dx$$

input
```
integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^5}\,dx$$

input
```
int((b*tan(e + f*x))^n/sin(e + f*x)^5,x)
```

output
```
int((b*tan(e + f*x))^n/sin(e + f*x)^5, x)
```

3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\sin^2(e+fx\right.}{bf(5+2n)}$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([5/4+1/2*n, 1/2+1/2*n],[9/4+1/2*n],
sin(f*x+e)^2)*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1+n)/b/f/(5+2*n)
```

3.185.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 32.84 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \frac{8(9+2n}{f(5+2n)\left(2(9+2n)\text{AppellF1}\left(\frac{5}{4}+\frac{n}{2},n,\frac{5}{2},\frac{9}{4}+\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\right.\right.\right.}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

output
```
(8*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*(a*Sin[e + f*x])^(3
/2)*(b*Tan[e + f*x])^n)/(f*(5 + 2*n)*(2*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5
/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2
 + 2*(5*AppellF1[9/4 + n/2, n, 7/2, 13/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] - 2*n*AppellF1[9/4 + n/2, 1 + n, 5/2, 13/4 + n/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

3.185.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n+\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n+\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\sin^2(e+f\right.}{bf(2n+5)}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4,
(9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 +
n))/(b*f*(5 + 2*n))
```

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.185.4 Maple [F]

$$\int (\sin (fx+e)\, a)^{\frac{3}{2}} (b\tan (fx+e))^n \, dx$$

input
```
int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^n,x)
```

output
```
int((sin(f*x+e)*a)^(3/2)*(b*tan(f*x+e))^n,x)
```

3.185.5 Fricas [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n*a*sin(f*x + e), x)
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**n,x)
```

output
```
Timed out
```

3.185.7 Maxima [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

3.185.8 Giac [F]

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\sin(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx = \int (a\ \sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n, x)
```

3.186 $\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}(b\tan(e-}{bf(3+2n)}$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([3/4+1/2*n, 1/2+1/2*n],[7/4+1/2*n],
sin(f*x+e)^2)*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1+n)/b/f/(3+2*n)
```

3.186.2 Mathematica [A] (verified)

Time = 11.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\sin(2(e}{f(3+2n)}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]*(b*Tan[e
 + f*x])^n)/(f*(3 + 2*n))
```

3.186.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{ 3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n+\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n+\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{2\sqrt{a\sin(e+fx)}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\sin^2(e+fx)\right)}{bf(2n+3)}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4,
(7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(3 + 2*n))
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.186.4 Maple [F]

$$\int \sqrt{\sin \left(f x +e \right) a}\, \left(b \tan \left(f x +e \right)\right)^{n}d x$$

input
```
int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^n,x)
```

output
```
int((sin(f*x+e)*a)^(1/2)*(b*tan(f*x+e))^n,x)
```

3.186.5 Fricas [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a \sin \left(f x + e\right)} \left(b \tan \left(f x + e\right)\right)^{n} \,d x$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

3.186.6 Sympy [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*sin(e + f*x))*(b*tan(e + f*x))**n, x)
```

3.186.7 Maxima [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin(fx+e)}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

3.186.8 Giac [F]

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\sin(fx+e)}(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sin(e+fx)}(b\tan(e+fx))^n\,dx = \int \sqrt{a\ \sin\left(e+f\,x\right)}\,\left(b\tan(e+f\,x)\right)^n\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n, x)
```

3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

output
```
2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/4+1/2*n, 1/2+1/2*n],[5/4+1/2*n],
sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+2*n)/(a*sin(f*x+e))^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 11.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\sin^2(e+fx)\right)\sin(2(e+fx))(b\tan(e}{(f+2fn)\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

output
```
((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*(b*Tan[e + f*x])^n)/((f + 2*f
*n)*Sqrt[a*Sin[e + f*x]])
```

3.187.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n-\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n-\frac{1}{2}}dx}{b}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4,
(5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt
[a*Sin[e + f*x]])
```

3.187.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.187.4 Maple [F]

$$\int \frac{(b \tan (fx+e))^n}{\sqrt{\sin (fx+e)\, a}} dx$$

input
```
int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(1/2),x)
```

output
```
int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(1/2),x)
```

3.187.5 Fricas [F]

$$\int \frac{(b \tan(e+fx))^n}{\sqrt{a \sin(e+fx)}}\, dx = \int \frac{(b \tan (fx+e))^n}{\sqrt{a \sin (fx+e)}}\, dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a*sin(f*x + e)), x)
```

3.187.6 Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan{(e+fx)})^n}{\sqrt{a\sin{(e+fx)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((b*tan(e + f*x))**n/sqrt(a*sin(e + f*x)), x)
```

3.187.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)})^n}{\sqrt{a\sin{(fx+e)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

3.187.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)})^n}{\sqrt{a\sin{(fx+e)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2), x)
```

3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

output
```
-2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([-1/4+1/2*n, 1/2+1/2*n],[3/4+1/2*n
],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-2*n)/(a*sin(f*x+e))^(3/2)
```

3.188.2 Mathematica [A] (verified)

Time = 9.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2b\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\sin^2(e+\right.}{a^2f(-1+2n)}$$

input
```
Integrate[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(2*b*(Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)
/4, (3 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(-1
 + n))/(a^2*f*(-1 + 2*n))
```

3.188.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3082$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos^{-n}(e+fx)(a\sin(e+fx))^{n-\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos^{n+1}(e+fx)(a\sin(e+fx))^{-n-1}(b\tan(e+fx))^{n+1}\int \cos(e+fx)^{-n}(a\sin(e+fx))^{n-\frac{3}{2}}dx}{b}$$

$$\Big\downarrow \; 3057$$

$$-\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4
, (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a
*Sin[e + f*x])^(3/2))
```

3.188.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

3.188.4 Maple [F]

$$\int \frac{(b \tan (fx+e))^{n}}{(\sin (fx+e) a)^{\frac{3}{2}}}dx$$

input
```
int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(3/2),x)
```

output
```
int((b*tan(f*x+e))^n/(sin(f*x+e)*a)^(3/2),x)
```

3.188.5 Fricas [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{n}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a^2*cos(f*x + e)^2 - a^
2), x)
```

3.188.6 Sympy [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(e+fx)})^n}{(a\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**n/(a*sin(e + f*x))**(3/2), x)
```

3.188.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^n}{(a\sin{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

3.188.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^n}{(a\sin{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^n}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2), x)
```

3.189 $\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$

3.189.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m\cos^2(e+fx)^{\frac{1}{2}(1-m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(b\tan}{bf(1+n)}$$

output
```
(a*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/
2-1/2*m+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

3.189.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{(a\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{2+m}{2},\frac{1+n}{2},\frac{3+n}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)(b\tan}{f(1+n)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n)/2, (3 + n)/2, -Ta
n[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x])^n)/(f*(
1 + n))
```

3.189.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3083, 3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \ 3083$$

$$(a\cos(e+fx))^m\left(\frac{\sec(e+fx)}{a}\right)^m\int\left(\frac{\sec(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \ 3042$$

$$(a\cos(e+fx))^m\left(\frac{\sec(e+fx)}{a}\right)^m\int\left(\frac{\sec(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \ 3097$$

$$\frac{(a\cos(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(-m+n+1)}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{n+3}{2},\sin^2(e}{bf(n+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1
+ n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + n))
```

3.189.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3083
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.189.4 Maple [F]

$$\int (a\cos(fx+e))^m (b\tan(fx+e))^n\, dx$$

input
```
int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

3.189.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\, dx = \int (a\cos(fx+e))^m (b\tan(fx+e))^n\, dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.189.6 Sympy [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos{(e+fx)})^m\,(b\tan{(e+fx)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.189.7 Maxima [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos{(fx+e)})^m\,(b\tan{(fx+e)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.189.8 Giac [F]

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cos{(fx+e)})^m\,(b\tan{(fx+e)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\,\cos(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.190 $\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$

3.190.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(e+fx))^{n}}{af(1+m+n)}$$

output
```
hypergeom([1, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],-tan(f*x+e)^2)*(a*tan(f*x
+e))^(1+m)*(b*tan(f*x+e))^n/a/f/(1+m+n)
```

3.190.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),-\tan^2(e+fx)\right)\tan(e+fx)(a\tan(e+fx))^m(b\tan(e+fx))^n}{f(1+m+n)}$$

input
```
Integrate[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*Tan[e
 + f*x]*(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n)/(f*(1 + m + n))
```

3.190.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2034, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int (a\tan(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int (a\tan(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{a(a\tan(e+fx))^{-n}(b\tan(e+fx))^n\int \frac{(a\tan(e+fx))^{m+n}}{\tan^2(e+fx)a^2+a^2}d(a\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),-\tan^2(e+fx)\right)}{af(m+n+1)}$$

input
```
Int[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Ta
n[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

3.190.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.190.4 Maple [F]

$$\int (a \tan (fx+e))^m (b \tan (fx+e))^n \, dx$$

input
```
int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

3.190.5 Fricas [F]

$$\int (a \tan(e+fx))^m (b \tan(e+fx))^n \, dx = \int (a \tan (fx+e))^m (b \tan (fx+e))^n \, dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.190.6 Sympy [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan(e+fx))^m\,(b\tan(e+fx))^n\,dx$$

input
```
integrate((a*tan(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*tan(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.190.7 Maxima [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.190.8 Giac [F]

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan(fx+e))^m\,(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\tan(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.191 $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

3.191.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\begin{aligned}\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx &= \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d^3}{5f(d\cot(e+fx))^{5/2}}\\ &\quad-\frac{2d}{f\sqrt{d\cot(e+fx)}}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\ &\quad+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

output
```
2/5*d^3/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2
))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+
e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*
cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-2*d/f/(d*cot(f*x+e))^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)\right.}{5f}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
-1/5*(Sqrt[d*Cot[e + f*x]]*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e + f*
x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[e
+ f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^3)/f
```

3.191.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^4}dx$$
$$\Big\downarrow \text{2030}$$
$$d^4\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$
$$\Big\downarrow \text{3955}$$
$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{ 3955}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{ 3957}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{ 266}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{ 826}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot}\right.\right.}{df}}{d^2}\right.$$

$$\Big\downarrow \text{ 1082}$$

3.191. $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2}{d^4\cot^4(e+}}{df}}{d^2}\right.$$

$$\Big\downarrow \ 217$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+}{d^2}\right.$$

$$\Big\downarrow \ 1479$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+}}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$$\Big\downarrow \ 25$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$$\Big\downarrow \ 27$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}}{2\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

3.191. $\int\sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

$$\Big\downarrow \text{1103}$$

$$d^4\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
d^4*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

3.191. $\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.191.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(179) = 358$.

Time = 17.60 (sec) , antiderivative size = 707, normalized size of antiderivative = 3.05

method	result
default	$-\frac{\csc(fx+e)\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\,(1-\cos(fx+e))\left(-40\left(\csc^7(fx+e)\right)(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(}{}\right.\right.}{}$

input
```
int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/20/f*csc(f*x+e)*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x
+e)))^(1/2)*(1-cos(f*x+e))*(-40*csc(f*x+e)^7*(1-cos(f*x+e))^7+5*ln(1/(1-co
s(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f
*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(csc(f*x
+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arctan(1/(1-cos(f*x
+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/
2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5
/2)-5*ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc
(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin
(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*a
rctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+
e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x
+e)+cot(f*x+e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-40*csc(f*x+e)^3*(1
-cos(f*x+e))^3)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e
)))^(1/2)/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)*2^(1
/2)
```

3.191.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx =$$

$$-\frac{5\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)+5i\,f\left(\cdots\right.}{\cdots}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
-1/10*(5*f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x +
e))) - 5*I*f*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*
x + e))) + 5*I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/t
an(f*x + e))) - 5*f*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/
tan(f*x + e))) - 4*(tan(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))
/f
```

3.191.6 Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot\left(e+fx\right)}\tan^4\left(e+fx\right)dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**4, x)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.89

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = -\frac{d^5\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\cdots\right.}{d^4}\cdots\right.}{20\,f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
-1/20*d^5*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

3.191.8 Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^4\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^4, x)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx = \frac{\frac{2d^3}{5} - \frac{2d^3}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^4*(d*cot(e + f*x))^(1/2),x)
```

output
```
((2*d^3)/5 - (2*d^3)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(
1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.192 $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

3.192.1 Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
2/3*d^2/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2
))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+
e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*
cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = -\frac{\sqrt{d\cot(e+fx)}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right.}{3f}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

output
```
-1/3*(Sqrt[d*Cot[e + f*x]]*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e
 + f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3
/4))*Tan[e + f*x]^2)/f
```

3.192.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$\downarrow$ 3955

$$d^3\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$\downarrow$ 3042

$$d^3\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$\downarrow$ 3957

$$d^3\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 266

$$d^3\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 755

$$d^3\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1476

$$d^3\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\ldots}}{2d}\ldots\right.}{df}\right.$$

$\downarrow$ 1082

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{\ldots}{3df(\ldots}\right.$$

3.192. $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

$$\Big\downarrow \; 217$$

$$d^3\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \; 1479$$

$$d^3\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots}{}\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d^3\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 1103$$

3.192. $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

$$d^3\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

output
```
d^3*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.192. $\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$

3.192.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(163) = 326$.

Time = 17.70 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.65

method	result
default	$-\frac{(\sec^2(fx+e))(\cos(fx+e)+1)\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)\right.}{}$

input
```
int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*sec(f*x+e)^2*(cos(f*x+e)+1)*(-6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f*x+e)*cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))+
3*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+
e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(
f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)
-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*(-sin(f*x+e)*cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot
(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)
^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f
*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+4*2^(1/2)*cos(f*x+e)-4*2^(1/2))*(cot(f
*x+e)*d)^(1/2)*2^(1/2)
```

3.192.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$

$$= \frac{4\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2 + 3f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}} + \sqrt{\frac{d}{\tan(fx+e)}}\right) + 3if\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(if\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}} +\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 3*f*(-d^2/f^4)^(1/4)*log(f*(-
d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 3*I*f*(-d^2/f^4)^(1/4)*log(I*f*(-
d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 3*I*f*(-d^2/f^4)^(1/4)*log(-I*f*(
-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 3*f*(-d^2/f^4)^(1/4)*log(-f*(-d^
2/f^4)^(1/4) + sqrt(d/tan(f*x + e))))/f
```

3.192.6 Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot\left(e+fx\right)}\tan^3\left(e+fx\right)dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**3,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**3, x)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx$$
$$= \frac{d^4\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log}{}\right.}{d^2}\right.}{12\,f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/12*d^4*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

3.192.8 Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)^3\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^3, x)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx = \frac{2d^2}{3f\left(\frac{d}{\tan(e+fx)}\right)^{3/2}} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)1\mathrm{i}}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d^2)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)
^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.193 $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

3.193.1 Optimal result

Integrand size = 21, antiderivative size = 210

$$\begin{aligned}\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx &= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}\\ &\quad+\frac{2d}{f\sqrt{d\cot(e+fx)}}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\\ &\quad-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2
^(1/2)+2*d/f/(d*cot(f*x+e))^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \frac{d\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}\right)}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
(d*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[
(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x]]
)
```

3.193.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^2\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$\downarrow$ 3042

$$d^2\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$\downarrow$ 3957

$$d^2\left(\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\downarrow$ 266

$$d^2\left(\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\downarrow$ 826

$$d^2\left(\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$\downarrow$ 1476

$$d^2\Bigg(\frac{2\Big(\frac{1}{2}\Big(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{df}$$

$\downarrow$ 1082

$$d^2\Bigg(\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot}}{df}$$

$\downarrow$ 217

$$d^2\Bigg(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d\,c}}$$

$\downarrow$ 1479

3.193. $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\cdots\right.}{}\right.\right.}{df}\right.$$

$\downarrow$ 25

$$d^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\cdots}\right.}{}\right.\right.}{df}\right.$$

$\downarrow$ 27

$$d^2\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\cdots}\right.}{}\right.\right.}{df}\right.$$

$\downarrow$ 1103

$$d^2\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}\cdots\right.}{}\right.\right.}{df}\right.$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
d^2*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f
*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

3.193. $\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.193.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(161) = 322$.

Time = 18.90 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.12

method	result
default	$\csc(fx+e)\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\,(1-\cos(fx+e))\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(}}{1-\cos(}\right.\right.$

input
```
int((cot(f*x+e)*d)^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/f*csc(f*x+e)*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e
)))^(1/2)*(1-cos(f*x+e))*(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2
+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+
2-2*cos(f*x+e)-sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(
f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f
*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(
f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(
1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot
(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-
csc(f*x+e)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*
x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f
*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-8*csc(f*x+e)+8*cot(f
*x+e))/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)
/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)*2^(1/2)
```

3.193.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \frac{f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)-i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d}{\tan(fx+e)}}\right)+i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}}{}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*(f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)))
 - I*f*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x + e)
)) + I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x +
 e))) - f*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt(d/tan(f*x +
e))) + 4*sqrt(d/tan(f*x + e))*tan(f*x + e))/f
```

3.193.6 Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**2, x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{d^3\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\cdots\right.}{}}{d^2}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
1/4*d^3*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

3.193.8 Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{d\cot\left(fx+e\right)}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^2, x)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = \frac{2\,d}{f\,\sqrt{\frac{d}{\tan(e+f\,x)}}} + \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f} - \frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d
/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.194 $\int \sqrt{d \cot(e+fx)} \tan(e+fx) \, dx$

3.194.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \sqrt{d \cot(e+fx)} \tan(e+fx) \, dx = \frac{\sqrt{d} \arctan\left(1 - \frac{\sqrt{2}\sqrt{d \cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2} f} - \frac{\sqrt{d} \arctan\left(1 + \frac{\sqrt{2}\sqrt{d \cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2} f} + \frac{\sqrt{d} \log\left(\sqrt{d} + \sqrt{d} \cot(e+fx) - \sqrt{2}\sqrt{d \cot(e+fx)}\right)}{2\sqrt{2} f} - \frac{\sqrt{d} \log\left(\sqrt{d} + \sqrt{d} \cot(e+fx) + \sqrt{2}\sqrt{d \cot(e+fx)}\right)}{2\sqrt{2} f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^
(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$$
$$= \frac{d\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

output
```
(d*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan
[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt
[2]*f*Sqrt[d*Cot[e + f*x]])
```

3.194.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {3042, 25, 2030, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}{\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \; 2030$$

$$d\int \frac{1}{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d^2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2d^2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2d^2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2d^2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2d^2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right.}{\sqrt{2}\sqrt{d}}}{}\right)}{f}$$

$$\Big\downarrow \text{25}$$

3.194. $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$

$$-\frac{2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{ar}}{2d}\right.}{f}$$

$\downarrow$ 27

$$-\frac{2d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{ar}}{2d}\right.}{f}$$

$\downarrow$ 1103

$$-\frac{2d^2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

output
```
(-2*d^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/f
```

3.194.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.194. $\int \sqrt{d \cot(e+fx)} \tan(e+fx) \, dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.194.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(145) = 290$.

Time = 14.92 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\sqrt{-\frac{d\left(\csc(fx+e)(1-\cos(fx+e))^2-\sin(fx+e)\right)}{1-\cos(fx+e)}}\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(fx+e))^3-\csc(fx+e)+\cot(}}{1-\cos(fx+e)}\right.\right.}{}$

input
```
int((cot(f*x+e)*d)^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin(f*x+e)))^(1/2)*
(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)
^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)
))+2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-cs
c(f*x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))-ln(-1/(1-cos(f*x+e))*(-csc(f*x+e
)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+
cot(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))+2*arctan(1/(1-cos(f*x+e))*(s
in(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+co
s(f*x+e))))/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^
(1/2)*(csc(f*x+e)-cot(f*x+e))*2^(1/2)
```

3.194.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = -\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) - \frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) + \frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right) + \frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) - 1/2
*I*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 1/2
*I*(-d^2/f^4)^(1/4)*log(-I*f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e))) + 1/
2*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(1/4) + sqrt(d/tan(f*x + e)))
```

3.194.6 Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*tan(e + f*x), x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx =$$
$$-\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+}\right.}{d^{\frac{3}{2}}}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

3.194.8 Giac [F]

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \int \sqrt{d\cot(fx+e)}\tan(fx+e)\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*tan(f*x + e), x)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} + \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/
f + ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))
*1i)/f
```

3.194. $\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx$

3.195 $\int \sqrt{d\cot(e+fx)}\,dx$

3.195.1 Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^
(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \sqrt{d\cot(e+fx)}\,dx$$
$$=\frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))
```

3.195.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\cot(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3957}$$
$$-\frac{d\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$
$$\Big\downarrow \text{266}$$
$$-\frac{2d\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$
$$\Big\downarrow \text{826}$$

$$-\frac{2d\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\downarrow$ 1476

$$-\frac{2d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)\right.}{f}$$

$\downarrow$ 1082

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot}\right.}{f}$$

$\downarrow$ 217

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\downarrow$ 1479

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{f}$$

$\downarrow$ 25

$$-\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$\downarrow$ 27

$$-\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$\downarrow$ 1103

3.195. $\int \sqrt{d\cot(e+fx)}\,dx$

$$-\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.}{f}$$

input
```
Int[Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Ar
cTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqr
t[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[
d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d])
)/2))/f
```

3.195.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.195. $\int \sqrt{d \cot(e+fx)}\, dx$

3.195.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{d\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$
default	$-\frac{d\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$

input
```
int((cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^
(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.195.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.21

$$\begin{aligned}\int\sqrt{d\cot(e+fx)}\,dx=&-\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)\\&+\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(if^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)\\&-\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-if^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)\\&+\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)\end{aligned}$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e
) + d)/sin(2*f*x + 2*e))) + 1/2*I*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3
/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-d^2/f^4
)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(
2*f*x + 2*e))) + 1/2*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt((
d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

3.195.6 Sympy [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\,dx$$

input
```
integrate((d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x)), x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e}\right.}{\sqrt{d}}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

3.195.8 Giac [F]

$$\int \sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(fx+e\right)}\,dx$$

input
```
integrate((d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e)), x)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.26

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int((d*cot(e + f*x))^(1/2),x)
```

output
```
-((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)) -
atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))))/f
```

3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.196.1 Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2
^(1/2)-2*(d*cot(f*x+e))^(1/2)/f
```

3.196.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx\ldots}{2\sqrt{2}}\right.}{df\cot^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

output
```
-(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2]
- ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] +
Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(d*f*Cot[e + f*x]^
(3/2)))
```

3.196.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 3042

$$\frac{d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 755

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right.}{f}}{d}$$

$\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+f}}{f}}{d}$$

$\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d}$$

$\downarrow$ 1479

3.196. $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

$$\frac{\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arc}\ldots}{2d}\ldots\right.}{f}\ldots}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\ldots\right.}{}}{2d}\ldots\right.}{f}\ldots}{d}$$

$\big\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\ldots\right.}{}}{2d}\ldots\right.}{f}\ldots}{d}$$

$\big\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right)}{f}\ldots}{d}$$

input
```
Int[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d
```

3.196. $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.196. $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.196.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-2\sqrt{\cot(fx+e)d}+\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right.\right.}{4}}{f}$
default	$\frac{-2\sqrt{\cot(fx+e)d}+\frac{\left(d^{2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\cdots\right.\right.}{4}}{f}$

input
```
int(cot(f*x+e)*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2*(cot(f*x+e)*d)^(1/2)+1/4*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/
4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2
)+1)))
```

3.196.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.21

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)+i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)-i\,f\left(\cdots\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(f*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e)
+ d)/sin(2*f*x + 2*e))) + I*f*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) +
sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - I*f*(-d^2/f^4)^(1/4)*lo
g(-I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
 - f*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) +
 d)/sin(2*f*x + 2*e))) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))
)/f
```

3.196.6 Sympy [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\cot(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x), x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\right.}{4f}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/f
```

3.196.8 Giac [F]

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e), x)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{2\sqrt{d\cot(e+f\,x)}}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(cot(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.196. $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-2/3*(d*cot(f*x+e))^(3/2)/d/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e
))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*c
ot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$
$$-\frac{\sqrt{d\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\right.}{3f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(Sqrt[d*Cot[e + f*x]]*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f
*x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*
Cot[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(3/4))
```

3.197.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^2}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 826

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\cdots}}{f}}{d^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cdots}{d^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\cdots\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$\Big\downarrow$ 25

3.197. $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

$\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d^2}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^2
```

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.197. $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.197.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^{2})^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}{8(d^{2})^{\frac{1}{4}}}\cdots\right)}{fd}\cdots$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^{2})^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right.\cdots}{8(d^{2})^{\frac{1}{4}}}\cdots\right)}{fd}\cdots$

input
```
int(cot(f*x+e)^2*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+
e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)
*d)^(1/2)+1)))
```

3.197.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.53

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{3\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin\left(2\,fx+2\,e\right)-3i\,f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*f*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x +
 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*f*(-d^2/f^4)^(1/4)*lo
g(I*f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e
)))*sin(2*f*x + 2*e) + 3*I*f*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4)
+ d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*
f*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + d*sqrt((d*cos(2*f*x + 2*e)
+ d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)
/sin(2*f*x + 2*e))*(cos(2*f*x + 2*e) + 1))/(f*sin(2*f*x + 2*e))
```

3.197.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot\left(e+fx\right)}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**2, x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e}\right.}{\sqrt{d}}\right.}{12\,df}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d*f)
```

3.197.8 Giac [F]

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^2, x)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d\,f} - \frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
 (2*(d*cot(e + f*x))^(3/2))/(3*d*f) - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4
)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.198.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-2/5*(d*cot(f*x+e))^(5/2)/d^2/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/
2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x
+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/f
```

3.198.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{\sqrt{d\cot(e+fx)}\left(10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+40\sqrt{\cot}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

output
```
(Sqrt[d*Cot[e + f*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] -
 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]]
 - 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + C
ot[e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]
]))/(20*f*Sqrt[Cot[e + f*x]])
```

3.198.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^3}$$

$$\Big\downarrow \; 3954$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 3042

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 3954

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 3042

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 3957

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 266

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 755

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^3}$$

$\downarrow$ 1476

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot}}{2d}\right.}{f}\right.}{d^3}$$

3.198. $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

$\Big\downarrow$ 1082

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\ldots}}{\ldots}\right.}{d^3}$$

$\Big\downarrow$ 217

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+\ldots}{5f}}{d^3}$$

$\Big\downarrow$ 1479

$$\frac{-d^2\left(\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\ldots\right.}{\sqrt{2}\sqrt{d}}\ldots\right.}{f}\ldots\right.}{d^3}$$

$\Big\downarrow$ 25

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\ldots}{2d}\ldots\right.}{f}\ldots\right.}{d^3}$$

$\Big\downarrow$ 27

3.198. $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\right.}{f}\right.}{d^3}$$

$\downarrow$ 1103

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\cdots\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}\right.}{d^3}$$

input
```
Int[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^3
```

3.198.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.198. $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.198.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f d^2}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f d^2}$

input
```
int(cot(f*x+e)^3*(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^
(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(
d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

3.198.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.47

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{5\,(f\cos(2\,fx+2\,e)-f)\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+5\,(i\,f\cos(2\,fx+2\,e)-i\,f)\Big(-$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/10*(5*(f*cos(2*f*x + 2*e) - f)*(-d^2/f^4)^(1/4)*log(f*(-d^2/f^4)^(1/4)
+ sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5*(I*f*cos(2*f*x + 2*
e) - I*f)*(-d^2/f^4)^(1/4)*log(I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x +
2*e) + d)/sin(2*f*x + 2*e))) + 5*(-I*f*cos(2*f*x + 2*e) + I*f)*(-d^2/f^4)^
(1/4)*log(-I*f*(-d^2/f^4)^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
+ 2*e))) - 5*(f*cos(2*f*x + 2*e) - f)*(-d^2/f^4)^(1/4)*log(-f*(-d^2/f^4)^(
1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 8*sqrt((d*cos(2*
f*x + 2*e) + d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(f*cos(2*f*x +
 2*e) - f)
```

3.198.6 Sympy [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(e+fx)}\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**3, x)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{\right.}$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^2*f)
```

3.198.8 Giac [F]

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \int \sqrt{d\cot(fx+e)}\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^3, x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.39

$$\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{2\,(d\cot(e+fx))^{5/2}}{5\,d^2\,f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d^2*f) + ((-1
)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f +
((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
f
```

3.198. $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

3.199.1 Optimal result

Integrand size = 21, antiderivative size = 234

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
2/5*d^4/f/(d*cot(f*x+e))^(5/2)+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))
^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2
)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*
cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(
1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-2*d^2/f/(d*cot(f*x+e))^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(-2+10\cot^2(e+fx)+5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}\cot^{\frac{9}{4}}(e+fx)\right.}{5f}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

output
```
-1/5*((d*Cot[e + f*x])^(3/2)*(-2 + 10*Cot[e + f*x]^2 + 5*ArcTan[(-Cot[e +
f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4)*Cot[e + f*x]^(9/4) + 5*ArcTanh[(-Cot[
e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(5/4))*Tan[e + f*x]^4)/f
```

3.199.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.98, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 25, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^5}dx$$

$$\Big\downarrow \; 2030$$

$$d^5\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int\frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int\frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3955}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{266}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{826}$$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \text{1476}$$

3.199. $\int(d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 1082

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2}{d^4\cot^4(e+\right.}{df}}{d^2}\right.$$

$\downarrow$ 217

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\ldots}{d^2}\right.$$

$\downarrow$ 1479

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 25

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f\right.\right.}{df}}{d^2}\right.$$

$\downarrow$ 27

3.199. $\int(d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}{2\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

$\Big\downarrow$ 1103

$$d^5\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

output
```
d^5*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

3.199.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.199. $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.199.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(181) = 362$.

Time = 3.41 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.70

method	result
default	$\frac{(\sec^3(fx+e))\csc(fx+e)\left(5(\cos^2(fx+e))\sin(fx+e)\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\left(\cot^3(fx+e)\right)+3\left(\cot^2(fx+e\right.}}{}\right.\right.}{}$

input
```
int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/20/f*sec(f*x+e)^3*csc(f*x+e)*(5*cos(f*x+e)^2*sin(f*x+e)*ln(-(cot(f*x+e)*
cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x
+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*
cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^2*sin(f*x+e)*ln(-(cot(f*x+e)*cos(f*
x+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*
csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*
x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)+10*cos(f*x+e)^2*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e
)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-
1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-10*cos(f*x+e)^2*sin(f*
x+e)*arctan((-2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)+24*2^(1/2)*cos(f*x+e)^3-24*2^(1/2)*cos(f*x+e)^2-4*2^(1/2)*cos(f
*x+e)+4*2^(1/2))*(cos(f*x+e)+1)*d*(cot(f*x+e)*d)^(1/2)*2^(1/2)
```

3.199.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.95

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)-5i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)+5i\left(-\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/10*(5*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(3/4
)*f^3) - 5*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4
)^(3/4)*f^3) + 5*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) - I*(-d
^6/f^4)^(3/4)*f^3) - 5*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt(d/tan(f*x + e)) - (
-d^6/f^4)^(3/4)*f^3) - 4*(d*tan(f*x + e)^3 - 5*d*tan(f*x + e))*sqrt(d/tan(
f*x + e)))/f
```

3.199.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^5(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**5,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**5, x)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx =$$

$$-\frac{d^6\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log(}{}\right.}{d^4}\right.}{20f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
-1/20*d^6*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

3.199.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^5\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^5, x)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.41

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = \frac{\frac{2d^4}{5}-\frac{2d^4}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^5*(d*cot(e + f*x))^(3/2),x)
```

output
```
((2*d^4)/5 - (2*d^4)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(
1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.200.1 Optimal result

Integrand size = 21, antiderivative size = 214

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
2/3*d^3/f/(d*cot(f*x+e))^(3/2)-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))
^(1/2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2
)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*
cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(
1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.43

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx =$$
$$-\frac{(d\cot(e+fx))^{3/2}\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\cdots\right.}{3f}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
-1/3*((d*Cot[e + f*x])^(3/2)*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot
[e + f*x]^2)^(3/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^
(3/4))*Tan[e + f*x]^3)/f
```

3.200.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \; 2030$$

$$d^4\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3955$$

$$d^4\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3042$$

$$d^4\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \ 3957$$

$$d^4\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 266$$

$$d^4\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 755$$

$$d^4\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \ 1476$$

$$d^4\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\ldots}}{2d}\right.}{df}\right.$$

$$\Big\downarrow \ 1082$$

$$d^4\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{}{3df(\ldots}\right.$$

$$\Big\downarrow \ 217$$

3.200. $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

$$d^4\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1479

$$d^4\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{}\right)}{df}\right)$$

$\downarrow$ 25

$$d^4\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right)}{df}\right)$$

$\downarrow$ 27

$$d^4\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right)}{df}\right)$$

$\downarrow$ 1103

3.200. $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

$$d^4\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
d^4*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.200. $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.200.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(163) = 326$.

Time = 2.09 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.66

method	result
default	$\frac{(\sec^2(fx+e))\left(6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)+6\sqrt{-\frac{\sin(fx+e)\cos}{(\cos(fx+e)}}\right.}{}$

input
```
int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12/f*sec(f*x+e)^2*(6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos
(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*si
n(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))+6*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-3*(-sin(f*x+e)*co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*c
ot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+
e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin
(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+3*ln((2*sin(f*x+e)*(-cot(f*x+e)^3+3*
cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-
csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+
e)-csc(f*x+e)-2)/(cos(f*x+e)-1))*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*cos(f*x+e)-4*2^(1/2)*cos(f*x+e)+4*2^(1/2))*(cos(f*x+e)+1)*d*(cot(f*
x+e)*d)^(1/2)*2^(1/2)
```

3.200.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{4\,d\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)^2+3\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right)+3i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\log\left(d\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/6*(4*d*sqrt(d/tan(f*x + e))*tan(f*x + e)^2 + 3*(-d^6/f^4)^(1/4)*f*log(d*
sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(1/4)*f) + 3*I*(-d^6/f^4)^(1/4)*f*log(d*
sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(1/4)*f) - 3*I*(-d^6/f^4)^(1/4)*f*log(
d*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(1/4)*f) - 3*(-d^6/f^4)^(1/4)*f*log(
d*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^(1/4)*f))/f
```

3.200.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^4(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**4, x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{d^5\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+\cdots}\right.}{d^{\frac{3}{2}}}\cdots\right.}{d^2}\cdots\right.}{12\,f}\cdots$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
1/12*d^5*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

3.200.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^4, x)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{2\,d^3}{3\,f\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int(tan(e + f*x)^4*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d^3)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)
^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

3.201.1 Optimal result

Integrand size = 21, antiderivative size = 212

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/
2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1
/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2
^(1/2)+2*d^2/f/(d*cot(f*x+e))^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^2\left(2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}\right)}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

output
```
(d^2*(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTan
h[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Cot[e + f*x
]])
```

3.201.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^3\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^3\left(\frac{\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{266}$$

$$d^3\left(\frac{2\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{826}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \text{1476}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right.\right.}{df}\right.$$

$$\Big\downarrow \text{1082}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot}\right.}{df}\right.$$

$$\Big\downarrow \text{217}$$

3.201. $\int(d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d\mathrm{c}}}\right.$$

$$\Big\downarrow \text{1479}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\right.}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \text{25}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \text{27}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{df}\right.$$

$$\Big\downarrow \text{1103}$$

$$d^3\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}\right.}{}\right.\right.}{df}\right.$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

3.201. $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

output
```
d^3*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f
*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

3.201.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.201. $\int (d \cot(e+fx))^{3/2} \tan^3(e+fx)\, dx$

3.201.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(163) = 326$.

Time = 3.09 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.70

method	result
default	$-\frac{\sec(fx+e)\csc(fx+e)\left(\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\left(\cot^3(fx+e)\right)+3\left(co\right.}}{}\right.\right.}{}$

input
```
int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*sec(f*x+e)*csc(f*x+e)*((-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*sin(f*x+e)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(
f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+
cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*
x+e)-1))+2*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arct
an((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos
(f*x+e)-1)/(cos(f*x+e)-1))+2*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-sin(f*x+e)*(-sin(f*x+e)*cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)-2*si
n(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e
)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*
x+e)+2)/(cos(f*x+e)-1))+4*2^(1/2)*cos(f*x+e)-4*2^(1/2))*(cos(f*x+e)+1)*d*(
cot(f*x+e)*d)^(1/2)*2^(1/2)
```

3.201.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{4\,d\sqrt{\frac{d}{\tan(fx+e)}}\tan(fx+e)+\left(-\frac{d^6}{f^4}\right)^{\frac14}f\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac34}f^3\right)-i\left(-\frac{d^6}{f^4}\right)^{\frac14}f\log\left(d^4\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/2*(4*d*sqrt(d/tan(f*x + e))*tan(f*x + e) + (-d^6/f^4)^(1/4)*f*log(d^4*sq
rt(d/tan(f*x + e)) + (-d^6/f^4)^(3/4)*f^3) - I*(-d^6/f^4)^(1/4)*f*log(d^4*
sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(3/4)*f^3) + I*(-d^6/f^4)^(1/4)*f*log(
d^4*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(3/4)*f^3) - (-d^6/f^4)^(1/4)*f*lo
g(d^4*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^(3/4)*f^3))/f
```

3.201.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^3(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**3,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**3, x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{d^4\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
1/4*d^4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

3.201.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^3\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^3, x)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx = \frac{2\,d^2}{f\,\sqrt{\frac{d}{\tan(e+f\,x)}}} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{f}$$

input
```
int(tan(e + f*x)^3*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d^2)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4
)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.202.1 Optimal result

Integrand size = 21, antiderivative size = 192

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/2
)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1/
4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^
(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx=\frac{d^2\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+}}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(d^2*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcT
an[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] +
Cot[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sq
rt[2]*f*Sqrt[d*Cot[e + f*x]])
```

3.202.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 2030, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \; 266$$

$$-\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 755$$

$$-\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \; 1476$$

$$-\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{}}{2d}\right.}{f}$$

$$\Big\downarrow \; 1082$$

$$-\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\Big\downarrow \; 217$$

$$-\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right.}{\sqrt{2}\sqrt{d}}\right.}{f}$$

$$\Big\downarrow \; 25$$

$$-\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{f}$$

3.202. $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

$$\Big\downarrow \text{27}$$

$$-\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{ar}}{2d}\right.}{f}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/f
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.202. $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.202.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(145) = 290$.

Time = 2.05 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.33

method	result
default	$\left(\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)+2\sin(fx+e)\sqrt{-\left(\cot^3(fx+e)\right)+3\left(\cot^2(fx+e)\right)\csc(fx+e)-3\left(\csc^2(fx+e)\right)\cot(fx+e)+\csc^3(fx+e)+\text{co}}}{\cos(fx+e)-1}\right.\right.$

input
```
int((cot(f*x+e)*d)^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3
+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+
e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))
-2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x
+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-ln((2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*
x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*
x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc
(f*x+e)-2)/(cos(f*x+e)-1))-2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*d*(cot(f*x+e)
*d)^(1/2)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)-csc(
f*x+e))*2^(1/2)
```

3.202.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) - \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) + \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}-i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right) + \frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d\sqrt{\frac{d}{\tan(fx+e)}}-\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right)$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/2*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(1/4)*f) - 1
/2*I*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(1/4)*f) +
 1/2*I*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4)^(1/4)*f)
 + 1/2*(-d^6/f^4)^(1/4)*log(d*sqrt(d/tan(f*x + e)) - (-d^6/f^4)^(1/4)*f)
```

3.202.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**2, x)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx =$$

$$-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
-1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

3.202.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^2, x)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.32

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1i}{f} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1i}{f}$$

input
```
int(tan(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/
f + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))
*1i)/f
```

3.202. $\int (d \cot(e + fx))^{3/2} \tan^2(e + fx) \, dx$

3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

3.203.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2
)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1/
4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^
(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.38

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx=\frac{d\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+f}}{f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

output
```
(d*(-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(
-Cot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(f*Cot[e + f*x]^(3/4))
```

3.203.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {3042, 25, 2030, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}{\tan\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3957}$$

$$-\frac{d^2\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$

$\Big\downarrow$ 266

$$-\frac{2d^2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 826

$$-\frac{2d^2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1476

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)\right.\right.}}{f}$$ (truncated)

$\Big\downarrow$ 1082

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\,\mathrm{co}}\right.}{f}$$ (truncated)

$\Big\downarrow$ 217

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\Big\downarrow$ 1479

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\arctan\left(\cdots\right.\right.\right.}{f}$$ (truncated)

$\Big\downarrow$ 25

$$-\frac{2d^2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\arctan\left(\sqrt{\cdots}\right.\right.\right.}{f}$$ (truncated)

$\Big\downarrow$ 27

3.203. $\int(d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

$$-\frac{2d^2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$\Big\downarrow$ 1103

$$-\frac{2d^2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

input
```
Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

output
```
(-2*d^2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Lo
g[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d
]))/2))/f
```

3.203.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.203. $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

3.203.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result	si
default	$-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4f\left(d^2\right)^{\frac{1}{4}}}$	13

input
```
int((cot(f*x+e)*d)^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^2/f/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d
)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+
1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.203.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right) + \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}+i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right) - \frac{1}{2}i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}-i\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right) + \frac{1}{2}\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(d^4\sqrt{\frac{d}{\tan(fx+e)}}-\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\right)$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) + (-d^6/f^4)^(3/4)*f^3)
 + 1/2*I*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) + I*(-d^6/f^4)^(3/4
)*f^3) - 1/2*I*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) - I*(-d^6/f^4
)^(3/4)*f^3) + 1/2*(-d^6/f^4)^(1/4)*log(d^4*sqrt(d/tan(f*x + e)) - (-d^6/f
^4)^(3/4)*f^3)
```

3.203.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\tan(e+fx)\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x), x)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right.}{4f}$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

output
```
-1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

3.203.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\tan(fx+e)\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e), x)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.28

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = -\frac{(-1)^{1/4}\,d^{3/2}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int(tan(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

output
```
-((-1)^(1/4)*d^(3/2)*(atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)) -
atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))))/f
```

3.204 $\int (d\cot(e+fx))^{3/2}\,dx$

3.204.1 Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*
d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/
2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1
/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2
^(1/2)-2*d*(d*cot(f*x+e))^(1/2)/f
```

3.204.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.77

$$\int (d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\text{c}\right.}{2\sqrt{2}}\right.}{f\cot^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[(d*Cot[e + f*x])^(3/2),x]
```

output
```
-(((d*Cot[e + f*x])^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2]
- ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] +
Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Cot[e + f*x]^(3
/2)))
```

3.204.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 3954$$

$$d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 3042$$

$$d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 3957$$

$$\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 266$$

$$\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 755$$

$$\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 1476$$

$$\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 1082$$

$$\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 217$$

$$\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$$\Big\downarrow \; 1479$$

3.204. $\int (d\cot(e+fx))^{3/2}\,dx$

$$\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 25

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arcta}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 27

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arcta}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

$\Big\downarrow$ 1103

$$\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

$$\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

input
```
Int[(d*Cot[e + f*x])^(3/2),x]
```

3.204. $\int (d\cot(e+fx))^{3/2}\,dx$

output
```
(-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[
e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(S
qrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*
Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.204. $\int (d\cot(e+fx))^{3/2}\,dx$

3.204.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2d\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right)}{f}$
default	$-\frac{2d\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right)}{f}$

input
```
int((cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*d*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/
4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2
)+1)))
```

3.204.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.25

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\right)+i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+i\left(-\right.\right.}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/2*((-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*
e)) + (-d^6/f^4)^(1/4)*f) + I*(-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x +
 2*e) + d)/sin(2*f*x + 2*e)) + I*(-d^6/f^4)^(1/4)*f) - I*(-d^6/f^4)^(1/4)*
f*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/f^4)^(1/
4)*f) - (-d^6/f^4)^(1/4)*f*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
 2*e)) - (-d^6/f^4)^(1/4)*f) - 4*d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
 + 2*e)))/f
```

3.204.6 Sympy [F]

$$\int (d\cot(e+fx))^{3/2}\,dx = \int (d\cot{(e+fx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2), x)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.85

$$\int (d\cot(e+fx))^{3/2}\,dx = \frac{\left(2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\cdots}{}$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))*d/f
```

3.204.8 Giac [F]

$$\int (d\cot(e+fx))^{3/2}\,dx=\int (d\cot(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2), x)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

$$\int (d\cot(e+fx))^{3/2}\,dx=-\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

input
```
int((d*cot(e + f*x))^(3/2),x)
```

output
```
- (2*d*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/
4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

3.205.1 Optimal result

Integrand size = 19, antiderivative size = 211

$$\begin{aligned}\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx &= -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} \\ &+ \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3f} \\ &+ \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} \\ &- \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}\end{aligned}$$

output
```
-2/3*(d*cot(f*x+e))^(3/2)/f-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1
/2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2
)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{(d\cot(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\right.}{3f\cot^{\frac{7}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*((d*Cot[e + f*x])^(3/2)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e +
 f*x])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) +
2*Cot[e + f*x]^(7/4)))/(f*Cot[e + f*x]^(7/4))
```

3.205.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 3957

$$\frac{\frac{d^3\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 826

$$\frac{\frac{2d^3\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\right.}{f}}{d}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d}{}}{d}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d}$$

$\Big\downarrow$ 25

3.205. $\int\cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d}$$

$\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d}$$

$\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d}$$

input
```
Int[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d
```

3.205.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.205. $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.205.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{-\frac{2(\cot(fx+e)d)^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{c}}{(}\right.\right.}{4(d^2)^{\frac{1}{4}}}}{f}$
default	$\frac{-\frac{2(\cot(fx+e)d)^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{c}}{(}\right.\right.}{4(d^2)^{\frac{1}{4}}}}{f}$

input
```
int(cot(f*x+e)*(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/3*(cot(f*x+e)*d)^(3/2)+1/4*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)
*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)
^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d
)^(1/2)+1)))
```

3.205.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.60

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{3\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\right)\sin\left(2\,fx+2\,e\right)-3i\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\log\left(d^4\sqrt{\ldots}\right.}{}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
+ 2*e)) + (-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e) - 3*I*(-d^6/f^4)^(1/4)*f*
log(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) + I*(-d^6/f^4)^(3/
4)*f^3)*sin(2*f*x + 2*e) + 3*I*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*
x + 2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e)
 - 3*(-d^6/f^4)^(1/4)*f*log(d^4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
2*e)) - (-d^6/f^4)^(3/4)*f^3)*sin(2*f*x + 2*e) - 4*(d*cos(2*f*x + 2*e) + d
)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(f*sin(2*f*x + 2*e))
```

3.205.6 Sympy [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\cot\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x), x)
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.87

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+\cdots}}\right.}{\sqrt{d}}\cdots}{12\,f}$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/f
```

3.205.8 Giac [F]

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e), x)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f} - \frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,f} - \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
 (2*(d*cot(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*
(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

3.206.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-2/5*(d*cot(f*x+e))^(5/2)/d/f+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^
(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)
/d^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*c
ot(f*x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1
/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+2*d*(d*cot(f*x+e))^(1/2)/f
```

3.206.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$
$$-\frac{(d\cot(e+fx))^{3/2}\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\sqrt{}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/20*((d*Cot[e + f*x])^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e +
 f*x]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[
e + f*x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e +
f*x]] + Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot
[e + f*x]]))/(f*Cot[e + f*x]^(3/2))
```

3.206.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \text{ 2030}$$
$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^2}$$
$$\Big\downarrow \text{ 3954}$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\mathrm{co}}}{2d}\right.}{f}\right.}{d^2}$$

3.206. $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

$$\Big\downarrow \text{1082}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d}}{}\right.}{d^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+}{5f}}{d^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\right.}{\sqrt{2}\sqrt{d}}\right.}{f}\right.}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{f}\right.}{d^2}$$

$$\Big\downarrow \text{27}$$

3.206. $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right.}{f}\right.}{d^2}$$

$\Big\downarrow$ 1103

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}\right.}{d^2}$$

input
```
Int[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^2
```

3.206.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.206. $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.206.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^{2}\sqrt{\cot(fx+e)d}+\frac{d^{2}(d^{2})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^{2})^{}}\right.\right.}{8}\right.}{fd}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^{2}\sqrt{\cot(fx+e)d}+\frac{d^{2}(d^{2})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^{2})^{}}\right.\right.}{8}\right.}{fd}$

input
```
int(cot(f*x+e)^2*(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^(1
/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^
2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/
2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2
)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

3.206.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.51

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$
$$-\frac{5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}(f\cos(2fx+2e)-f)\log\left(d\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}+\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\right)+5\left(-\frac{d^6}{f^4}\right)^{\frac{1}{4}}(if\cos(2fx+2e)-$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(5*(-d^6/f^4)^(1/4)*(f*cos(2*f*x + 2*e) - f)*log(d*sqrt((d*cos(2*f*x
 + 2*e) + d)/sin(2*f*x + 2*e)) + (-d^6/f^4)^(1/4)*f) + 5*(-d^6/f^4)^(1/4)*
(I*f*cos(2*f*x + 2*e) - I*f)*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
 + 2*e)) + I*(-d^6/f^4)^(1/4)*f) + 5*(-d^6/f^4)^(1/4)*(-I*f*cos(2*f*x + 2*
e) + I*f)*log(d*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - I*(-d^6/
f^4)^(1/4)*f) - 5*(-d^6/f^4)^(1/4)*(f*cos(2*f*x + 2*e) - f)*log(d*sqrt((d*
cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)) - (-d^6/f^4)^(1/4)*f) - 8*(3*d*cos
(2*f*x + 2*e) - 2*d)*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(f*c
os(2*f*x + 2*e) - f)
```

3.206.6 Sympy [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(e+fx))^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x)**2, x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{}\right.}{}$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d*f)
```

3.206.8 Giac [F]

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \int (d\cot(fx+e))^{\frac{3}{2}}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e)^2, x)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.39

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f}-\frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d\,f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

input
```
int(cot(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*d*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d*f) + ((-1
)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f +
((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
f
```

3.206. $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.207.1 Optimal result

Integrand size = 21, antiderivative size = 231

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
2/5*d^2/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)/d^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2
))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+
e))^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*
cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-2/f/(d*cot(f*x+e))^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.41

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+2(}{5f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-
Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/
(5*f*Sqrt[d*Cot[e + f*x]])
```

3.207.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 25, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^3\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)^3\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3 \int \frac{1}{\left(-d \tan \left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \; 3955$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3955$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3957$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df} + \frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 266$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df} + \frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 826$$

$$d^3 \left(\frac{2}{5df(d\cot(e+fx))^{5/2}} - \frac{\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)} - \frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df} + \frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 1476$$

3.207. $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}dx$

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot\cdots}\right.\right.}{df}\cdots}{d^2}\cdots\right.$$

$\Big\downarrow$ 1082

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2\cdots}{d^4\cot^4(e+\cdots}\cdots\right.}{df}\cdots}{d^2}\cdots\right.$$

$\Big\downarrow$ 217

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\cdots}{d^2}\cdots\right.$$

$\Big\downarrow$ 1479

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+\cdots}}{2\sqrt{2}\sqrt{d}}\cdots\right.\right.}{df}\cdots}{d^2}\cdots\right.$$

$\Big\downarrow$ 25

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f\cdots}}{2\sqrt{2}\sqrt{d}}\cdots\right.\right.}{df}\cdots}{d^2}\cdots\right.$$

$\Big\downarrow$ 27

3.207. $\displaystyle\int\frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}{2\sqrt{d}}}{df}}{d^2}\right.$$

$\downarrow$ 1103

$$d^3\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}\right.}{df}}{d^2}\right.$$

input
```
Int[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d^3*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

3.207.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.207. $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.207.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(178) = 356$.

Time = 3.46 (sec) , antiderivative size = 714, normalized size of antiderivative = 3.09

method	result
default	$\frac{\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(-40\left(\csc^7(fx+e)\right)(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(\cdots}}{1\cdots}\right.\right.}{}$

input
```
int(tan(f*x+e)^3/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(-40*csc(f*x+e)^7*(1-cos(f*x+e))^
7+5*ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x
+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x
+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*arcta
n(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+c
ot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+
cot(f*x+e))^(5/2)-5*ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*s
in(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*
cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+
e))^(5/2)-10*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+
e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x
+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-40*c
sc(f*x+e)^3*(1-cos(f*x+e))^3)/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e)
)^2-sin(f*x+e)))^(1/2)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-co
s(f*x+e)))^(1/2)/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/
2)*2^(1/2)
```

3.207.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.97

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{5\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right.}{}$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/10*(5*d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(
d/tan(f*x + e))) - 5*I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4
))^(3/4) + sqrt(d/tan(f*x + e))) + 5*I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d^2
*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 5*d*f*(-1/(d^2*f^4))^(
1/4)*log(-d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 4*(tan(f*
x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d*f)
```

3.207.6 Sympy [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^3{(e+fx)}}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(tan(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d^4\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\ \cdots}{\cdots}\right.}{d^4}\ \cdots\right.}{20\,f}$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/20*d^4*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

3.207.8 Giac [F]

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)^3}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\frac{2\,d^2}{5}-\frac{2\,d^2}{\tan(e+f\,x)^2}}{f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{5/2}}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

output
```
((2*d^2)/5 - (2*d^2)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1
/4)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) + ((-1)
^(1/4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.208.1 Optimal result

Integrand size = 21, antiderivative size = 212

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx =& -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} \\ &+ \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2d}{3f(d\cot(e+fx))^{3/2}} \\ &- \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} \\ &+ \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}\end{aligned}$$

output
```
2/3*d/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*co
t(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.40

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+\right.}{3f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(d*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) +
3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4)))/(f*(d*Cot[e +
 f*x])^(3/2))
```

3.208.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \; 2030$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3955$$

$$d^2\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3957}$$

$$d^2\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{266}$$

$$d^2\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{755}$$

$$d^2\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$d^2\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\vphantom{d}}}{2d}\right.}{df}\right.$$

$$\Big\downarrow \text{1082}$$

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{}{3df(\right.$$

$$\Big\downarrow \text{217}$$

3.208. $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

$$d^2\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 1479

$$d^2\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots}{\cdots}\right.}{df}\cdots\right.$$

$\Big\downarrow$ 25

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{df}\cdots\right.$$

$\Big\downarrow$ 27

$$d^2\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{df}\cdots\right.$$

$\Big\downarrow$ 1103

3.208. $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

$$d^2\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d^2*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*
Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]
]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] +
d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*
x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

3.208.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.208. $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.208.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(161) = 322$.

Time = 2.27 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sec(fx+e)\csc(fx+e)(\cos(fx+e)+1)\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx\text{-}}{\cos(fx+e)-1}\right.\right.}{}$

input
```
int(tan(f*x+e)^2/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*sec(f*x+e)*csc(f*x+e)*(cos(f*x+e)+1)*(-6*(-sin(f*x+e)*cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f
*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)*arctan((2^(1/2)*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*
x+e)-1))+3*cos(f*x+e)*ln(2*cot(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(c
os(f*x+e)-1)^2)^(1/2)+2*csc(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos(
f*x+e)-1)^2)^(1/2)-2*cot(f*x+e)+2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-3*cos(f*x+e)*ln(-2*cot(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(
cos(f*x+e)-1)^2)^(1/2)-2*csc(f*x+e)*2^(1/2)*(-csc(f*x+e)^2*cot(f*x+e)*(cos
(f*x+e)-1)^2)^(1/2)-2*cot(f*x+e)+2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)+4*2^(1/2)*cos(f*x+e)-4*2^(1/2))/(cot(f*x+e)*d)^(1/2)*2^(1/2)
```

3.208.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+3i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-3i}{}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*d*f*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt(d/tan(
f*x + e))) + 3*I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d*f*(-1/(d^2*f^4))^(1/4) +
 sqrt(d/tan(f*x + e))) - 3*I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*
f^4))^(1/4) + sqrt(d/tan(f*x + e))) - 3*d*f*(-1/(d^2*f^4))^(1/4)*log(-d*f*
(-1/(d^2*f^4))^(1/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/tan(f*x + e))*tan(
f*x + e)^2)/(d*f)
```

3.208.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^3\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\cdots}{\cdots}\right)}{d^2}\cdots\right)}{12\,f}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*d^3*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

3.208.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2d}{3f\left(\frac{d}{\tan(e+fx)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

output
```
(2*d)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e
 + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.209.1 Optimal result

Integrand size = 19, antiderivative size = 209

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d
^(1/2)+2/f/(d*cot(f*x+e))^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(f*Sqrt[d*Cot[e + f*x]])
```

3.209.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\sqrt{-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{d\cot(e+fx)}dx}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d\left(\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}\right)$$

$$\Big\downarrow \; 3957$$

$$d\left(\frac{\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \; 266$$

$$d\left(\frac{2\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \; 826$$

$$d\left(\frac{2\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}\right)$$

$$\Big\downarrow \; 1476$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right.\right.}{df}\right.$$

$$\Big\downarrow \; 1082$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot\right.}{df}\right.$$

$$\Big\downarrow \; 217$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{}{df\sqrt{d\,\mathrm{co}}}\right.$$

3.209. $\int\frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

$$\Big\downarrow \; 1479$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\cdots}\right.}{\cdots}\right.\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$d\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\cdots\right.}{\cdots}\right.\right.}{df}\cdots\right.$$

$$\Big\downarrow \; 1103$$

$$d\left(\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\cdots\right.}{\cdots}\right.\right.}{df}\cdots\right.$$

input
```
Int[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
d*(2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e
+ f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqr
t[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x
]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[
e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f))
```

3.209. $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.209.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.209. $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(160) = 320$.

Time = 3.13 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.17

method	result
default	$-\frac{\left(\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1\right)\left(\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{\left(\csc^3(fx+e)\right)(1-\cos(fx+e))^3-\csc(fx+e)+\cot(fx+e)}}{1-\cos(fx+e)}\right.\right.}{}$

input
```
int(tan(f*x+e)/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(ln(1/(1-cos(f*x+e))*(csc(f*x+e)*
(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+co
t(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3
-csc(f*x+e)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f
*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(
f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-ln(-1/(1-cos(f*x+e)
)*(-csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^
3-csc(f*x+e)+cot(f*x+e))^(1/2)-2+2*cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(
1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2*arctan(1/(1-cos(f*x+e))*(si
n(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos
(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-8*cs
c(f*x+e)+8*cot(f*x+e))/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2-sin
(f*x+e)))^(1/2)/(csc(f*x+e)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e
)))^(1/2)/(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)*2^(1
/2)
```

3.209.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+i}{}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/ta
n(f*x + e))) - I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4))^(3/
4) + sqrt(d/tan(f*x + e))) + I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d^2*f^3*(-1
/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - d*f*(-1/(d^2*f^4))^(1/4)*log(-
d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/tan(f*x +
e))*tan(f*x + e))/(d*f)
```

3.209.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{d^2\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{}\right.}{}}{d^2}\right.}{4f}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*d^2*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
- 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sq
rt(d/tan(f*x + e))))/f
```

3.209.8 Giac [F]

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\tan(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{2}{f\sqrt{\frac{d}{\tan(e+fx)}}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(tan(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

output
```
2/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*x
))^(1/2))/d^(1/2)))/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(e +
 f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

3.210.1 Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^
(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot\right.\right.}{2\sqrt{2}f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[1/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2
]*f*Sqrt[d*Cot[e + f*x]])
```

3.210.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \ 3957$$
$$-\frac{d\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}$$
$$\Big\downarrow \ 266$$
$$-\frac{2d\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$\downarrow$ 755

$$-\frac{2d\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\downarrow$ 1476

$$-\frac{2d\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d}}{2d}\right.}{f}$$

$\downarrow$ 1082

$$-\frac{2d\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$\downarrow$ 217

$$-\frac{2d\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$\downarrow$ 1479

$$-\frac{2d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{}\right.}{f}$$

$\downarrow$ 25

$$-\frac{2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arc}}{2d}\right.}{f}$$

$\downarrow$ 27

3.210. $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

$$-\frac{2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\text{arc}}{}}{2d}\right.}{f}$$

$$\Big\downarrow \ 1103$$

$$-\frac{2d\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[1/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Ar
cTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Lo
g[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d])
 + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*S
qrt[d]))/(2*d)))/f
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.210. $\int \frac{1}{\sqrt{d \cot(e+fx)}} \, dx$

3.210.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$
default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$

input
```
int(1/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/d*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^
(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.210.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.15

$$
\begin{aligned}
\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = & -\frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) \\
& -\frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) \\
& +\frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right) \\
& +\frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)
\end{aligned}
$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x
 + 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-1/(d^2*f^4))^(1/4)*log(I*d*f*(-1
/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2
*I*(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*
x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^2*f^4))^(1/4)*log(-d*f*(-1/(
d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

3.210.6 Sympy [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot{(e+fx)}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(1/sqrt(d*cot(e + f*x)), x)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e}\right.}{d^{\frac{3}{2}}}\right.}{4f}$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

3.210.8 Giac [F]

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d*cot(f*x + e)), x)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(1/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)
*f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(
d^(1/2)*f)
```

3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.211.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^
(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.39

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+fx)}\sqrt{d\cot(e+fx)}}{df\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d*f*Cot[e + f*x]^(3/4))
```

3.211.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int \sqrt{d\cot(e+fx)}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d}$$

$$\Big\downarrow \; 3957$$

$$-\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \; 266$$

$$-\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$\downarrow$ 826

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)\right.}{f}$$

$\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(\right.}{f}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{f}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{f}$$

3.211. $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

$$\Big\downarrow \text{1103}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/}\right.}{}\right.}{f}$$

input
```
Int[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/f
```

3.211.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.211. $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.211.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+\left(d^{2}\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{\left(d^{2}\right)^{\frac{1}{4}}}+1\right.}{4f\left(d^{2}\right)^{\frac{1}{4}}}$

input
```
int(cot(f*x+e)/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1
/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2
*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.211.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.23

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) + \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}} + \sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) - \frac{1}{2}i\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}} + \sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) + \frac{1}{2}\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(-d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}} + \sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2
*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*I*(-1/(d^2*f^4))^(1/4)*log(I*d^2
*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
)) - 1/2*I*(-1/(d^2*f^4))^(1/4)*log(-I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt
((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^2*f^4))^(1/4)*lo
g(-d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
+ 2*e)))
```

3.211.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}} -}{4f}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

3.211.8 Giac [F]

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 3.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.30

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2
)*f)
```

3.211. $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.212.1 Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d
^(1/2)-2*(d*cot(f*x+e))^(1/2)/d/f
```

3.212.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx =$$
$$-\frac{\sqrt{\cot(e+fx)}\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\mathrm{co}\right.}{2\sqrt{2}}\right.}{f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-((Sqrt[Cot[e + f*x]]*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[
1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[
2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*Sqrt[d*Cot[e + f*x]
]))
```

3.212.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$
$$\Big\downarrow \; 2030$$
$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d^2}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d^2}$$
$$\Big\downarrow \; 3954$$

$$\frac{d^2\left(-\int\frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{d^2\left(-\int\frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \ 3957$$

$$\frac{\frac{d^3\int\frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2d^3\int\frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \ 755$$

$$\frac{\frac{2d^3\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right.}{f}}{d^2}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{2d^3\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+f}}{f}}{d^2}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{2d^3\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^2}$$

3.212. $\int\frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}dx$

$\downarrow$ 1479

$$\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\ldots}{2d}\ldots\right.}{f}}{d^2}$$

$\downarrow$ 25

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\ldots\right.}{\ldots}}{2d}\ldots\right.}{f}}{d^2}$$

$\downarrow$ 27

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\ldots\right.}{\ldots}}{2d}\ldots\right.}{f}}{d^2}$$

$\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right)}{f}}{d^2}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d^2
```

3.212. $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.212.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.212. $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.212.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right)\right)}{8}\right)}{fd}$
default	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\cdots\right)\right)}{8}\right)}{fd}$

input
```
int(cot(f*x+e)^2/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^
2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/
4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)
*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2
)+1)))
```

3.212.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.25

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+}{\sin(2\,fx+2\,e)}}\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(d*f*(-1/(d^2*f^4))^(1/4)*log(d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2
*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + I*d*f*(-1/(d^2*f^4))^(1/4)*log(I*d*f
*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) -
 I*d*f*(-1/(d^2*f^4))^(1/4)*log(-I*d*f*(-1/(d^2*f^4))^(1/4) + sqrt((d*cos(
2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - d*f*(-1/(d^2*f^4))^(1/4)*log(-d*f*(
-1/(d^2*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 4
*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d*f)
```

3.212.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\right.}{4\,df}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/(d*f)
```

3.212.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.36

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{2\sqrt{d\cot(e+fx)}}{d\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$
$$-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/(d*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e
 + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.212. $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

3.213.1 Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}f}$$

output
```
-2/3*(d*cot(f*x+e))^(3/2)/d^2/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/
2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x
+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{\sqrt[4]{\cot(e+fx)}\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-}\right.}{3f\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
-1/3*(Cot[e + f*x]^(1/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x
])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Co
t[e + f*x]^(7/4)))/(f*Sqrt[d*Cot[e + f*x]])
```

3.213.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$\Big\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$\Big\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$\Big\downarrow$ 826

$$\frac{\frac{2d^3\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$\Big\downarrow$ 1476

$$\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{}\right.}{\dfrac{f}{d^3}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-2d(d}{d^3}$$

$\Big\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^3}$$

$\Big\downarrow$ 1479

$$\frac{2d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{\dfrac{f}{d^3}}$$

$\Big\downarrow$ 25

3.213. $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}dx$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots\right)\right)}{f}}{d^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\cdots\right)\right)}{f}}{d^3}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\cdots\right)}{2\sqrt{2}\sqrt{d}}\cdots\right)\right)}{f}}{d^3}$$

input
```
Int[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^3
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.213. $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.213. $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.213.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{}\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^2}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{}\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^2}$

input
```
int(cot(f*x+e)^3/(cot(f*x+e)*d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*
x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d
+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+
e)*d)^(1/2)+1)))
```

3.213.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.58

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$
$$= \frac{3\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(d^2f^3\left(-\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin\left(2\,fx+2\,e\right)-3i\,df\left(-\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f^3\left(-\frac{1}{d^2 ...}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*d*f*(-1/(d^2*f^4))^(1/4)*log(d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d
*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*d*f*(-1/(
d^2*f^4))^(1/4)*log(I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2
*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) + 3*I*d*f*(-1/(d^2*f^4))^(1/4
)*log(-I*d^2*f^3*(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(
2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*d*f*(-1/(d^2*f^4))^(1/4)*log(-d^2*f^3*
(-1/(d^2*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*si
n(2*f*x + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(cos(2*
f*x + 2*e) + 1))/(d*f*sin(2*f*x + 2*e))
```

3.213.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot^3\left(e+fx\right)}{\sqrt{d\cot\left(e+fx\right)}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{3\,d^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right.}{12\,d^2 f}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d^2*f)
```

3.213.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \int \frac{\cot(fx+e)^3}{\sqrt{d\cot(fx+e)}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

3.213.9 Mupad [B] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f} - \frac{2\,(d\cot(e+fx))^{3/2}}{3\,d^2\,f} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

input
```
int(cot(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
 - (2*(d*cot(e + f*x))^(3/2))/(3*d^2*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.214.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
2/5*d/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*co
t(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-2/d/f/(d*cot(f*x+e))^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.42

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{-5\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}+5\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right.}{5df\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-5*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 5*ArcTanh[(-
Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) + 2*(-5 + Tan[e + f*x]^2))/
(5*d*f*Sqrt[d*Cot[e + f*x]])
```

3.214.3 Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 2030, 3955, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)^2\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{(d\cot(e+fx))^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{d^2}\right)$$

$$\Big\downarrow \; 3955$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3042$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}}{d^2}\right)$$

$$\Big\downarrow \; 3957$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 266$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 826$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}}{d^2}\right)$$

$$\Big\downarrow \; 1476$$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot\right.\right.}{df}}{d^2}\right.$$

$$\Big\downarrow \; 1082$$

3.214. $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2}{d^4\cot^4(e+}\cdots}{df}}{d^2}\cdots\right.$$

$\downarrow$ 217

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\cdots}{d^2}\cdots\right.$$

$\downarrow$ 1479

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+}\cdots}{2\sqrt{2}\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

$\downarrow$ 25

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}\cdots}{2\sqrt{2}\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

$\downarrow$ 27

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+f}\cdots}{2\sqrt{d}}\cdots}{df}}{d^2}\cdots\right.$$

3.214. $\int\frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$\Big\downarrow$ 1103

$$d^2\left(\frac{2}{5df(d\cot(e+fx))^{5/2}}-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}\right.\right.}{df}}{d^2}\right.$$

input
```
Int[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
d^2*(2/(5*d*f*(d*Cot[e + f*x])^(5/2)) - (2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2
*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[
1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*
d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + S
qrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))
/(d*f))/d^2)
```

3.214.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

3.214. $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.214.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(179) = 358$.

Time = 3.28 (sec) , antiderivative size = 734, normalized size of antiderivative = 3.16

method	result
default	$-\frac{\left((\csc^2(fx+e))(1-\cos(fx+e))^2-1\right)^2\left(-40(\csc^7(fx+e))(1-\cos(fx+e))^7+5\ln\left(\frac{\csc(fx+e)(1-\cos(fx+e))^2+2\sin(fx+e)\sqrt{(\csc^3(fx+e}}{}\right.\right.}{}$

input
```
int(tan(f*x+e)^2/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/f*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)^2*(-40*csc(f*x+e)^7*(1-cos(f*x+e
))^7+5*ln(1/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x+e))^2+2*sin(f*x+e)*(csc(
f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)+2-2*cos(f*x+e)-sin(
f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)-10*ar
ctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e
)+cot(f*x+e))^(1/2)+1-cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+
e)+cot(f*x+e))^(5/2)-5*ln(-1/(1-cos(f*x+e))*(-csc(f*x+e)*(1-cos(f*x+e))^2+
2*sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-2
+2*cos(f*x+e)+sin(f*x+e)))*(csc(f*x+e)^3*(1-cos(f*x+e))^3-csc(f*x+e)+cot(f
*x+e))^(5/2)-10*arctan(1/(1-cos(f*x+e))*(sin(f*x+e)*(csc(f*x+e)^3*(1-cos(f
*x+e))^3-csc(f*x+e)+cot(f*x+e))^(1/2)-1+cos(f*x+e)))*(csc(f*x+e)^3*(1-cos(
f*x+e))^3-csc(f*x+e)+cot(f*x+e))^(5/2)+112*csc(f*x+e)^5*(1-cos(f*x+e))^5-4
0*csc(f*x+e)^3*(1-cos(f*x+e))^3)/(-d/(1-cos(f*x+e))*(csc(f*x+e)*(1-cos(f*x
+e))^2-sin(f*x+e)))^(3/2)/(1-cos(f*x+e))*sin(f*x+e)/(csc(f*x+e)*(csc(f*x+e
)^2*(1-cos(f*x+e))^2-1)*(1-cos(f*x+e)))^(1/2)/(csc(f*x+e)^3*(1-cos(f*x+e))
^3-csc(f*x+e)+cot(f*x+e))^(5/2)*2^(1/2)
```

3.214.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$
$$-\frac{5\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)-5i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right.}{}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(5*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqr
t(d/tan(f*x + e))) - 5*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/(d^6
*f^4))^(3/4) + sqrt(d/tan(f*x + e))) + 5*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(
-I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 5*d^2*f*(-1/(d^6
*f^4))^(1/4)*log(-d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt(d/tan(f*x + e))) - 4
*(tan(f*x + e)^3 - 5*tan(f*x + e))*sqrt(d/tan(f*x + e)))/(d^2*f)
```

3.214.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.89

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log(}{}\right)}{d^4}\right)}{20\,f}$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*d^3*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(
2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^
2 - 5*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

3.214.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^2}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\frac{2d}{5}-\frac{2d}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{d^{3/2}\,f} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(tan(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

output
```
((2*d)/5 - (2*d)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/4)*
atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) + ((-1)^(1/
4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.215.1 Optimal result

Integrand size = 19, antiderivative size = 211

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{2}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
2/3/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/
2))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d
^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^
(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$
$$-\frac{-2+3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)(-\cot^2(e+fx))}{3f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*(-2 + 3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4) + 3*A
rcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(3/4))/(f*(d*Cot[e + f*x
])^(3/2))
```

3.215.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {3042, 25, 2030, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{\tan\left(e+fx+\frac{\pi}{2}\right)\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{2030}$$
$$d\int \frac{1}{\left(-d\tan\left(\frac{1}{2}(2e+\pi)+fx\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3955}$$

$$d\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d^2}\right)$$

$\Big\downarrow$ 3042

$$d\left(\frac{2}{3df(d\cot(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d^2}\right)$$

$\Big\downarrow$ 3957

$$d\left(\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}(\cot^2(e+fx)d^2+d^2)}d(d\cot(e+fx))}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 266

$$d\left(\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 755

$$d\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 1476

$$d\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\vphantom{x}}}{2d}\right.}{df}\right.$$

$\Big\downarrow$ 1082

$$d\left(\frac{2\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{df}+\frac{}{3df(d}\right.$$

$\Big\downarrow$ 217

3.215. $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$d\left(\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{df}+\frac{2}{3df(d\cot(e+fx))^{3/2}}\right)$$

$\downarrow$ 1479

$$d\left(\frac{2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{}\right)}{df}\right)$$

$\downarrow$ 25

$$d\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right)}{df}\right)$$

$\downarrow$ 27

$$d\left(\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-}{2d}\right)}{df}\right)$$

$\downarrow$ 1103

3.215. $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$d\left(\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{df}\right.$$

input
```
Int[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
d*(2/(3*d*f*(d*Cot[e + f*x])^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Co
t[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/
(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^
2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
 + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/(d*f))
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.215. $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.215.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(160) = 320$.

Time = 2.72 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.71

method	result
default	$\frac{\left(-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+\cos(fx+e)-1}{\cos(fx+e)-1}\right)-6\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(\right.}{}$

input
```
int(tan(f*x+e)/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f/(cos(f*x+e)-1)/d/(cot(f*x+e)*d)^(1/2)*(-6*(-sin(f*x+e)*cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1))-6*(-sin(f
*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*arctan((2^(1/2)*(-sin(
f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*
x+e)-1))+3*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)*ln(-
(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+
e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+
e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*sin(f*x+
e)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(cot(f*x+e)*cos(f*x
+e)-2*cot(f*x+e)-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*c
sc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x
+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+4*2^(1/2)*sin(f*x+e)-4*tan(f*
x+e)*2^(1/2))*2^(1/2)
```

3.215.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2 f\left(-\frac{1}{d^6 f^4}\right)^{\frac{1}{4}}\log\left(d^2 f\left(-\frac{1}{d^6 f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d}{\tan(fx+e)}}\right)+3i\,d^2 f\left(-\frac{1}{d^6 f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2 f\left(\right.\right.}{}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt(d/
tan(f*x + e))) + 3*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f*(-1/(d^6*f^4))
^(1/4) + sqrt(d/tan(f*x + e))) - 3*I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-I*d^2
*f*(-1/(d^6*f^4))^(1/4) + sqrt(d/tan(f*x + e))) - 3*d^2*f*(-1/(d^6*f^4))^(
1/4)*log(-d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt(d/tan(f*x + e))) + 4*sqrt(d/ta
n(f*x + e))*tan(f*x + e)^2)/(d^2*f)
```

3.215.6 Sympy [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d^2\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\cdots}\right.}{\cdots}\cdots\right)}{d^2}\cdots\right)}{12\,f}$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*d^2*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2
*(d/tan(f*x + e))^(3/2)))/f
```

3.215.8 Giac [F]

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.38

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{3f\left(\frac{d}{\tan(e+fx)}\right)^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)1\mathrm{i}}{d^{3/2}f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)1\mathrm{i}}{d^{3/2}f}$$

input
```
int(tan(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

output
```
2/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f
*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/ta
n(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

3.216.1 Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2
^(1/2)+2/d/f/(d*cot(f*x+e))^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot^2(e+fx)}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)}{df\sqrt{d\cot(e+fx)}}$$

input
```
Integrate[(d*Cot[e + f*x])^(-3/2),x]
```

output
```
(2 + ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4) - ArcTanh[(-C
ot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x]^2)^(1/4))/(d*f*Sqrt[d*Cot[e + f*x]])
```

3.216.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3955}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}-\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 266

$$\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 826

$$\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 1476

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 217

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}+$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d\right.}{\sqrt{}}\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

3.216. $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

$\big\downarrow$ 25

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\right.}{\sqrt{2}}\right.\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\,c\right.}{\sqrt{2}}\right.\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

$\big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\right.}{}\right.\right.}{df}$$

$$\frac{2}{df\sqrt{d\cot(e+fx)}}$$

input
```
Int[(d*Cot[e + f*x])^(-3/2),x]
```

output
```
2/(d*f*Sqrt[d*Cot[e + f*x]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2
]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2
]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e +
 f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/(d*f)
```

3.216. $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.216.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2d\left(-\frac{1}{d^{2}\sqrt{\cot(fx+e)d}}-\frac{\sqrt{2}\,(\ln\left(\frac{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^{2})^{\frac{1}{4}}}+1\right)-2\arctan(-\cdots}{8d^{2}(d^{2})^{\frac{1}{4}}}\cdots}{f}$
default	$-\frac{2d\left(-\frac{1}{d^{2}\sqrt{\cot(fx+e)d}}-\frac{\sqrt{2}\,(\ln\left(\frac{\cot(fx+e)d-(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}{\cot(fx+e)d+(d^{2})^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^{2})^{\frac{1}{4}}}+1\right)-2\arctan(-\cdots}{8d^{2}(d^{2})^{\frac{1}{4}}}\cdots}{f}$

input
```
int(1/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*d*(-1/d^2/(cot(f*x+e)*d)^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f
*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*
d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/
(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x
+e)*d)^(1/2)+1)))
```

3.216.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.79

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(d^2f\cos(2fx+2e)+d^2f)\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos(2fx+2e)+d}{\sin(2fx+2e)}}\right)+\cdots}{\cdots}$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/2*((d^2*f*cos(2*f*x + 2*e) + d^2*f)*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1
/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + (-I
*d^2*f*cos(2*f*x + 2*e) - I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/
(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + (I*d
^2*f*cos(2*f*x + 2*e) + I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-I*d^5*f^3*(-1/(
d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - (d^2*
f*cos(2*f*x + 2*e) + d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-d^5*f^3*(-1/(d^6*f^4
))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 4*sqrt((d*co
s(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*sin(2*f*x + 2*e))/(d^2*f*cos(2*f*x +
 2*e) + d^2*f)
```

3.216.6 Sympy [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral((d*cot(e + f*x))**(-3/2), x)
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(f}}\right.}{}}{d^2}\right.}{4f}$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*d*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
 e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqr
t(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt
(d/tan(f*x + e))))/f
```

3.216.8 Giac [F]

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cot(f*x + e))^(-3/2), x)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2}{d\,f\,\sqrt{d\cot(e+f\,x)}} + \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(1/(d*cot(e + f*x))^(3/2),x)
```

output
```
2/(d*f*(d*cot(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f
*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e
 + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.216. $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.217.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^
(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.70

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\sqrt{\cot(e+fx)}\left(2\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot(e+f\right.\right.}{2\sqrt{}}$$

input
```
Integrate[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2
]*d*f*Sqrt[d*Cot[e + f*x]])
```

3.217.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2030, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx}{d}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}$$

$$\downarrow \text{755}$$

$$-\frac{2\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\downarrow \text{1476}$$

$$-\frac{2\left(\frac{\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cdots}}{2d}\cdots\right)}{f}$$

$$\downarrow \text{1082}$$

$$-\frac{2\left(\frac{\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}$$

$$\downarrow \text{217}$$

$$-\frac{2\left(\frac{\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}$$

$$\downarrow \text{1479}$$

$$-\frac{2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d\cdots}\right)}{f}$$

$$\downarrow \text{25}$$

$$-\frac{2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arcta}\cdots}{2d}\cdots\right)}{f}$$

$$\downarrow \text{27}$$

3.217. $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arcta}}{2d}\right.}{f}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{f}$$

input
```
Int[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[
d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) +
 Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqr
t[d]))/(2*d)))/f
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.217. $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.217.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$
default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$

input
```
int(cot(f*x+e)/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d
)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2
)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+
1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.217.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.19

$$
\begin{aligned}
&\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \\
&-\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) \\
&-\frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) \\
&+\frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right) \\
&+\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)
\end{aligned}
$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f
*x + 2*e) + d)/sin(2*f*x + 2*e))) - 1/2*I*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f
*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) +
 1/2*I*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*co
s(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^6*f^4))^(1/4)*log(-d^2
*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))
```

3.217.6 Sympy [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\ldots}{4f}$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

3.217.8 Giac [F]

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 2.85 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} + \frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)
*f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(
d^(3/2)*f)
```

3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.218.1 Optimal result

Integrand size = 21, antiderivative size = 192

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(
1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1/
4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^
(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.39

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\right)\sqrt[4]{-\cot(e+f}}{d^2 f\cot^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-ArcTan[(-Cot[e + f*x]^2)^(1/4)] + ArcTanh[(-Cot[e + f*x]^2)^(1/4)])*(-C
ot[e + f*x])^(1/4)*Sqrt[d*Cot[e + f*x]])/(d^2*f*Cot[e + f*x]^(3/4))
```

3.218.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2030, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{d\cot(e+fx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx}{d^2}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{df}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\int \frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{df}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)\right.}{df}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(\right.}{df}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{df}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{df}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{df}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}}{\sqrt{}}\right.\right.}{df}$$

$$\Big\downarrow \text{1103}$$

3.218. $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/}\right.}{}\right.\right.}{df}$$

input
```
Int[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/(d*f)
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.218. $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.218.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4fd\left(d^2\right)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+\left(d^2\right)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\,\sqrt{\cot(fx+e)d}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)\right)}{4fd\left(d^2\right)^{\frac{1}{4}}}$

input
```
int(cot(f*x+e)^2/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f/d/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^
(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)
-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1))
```

3.218.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.23

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = & -\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) \\
& +\frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) \\
& -\frac{1}{2}i\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-i\,d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right) \\
& +\frac{1}{2}\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(-d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac{3}{4}}+\sqrt{\frac{d\cos\left(2\,fx+2\,e\right)+d}{\sin\left(2\,fx+2\,e\right)}}\right)
\end{aligned}
$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2
*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*I*(-1/(d^6*f^4))^(1/4)*log(I*d^5
*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)
)) - 1/2*I*(-1/(d^6*f^4))^(1/4)*log(-I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt
((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 1/2*(-1/(d^6*f^4))^(1/4)*lo
g(-d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x
+ 2*e)))
```

3.218.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^2\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}} + \cdots}{4\,df}$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(
d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/(d*f)
```

3.218.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

3.218.9 Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.30

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2
)*f)
```

3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.219.1 Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2\sqrt{d\cot(e+fx)}}{d^2f}-\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*
arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^
(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1
/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2
^(1/2)-2*(d*cot(f*x+e))^(1/2)/d^2/f
```

3.219.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.76

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\cot^{\frac{3}{2}}(e+fx)\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)}{\sqrt{2}}+2\sqrt{\cot(e+fx)}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+\right.}{2\sqrt{2}}\right.}{f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-((Cot[e + f*x]^(3/2)*(ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] - Ar
cTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]]/Sqrt[2] + 2*Sqrt[Cot[e + f*x]] + Log[
1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2]) - Log[1 + Sqrt[
2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]/(2*Sqrt[2])))/(f*(d*Cot[e + f*x])^(3
/2)))
```

3.219.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\cot(e+fx))^{3/2}dx}{d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx}{d^3}$$

$$\Big\downarrow \; 3954$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 3042

$$\frac{d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 3957

$$\frac{\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 266

$$\frac{\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 755

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 1476

$$\frac{\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right.}{f}}{d^3}$$

$\downarrow$ 1082

$$\frac{\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+f}}{f}}{d^3}$$

$\downarrow$ 217

$$\frac{\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}}{d^3}$$

$\downarrow$ 1479

3.219. $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{f}}{d^3}$$

$\downarrow$ 25

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\cdots\right.}{\cdots}}{2d}\cdots\right.}{f}}{d^3}$$

$\downarrow$ 27

$$\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1\cdots\right.}{\cdots}}{2d}\cdots\right.}{f}}{d^3}$$

$\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right)}{f}}{d^3}$$

input
```
Int[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*Sqrt[d*Cot[e + f*x]])/f + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot
[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(
Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2
*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x]
+ d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/(2*d)))/f)/d^3
```

3.219. $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.219. $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.219.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right.}{f\,d^2}$
default	$-\frac{2\left(\sqrt{\cot(fx+e)d}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(\right.}{8}\right.}{f\,d^2}$

input
```
int(cot(f*x+e)^3/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^2*((cot(f*x+e)*d)^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(
1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/
4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1
/2)+1)))
```

3.219.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.32

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+i\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(i\,d^2\right.}{}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*c
os(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + I*d^2*f*(-1/(d^6*f^4))^(1/4)*log
(I*d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x +
2*e))) - I*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1/(d^6*f^4))^(1/4) +
sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - d^2*f*(-1/(d^6*f^4))^(1
/4)*log(-d^2*f*(-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*
f*x + 2*e))) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))/(d^2*f)
```

3.219.6 Sympy [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**3/(d*cot(e + f*x))**(3/2), x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right.}{}$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x +
e)))/(d^2*f)
```

3.219.8 Giac [F]

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^3}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^3/(d*cot(f*x + e))^(3/2), x)
```

3.219.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.36

$$\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2} f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2} f}$$

input
```
int(cot(e + f*x)^3/(d*cot(e + f*x))^(3/2),x)
```

output
```
- (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot
(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.219. $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.220.1 Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2(d\cot(e+fx))^{3/2}}{3d^3f}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
-2/3*(d*cot(f*x+e))^(3/2)/d^3/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/
2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x
+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.47

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{\cot^{\frac{5}{4}}(e+fx)\left(-3\arctan\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(e+fx)}\right)\sqrt[4]{-\cot(}\right.}{3f(d\cot(e+fx))^{3/2}}$$

input
```
Integrate[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/3*(Cot[e + f*x]^(5/4)*(-3*ArcTan[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x
])^(1/4) + 3*ArcTanh[(-Cot[e + f*x]^2)^(1/4)]*(-Cot[e + f*x])^(1/4) + 2*Co
t[e + f*x]^(7/4)))/(f*(d*Cot[e + f*x])^(3/2))
```

3.220.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2030, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{3954}$$

$$\frac{d^2\left(-\int \sqrt{d\cot(e+fx)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}dx\right)-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \; 3957$$

$$\frac{\frac{d^3\int\frac{\sqrt{d\cot(e+fx)}}{\cot^2(e+fx)d^2+d^2}d(d\cot(e+fx))}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2d^3\int\frac{d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \; 826$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\int\frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{\cdots}}{f}}{d^4}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cdots}{}}{d^4}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}\right)}{f}-\frac{2d(d\cot(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+\cdots\right.}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

$$\Big\downarrow \; 25$$

3.220. $\displaystyle\int\frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

$\downarrow$ 27

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

$\downarrow$ 1103

$$\frac{\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\right.}{2\sqrt{2}\sqrt{d}}\right.}{f}}{d^4}$$

input
```
Int[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(3/2))/(3*f) + (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[
d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f
*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Co
t[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] +
 d^2*Cot[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))/2))/f)/d^4
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.220. $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.220.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^3}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(fx+e)d}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)\right)}{8(d^2)^{\frac{1}{4}}}\right)}{f\,d^3}$

input
```
int(cot(f*x+e)^4/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^3*(1/3*(cot(f*x+e)*d)^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((cot(f*
x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2))/(cot(f*x+e)*d
+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(cot(f*x+
e)*d)^(1/2)+1)))
```

3.220.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac14}\log\left(d^5f^3\left(-\frac{1}{d^6f^4}\right)^{\frac34}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)\sin\left(2\,fx+2\,e\right)-3i\,d^2f}{}$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/6*(3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt(
(d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*I*d^2*f*(
-1/(d^6*f^4))^(1/4)*log(I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x
 + 2*e) + d)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) + 3*I*d^2*f*(-1/(d^6*f^4)
)^(1/4)*log(-I*d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d
)/sin(2*f*x + 2*e)))*sin(2*f*x + 2*e) - 3*d^2*f*(-1/(d^6*f^4))^(1/4)*log(-
d^5*f^3*(-1/(d^6*f^4))^(3/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2
*e)))*sin(2*f*x + 2*e) - 4*sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))
*(cos(2*f*x + 2*e) + 1))/(d^2*f*sin(2*f*x + 2*e))
```

3.220.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac32}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(d*cot(e + f*x))**(3/2), x)
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log}{}\right.}{12}$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d
)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2
)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f
*x + e))^(3/2))/(d^3*f)
```

3.220.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^4}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(d*cot(f*x + e))^(3/2), x)
```

3.220.9 Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}\,f} - \frac{2\,(d\cot(e+fx))^{3/2}}{3\,d^3\,f} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^4/(d*cot(e + f*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
 - (2*(d*cot(e + f*x))^(3/2))/(3*d^3*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.220. $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.221.1 Optimal result

Integrand size = 21, antiderivative size = 234

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f}$$

output
```
-2/5*(d*cot(f*x+e))^(5/2)/d^4/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/
2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x
+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/d^2/f
```

3.221.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$
$$-\frac{\cot^{\frac{3}{2}}(e+fx)\left(-10\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)+10\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)-40\sqrt{\cot(e+fx)}\right.}{}$$

input
```
Integrate[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

output
```
-1/20*(Cot[e + f*x]^(3/2)*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x
]]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] - 40*Sqrt[Cot[e +
f*x]] + 8*Cot[e + f*x]^(5/2) - 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]
] + Cot[e + f*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e +
 f*x]]))/(f*(d*Cot[e + f*x])^(3/2))
```

3.221.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {2030, 3042, 3954, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\cot(e+fx))^{7/2}dx}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}dx}{d^5}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-d^2\int (d\cot(e+fx))^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{-d^2\int \left(-d\tan\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 3954$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\cot(e+fx)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 3042$$

$$\frac{-d^2\left(d^2\left(-\int \frac{1}{\sqrt{-d\tan\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 3957$$

$$\frac{-d^2\left(\frac{d^3\int \frac{1}{\sqrt{d\cot(e+fx)}\left(\cot^2(e+fx)d^2+d^2\right)}d(d\cot(e+fx))}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 266$$

$$\frac{-d^2\left(\frac{2d^3\int \frac{1}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 755$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d^2\cot^2(e+fx)+d}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \ 1476$$

$$\frac{-d^2\left(\frac{2d^3\left(\frac{\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\mathrm{co}}}{2d}\right.}{f}\right.}{d^5}$$

3.221. $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$\Big\downarrow$ 1082

$$-\frac{d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\cot^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}\right)}{f}-\frac{2d\sqrt{d\cdots}}{\cdots}\right.}{d^5}$$

$\Big\downarrow$ 217

$$-\frac{d^2\left(\frac{2d^3\left(\frac{\int \frac{d-d^2\cot^2(e+fx)}{d^4\cot^4(e+fx)+d^2}d\sqrt{d\cot(e+fx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}\right)-\frac{2d(d\cot(e+\cdots}{5f}}{d^5}$$

$\Big\downarrow$ 1479

$$-\frac{d^2\left(\frac{2d^3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)\cdots\right.}{\sqrt{2}\sqrt{d}}\cdots\right.}{f}\cdots\right.}{d^5}$$

$\Big\downarrow$ 25

$$-\frac{d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\cdots}{2d}\cdots\right.}{f}\cdots\right.}{d^5}$$

$\Big\downarrow$ 27

3.221. $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)-\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\cot(e+fx)}}{d^2\cot^2(e+fx)+\sqrt{2}d^{3/2}\cot(e+fx)+d}d\sqrt{d\cot(e+fx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\ldots}{2d}\right)}{f}\ldots\right)$$
$$\overline{\qquad\qquad d^5\qquad\qquad}$$

$\downarrow$ 1103

$$-d^2\left(\frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\cot(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\cot(e+fx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\cot(e+fx)+d^2\cot^2(e+f\ldots\right)}{2\sqrt{2}\sqrt{d}}}{2d}\right)}{f}\ldots\right)$$
$$\overline{\qquad\qquad d^5\qquad\qquad}$$

input
```
Int[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

output
```
((-2*d*(d*Cot[e + f*x])^(5/2))/(5*f) - d^2*((-2*d*Sqrt[d*Cot[e + f*x]])/f
+ (2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
 ArcTan[1 + Sqrt[2]*Sqrt[d]*Cot[e + f*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2
*Log[d - Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(Sqrt[2]*Sqrt[
d]) + Log[d + Sqrt[2]*d^(3/2)*Cot[e + f*x] + d^2*Cot[e + f*x]^2]/(2*Sqrt[2
]*Sqrt[d]))/(2*d)))/f))/d^5
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.221. $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.221.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f\,d^4}$
default	$-\frac{2\left(\frac{(\cot(fx+e)d)^{\frac{5}{2}}}{5}-d^2\sqrt{\cot(fx+e)d}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{\cot(fx+e)d+(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}{\cot(fx+e)d-(d^2)^{\frac{1}{4}}\sqrt{\cot(fx+e)d}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{\cot(f}}{(d^2)^{\frac{1}{4}}}\right.\right.}{8}\right.}{f\,d^4}$

input
```
int(cot(f*x+e)^5/(cot(f*x+e)*d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f/d^4*(1/5*(cot(f*x+e)*d)^(5/2)-d^2*(cot(f*x+e)*d)^(1/2)+1/8*d^2*(d^2)^
(1/4)*2^(1/2)*(ln((cot(f*x+e)*d+(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(
d^2)^(1/2))/(cot(f*x+e)*d-(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)*2^(1/2)+(d^2)^(
1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)-2*arctan(-2^(1
/2)/(d^2)^(1/4)*(cot(f*x+e)*d)^(1/2)+1)))
```

3.221.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.63

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{5\left(d^2f\cos\left(2\,fx+2\,e\right)-d^2f\right)\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}\log\left(d^2f\left(-\frac{1}{d^6f^4}\right)^{\frac{1}{4}}+\sqrt{\frac{d\cos(2\,fx+2\,e)+d}{\sin(2\,fx+2\,e)}}\right)+5\left(i\,d^2f\cos\left(2\,fx+2\,e\right)\right.}{}$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(5*(d^2*f*cos(2*f*x + 2*e) - d^2*f)*(-1/(d^6*f^4))^(1/4)*log(d^2*f*(
-1/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5
*(I*d^2*f*cos(2*f*x + 2*e) - I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(I*d^2*f*(-1
/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) + 5*(
-I*d^2*f*cos(2*f*x + 2*e) + I*d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-I*d^2*f*(-1
/(d^6*f^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 5*(
d^2*f*cos(2*f*x + 2*e) - d^2*f)*(-1/(d^6*f^4))^(1/4)*log(-d^2*f*(-1/(d^6*f
^4))^(1/4) + sqrt((d*cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))) - 8*sqrt((d*
cos(2*f*x + 2*e) + d)/sin(2*f*x + 2*e))*(3*cos(2*f*x + 2*e) - 2))/(d^2*f*c
os(2*f*x + 2*e) - d^2*f)
```

3.221.6 Sympy [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot^5\left(e+fx\right)}{\left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**5/(d*cot(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**5/(d*cot(e + f*x))**(3/2), x)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.85

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx =$$

$$-\frac{10\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{}\right.}{}$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/t
an(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*s
qrt(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*
sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log
(-sqrt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt
(d/tan(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^4*f)
```

3.221.8 Giac [F]

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^5}{(d\cot(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^5/(d*cot(f*x + e))^(3/2), x)
```

3.221.9 Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.40

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{2\sqrt{d\cot(e+f\,x)}}{d^2\,f} - \frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d^4\,f} + \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f} + \frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

input
```
int(cot(e + f*x)^5/(d*cot(e + f*x))^(3/2),x)
```

output
```
(2*(d*cot(e + f*x))^(1/2))/(d^2*f) - (2*(d*cot(e + f*x))^(5/2))/(5*d^4*f)
+ ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/
2)*f) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/
(d^(3/2)*f)
```

3.221. $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

output
```
cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^
2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

3.222.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

input
```
Integrate[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

3.222.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(e+fx)^m\tan(e+fx)^n dx$$

$$\Big\downarrow \text{3084}$$

$$\tan^m(e+fx)\cot^m(e+fx)\int \tan^{n-m}(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\tan^m(e+fx)\cot^m(e+fx)\int \tan(e+fx)^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{\tan^m(e+fx)\cot^m(e+fx)\int \frac{\tan^{n-m}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

input
```
Int[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

3.222.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.222.4 Maple [F]

$$\int (\cot^m (fx+e)) (\tan^n (fx+e)) \, dx$$

input
```
int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

output
```
int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

3.222.5 Fricas [F]

$$\int \cot^m(e+fx) \tan^n(e+fx) \, dx = \int \cot (fx+e)^m \tan (fx+e)^n \, dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

3.222.6 Sympy [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \tan^n(e+fx)\cot^m(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**m*tan(f*x+e)**n,x)
```

output
```
Integral(tan(e + f*x)**n*cot(e + f*x)**m, x)
```

3.222.7 Maxima [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(fx+e)^m\tan(fx+e)^n\,dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

3.222.8 Giac [F]

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(fx+e)^m\tan(fx+e)^n\,dx$$

input
```
integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)\tan^n(e+fx)\,dx = \int \cot(e+f\,x)^m \tan(e+f\,x)^n\,dx$$

input
```
int(cot(e + f*x)^m*tan(e + f*x)^n,x)
```

output
```
int(cot(e + f*x)^m*tan(e + f*x)^n, x)
```

3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$

3.223.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^m(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-m+n)}$$

output
```
cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^
2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

3.223.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \frac{\cot^{-1+m}(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^n}{f(1-m+n)}$$

input
```
Integrate[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[e + f*x]^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2,
-Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

3.223.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(e+fx)^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3084}$$

$$\cot^m(e+fx)(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3042}$$

$$\cot^m(e+fx)(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b\cot^m(e+fx)(b\tan(e+fx))^m\int \frac{(b\tan(e+fx))^{n-m}}{\tan^2(e+fx)b^2+b^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

input
```
Int[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

output
```
(Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e
+ f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

3.223.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.223.4 Maple [F]

$$\int (\cot^m (fx+e)) (b\tan (fx+e))^n \, dx$$

input
```
int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

output
```
int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

3.223.5 Fricas [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n \, dx = \int (b\tan (fx+e))^n \cot (fx+e)^m \, dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

3.223.6 Sympy [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(e+fx))^n \cot^m(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((b*tan(e + f*x))**n*cot(e + f*x)**m, x)
```

3.223.7 Maxima [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

3.223.8 Giac [F]

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = \int (b\tan(fx+e))^n \cot(fx+e)^m\,dx$$

input
```
integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx=\int \cot(e+f\,x)^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int(cot(e + f*x)^m*(b*tan(e + f*x))^n,x)
```

output
```
int(cot(e + f*x)^m*(b*tan(e + f*x))^n, x)
```

3.224 $\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$

3.224.1 Optimal result

Integrand size = 19, antiderivative size = 64

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

output
```
(a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x
+e)^2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

3.224.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

input
```
Integrate[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

3.224.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^n(e+fx)(a\cot(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(e+fx)^n(a\cot(e+fx))^m dx$$

$$\Big\downarrow \ 3084$$

$$\tan^m(e+fx)(a\cot(e+fx))^m\int \tan^{n-m}(e+fx)dx$$

$$\Big\downarrow \ 3042$$

$$\tan^m(e+fx)(a\cot(e+fx))^m\int \tan(e+fx)^{n-m}dx$$

$$\Big\downarrow \ 3957$$

$$\frac{\tan^m(e+fx)(a\cot(e+fx))^m\int \frac{\tan^{n-m}(e+fx)}{\tan^2(e+fx)+1}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 278$$

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

input
```
Int[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

3.224.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.224.4 Maple [F]

$$\int (a\cot(fx+e))^m (\tan^n(fx+e))\,dx$$

input
```
int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

output
```
int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

3.224.5 Fricas [F]

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m \tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

3.224.6 Sympy [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx = \int (a\cot(e+fx))^m\tan^n(e+fx)\,dx$$

input
```
integrate((a*cot(f*x+e))**m*tan(f*x+e)**n,x)
```

output
```
Integral((a*cot(e + f*x))**m*tan(e + f*x)**n, x)
```

3.224.7 Maxima [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m\tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

3.224.8 Giac [F]

$$\int (a\cot(e+fx))^m\tan^n(e+fx)\,dx = \int (a\cot(fx+e))^m\tan(fx+e)^n\,dx$$

input
```
integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = \int \tan(e+f\,x)^n\,(a\cot(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^n*(a*cot(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^n*(a*cot(e + f*x))^m, x)
```

3.225 $\int (a\cot(e+fx))^m (b\tan(e+fx))^n\,dx$

3.225.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{(a\cot(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+}}{bf(1-m+n)}$$

output
```
(a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x
+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

3.225.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{a(a\cot(e+fx))^{-1+m} \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),-\tan^2(e+fx)\right)(b\tan(e+fx}{f(1-m+n)}$$

input
```
Integrate[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(a*(a*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m +
n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

3.225.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3084, 3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n dx$$

$$\Big\downarrow \text{3084}$$

$$(a\cot(e+fx))^m(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cot(e+fx))^m(b\tan(e+fx))^m\int (b\tan(e+fx))^{n-m}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{b(a\cot(e+fx))^m(b\tan(e+fx))^m\int \frac{(b\tan(e+fx))^{n-m}}{\tan^2(e+fx)b^2+b^2}d(b\tan(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(a\cot(e+fx))^m(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

input
```
Int[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Ta
n[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

3.225.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3084
```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m   Int[(b*Tan[e
+ f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
  !IntegerQ[n]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.225.4 Maple [F]

$$\int \left(a \cot \left(f x+e\right)\right)^{m} \left(b \tan \left(f x+e\right)\right)^{n}d x$$

input
```
int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

3.225.5 Fricas [F]

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = \int \left(a \cot \left(f x+e\right)\right)^{m} \left(b \tan \left(f x+e\right)\right)^{n}\,dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.225.6 Sympy [F]

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx$$

input
```
integrate((a*cot(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*cot(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.225.7 Maxima [F]

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cot(fx+e))^m (b\tan(fx+e))^n \, dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.225.8 Giac [F]

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\cot(fx+e))^m (b\tan(fx+e))^n \, dx$$

input
```
integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\cot(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

input
```
int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

output
```
int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.226.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f+4/7*(d*tan(f*x+e))^(7/2)/d^3/f+2/11*(d*tan(f*
x+e))^(11/2)/d^5/f
```

3.226.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(45+28\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{3/2}}{231df}$$

input
```
Integrate[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(45 + 28*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
 + f*x])^(3/2))/(231*d*f)
```

3.226.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \sqrt{d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{9/2}}{d^4}+\frac{2(d\tan(e+fx))^{5/2}}{d^2}+\sqrt{d\tan(e+fx)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{11/2}}{11d^5}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3}+\frac{2(d\tan(e+fx))^{3/2}}{3d}}{f}$$

input
```
Int[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((2*(d*Tan[e + f*x])^(3/2))/(3*d) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3) + (
2*(d*Tan[e + f*x])^(11/2))/(11*d^5))/f
```

3.226.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.226.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^{2}(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^{4}(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^{5}}$	52
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{4d^{2}(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^{4}(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^{5}}$	52

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^5*(1/11*(d*tan(f*x+e))^(11/2)+2/7*d^2*(d*tan(f*x+e))^(7/2)+1/3*d^4*(
d*tan(f*x+e))^(3/2))
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(32\cos\left(fx+e\right)^4+24\cos\left(fx+e\right)^2+21\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{231\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/231*(32*cos(f*x + e)^4 + 24*cos(f*x + e)^2 + 21)*sqrt(d*sin(f*x + e)/cos
(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

3.226.6 Sympy [F]

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\sec^6\left(e+fx\right)\,dx$$

input
```
integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**6, x)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(21\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+66\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2+77\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^4\right)}{231\,d^5 f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/231*(21*(d*tan(f*x + e))^(11/2) + 66*(d*tan(f*x + e))^(7/2)*d^2 + 77*(d*
tan(f*x + e))^(3/2)*d^4)/(d^5*f)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\left(21\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+66\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3+77\sqrt{d\tan(fx+e)}d^5\tan\right.}{231\,d^5f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/231*(21*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 66*sqrt(d*tan(f*x + e)
)*d^5*tan(f*x + e)^3 + 77*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e))/(d^5*f)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.99

$$\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,f} - \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{77\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,768\mathrm{i}}{77\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3} - \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{11\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^6,x)
```

output
```
((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*768i)/
(77*f*(exp(e*2i + f*x*2i) + 1)^3) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(e
xp(e*2i + f*x*2i) + 1))^(1/2)*64i)/(231*f*(exp(e*2i + f*x*2i) + 1)) - ((-(
d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(77*f
*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*
2i + f*x*2i) + 1))^(1/2)*64i)/(231*f) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i)
)/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i) + 1)^4)
+ ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i)
/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.227.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(d\tan(e+fx))^{3/2}}{3df} + \frac{2(d\tan(e+fx))^{7/2}}{7d^3f}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f+2/7*(d*tan(f*x+e))^(7/2)/d^3/f
```

3.227.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2(4+3\sec^2(e+fx))\,(d\tan(e+fx))^{3/2}}{21df}$$

input
```
Integrate[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(4 + 3*Sec[e + f*x]^2)*(d*Tan[e + f*x])^(3/2))/(21*d*f)
```

3.227.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \sqrt{d\tan(e+fx)}\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{5/2}}{d^2}+\sqrt{d\tan(e+fx)}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{7/2}}{7d^3}+\frac{2(d\tan(e+fx))^{3/2}}{3d}}{f}$$

input
```
Int[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((2*(d*Tan[e + f*x])^(3/2))/(3*d) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3))/f
```

3.227.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.227.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$	37
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7}+\frac{2d^2(d\tan(fx+e))^{\frac{3}{2}}}{3}}{f\,d^3}$	37

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^3*(1/7*(d*tan(f*x+e))^(7/2)+1/3*d^2*(d*tan(f*x+e))^(3/2))
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2\left(4\cos\left(fx+e\right)^2+3\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{21\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(4*cos(f*x + e)^2 + 3)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)
/(f*cos(f*x + e)^3)
```

3.227.6 Sympy [F]

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\sec^4{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**4, x)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(3\,(d\tan{(fx+e)})^{\frac{7}{2}}+7\,(d\tan{(fx+e)})^{\frac{3}{2}}\,d^2\right)}{21\,d^3 f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/21*(3*(d*tan(f*x + e))^(7/2) + 7*(d*tan(f*x + e))^(3/2)*d^2)/(d^3*f)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\left(3\,\sqrt{d\tan{(fx+e)}}d^3\tan{(fx+e)}^3+7\,\sqrt{d\tan{(fx+e)}}d^3\tan{(fx+e)}\right)}{21\,d^3 f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/21*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 + 7*sqrt(d*tan(f*x + e))*d
^3*tan(f*x + e))/(d^3*f)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 7.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.84

$$\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{-\frac{d\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{21\,f} - \frac{\sqrt{-\frac{d\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{21\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} + \frac{\sqrt{-\frac{d\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} - \frac{\sqrt{-\frac{d\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^4,x)
```

output
```
((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(
7*f*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(e
xp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f) - (
(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*16i)/(7
*f*(exp(e*2i + f*x*2i) + 1)^3)
```

3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f
```

3.228.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx=\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

input
```
Integrate[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.228.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(e+fx)^2\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \; 3087$$

$$\frac{\int \sqrt{d\tan(e+fx)}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 17$$

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

input
```
Int[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.228.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.228.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*tan(f*x+e))^(3/2)/d/f
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin(fx+e)}{3f\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e))
```

3.228.6 Sympy [F]

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\sec^2{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**2, x)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,(d\tan{(fx+e)})^{\frac{3}{2}}}{3\,df}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(d*tan(f*x + e))^(3/2)/(d*f)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sqrt{d\tan{(fx+e)}}\tan{(fx+e)}}{3\,f}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(d*tan(f*x + e))*tan(f*x + e)/f
```

3.228.9 Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\,\sin\left(2\,e+2\,f\,x\right)\,\sqrt{\frac{d\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^2,x)
```

output
```
(2*sin(2*e + 2*f*x)*((d*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(
3*f*(cos(2*e + 2*f*x) + 1))
```

3.229 $\int \sqrt{d\tan(e+fx)}\,dx$

3.229.1 Optimal result

Integrand size = 12, antiderivative size = 192

$$\int \sqrt{d\tan(e+fx)}\,dx = -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}$$

output
```
-1/2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*
arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^
(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-1
/4*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2
^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.37

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$=\frac{\left(\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(e+fx)}\right)\right)\sqrt[4]{-\tan(e+fx)}\sqrt{d\tan(e+fx)}}{f\tan^{\frac{3}{4}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]],x]
```

output
```
((ArcTan[(-Tan[e + f*x]^2)^(1/4)] - ArcTanh[(-Tan[e + f*x]^2)^(1/4)])*(-Ta
n[e + f*x])^(1/4)*Sqrt[d*Tan[e + f*x]])/(f*Tan[e + f*x]^(3/4))
```

3.229.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \sqrt{d\tan(e+fx)}dx$$
$$\Big\downarrow \; 3957$$
$$\frac{d\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)d^2+d^2}d(d\tan(e+fx))}{f}$$
$$\Big\downarrow \; 266$$
$$\frac{2d\int \frac{d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}}{f}$$
$$\Big\downarrow \; 826$$

$$\frac{2d\left(\frac{1}{2}\int \frac{d^2\tan^2(e+fx)+d}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}\right)\right.}{f}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(\right.}{f}$$

$$\Big\downarrow \text{217}$$

$$\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{f}$$

$$\Big\downarrow \text{1103}$$

3.229. $\int \sqrt{d\tan(e+fx)}\,dx$

$$\frac{2d\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(e+fx)+d^2\tan^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/}\right.}{}\right.}{f}$$

input
```
Int[Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*d*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d])) + Arc
Tan[1 + Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt
[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
 + Sqrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]))
/2))/f
```

3.229.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.229. $\int \sqrt{d \tan(e+fx)}\,dx$

3.229.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$
default	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+\left(d^2\right)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{\left(d^2\right)^{\frac{1}{4}}}+\right.\right.}{4f\left(d^2\right)^{\frac{1}{4}}}$

input
```
int((d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2
^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
```

3.229.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \sqrt{d\tan(e+fx)}\,dx = {} & \frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\ & -\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\ & +\frac{1}{2}i\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-i\,f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\\ & -\frac{1}{2}\left(-\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-f^3\left(-\frac{d^2}{f^4}\right)^{\frac{3}{4}}+\sqrt{d\tan\left(fx+e\right)}d\right)\end{aligned}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-d^2/f^4)^(1/4)*log(f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))*d) -
1/2*I*(-d^2/f^4)^(1/4)*log(I*f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x + e))*d
) + 1/2*I*(-d^2/f^4)^(1/4)*log(-I*f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x +
e))*d) - 1/2*(-d^2/f^4)^(1/4)*log(-f^3*(-d^2/f^4)^(3/4) + sqrt(d*tan(f*x +
 e))*d)
```

3.229.6 Sympy [F]

$$\int \sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan{(e+fx)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x)), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+\right.}{\sqrt{d}}\right.}{4f}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqr
t(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e
) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/f
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - \frac{\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(}{f}}{4d}$$

input
```
integrate((d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f -
sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(abs(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*
sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

3.229.9 Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.26

$$\int \sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right) - \operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)\right)}{f}$$

input
```
int((d*tan(e + f*x))^(1/2),x)
```

output
```
((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - a
tanh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))))/f
```

3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.230.1 Optimal result

Integrand size = 21, antiderivative size = 227

$$\begin{aligned}\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx &= -\frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}\\ &+\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}\\ &-\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}\\ &+\frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}\end{aligned}$$

output
```
-1/8*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/8*
arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/16*ln(d
^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-
1/16*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f
*2^(1/2)+1/2*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/d/f
```

3.230.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.45

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx =$$

$$-\frac{\left(\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(\right.\right.}{8f}$$

input
```
Integrate[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
-1/8*((ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + Csc[e + f*x]*Log
[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] - 2*Sqrt[Sin[2*(e +
 f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

3.230.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {3042, 3087, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \ 3087$$

$$\frac{\int \frac{\sqrt{d\tan(e+fx)}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \ 253$$

$$\frac{\frac{1}{4}\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}}{2d}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{2}d\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$\big\downarrow$ 826

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}}{f}$$

$\big\downarrow$ 1476

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}\right)\right.}{f}$$

$\big\downarrow$ 1082

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+\right.}{f}$$

$\big\downarrow$ 217

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+fx}{2d(\tan^2(e+f}}{f}$$

$\big\downarrow$ 1479

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\right.}{}\right.\right.}{f}$$

$\big\downarrow$ 25

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d\right.}{\sqrt{2}}\right.\right.}{f}$$

3.230. $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}}{}\right.}{\sqrt{2}}\right.\right.}{f}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(e+\right.}{}\right.\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

output
```
((d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d
])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])
)/2 + (Log[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2*S
qrt[2]*Sqrt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e +
f*x]]]/(2*Sqrt[2]*Sqrt[d]))/2))/2 + (d*Tan[e + f*x])^(3/2)/(2*d*(1 + Tan[e
 + f*x]^2)))/f
```

3.230.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.230. $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.230.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(171) = 342$.

Time = 1.17 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.33

method	result
default	$\left(4\sqrt{2}\,\cos(fx+e)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+4\sqrt{2}\,\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)-\ln\left(-\frac{\cot(fx+e)\cos(fx+e)-2\cot(fx+e)}{}\right.\right.$

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f*(4*2^(1/2)*cos(f*x+e)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*sin(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*si
n(f*x+e)-ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)
^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*
x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1
))+ln((2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^
2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+
e)+sin(f*x+e)+2*cos(f*x+e)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))+2*ar
ctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-c
os(f*x+e)+1)/(cos(f*x+e)-1))+2*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(d*tan(f*x+
e))^(1/2)*cos(f*x+e)/(cos(f*x+e)+1)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*2^(1/2)
```

3.230.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 924, normalized size of antiderivative = 4.07

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/32*(16*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + f*(
-d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*x + e)*sin(f*x + e) + 1/2*(f^3*(-d^2/f^4
)^(3/4)*cos(f*x + e)^2 - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*s
qrt(d*sin(f*x + e)/cos(f*x + e)) - 1/4*(2*d*f^2*cos(f*x + e)^2 - d*f^2)*sq
rt(-d^2/f^4)) - f*(-d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*x + e)*sin(f*x + e) -
 1/2*(f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)^2 - d*f*(-d^2/f^4)^(1/4)*cos(f*x +
 e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 1/4*(2*d*f^2*cos(f*x
 + e)^2 - d*f^2)*sqrt(-d^2/f^4)) - I*f*(-d^2/f^4)^(1/4)*log(1/2*d^2*cos(f*
x + e)*sin(f*x + e) + 1/2*(I*f^3*(-d^2/f^4)^(3/4)*cos(f*x + e)^2 + I*d*f*(
-d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e
)) + 1/4*(2*d*f^2*cos(f*x + e)^2 - d*f^2)*sqrt(-d^2/f^4)) + I*f*(-d^2/f^4)
^(1/4)*log(1/2*d^2*cos(f*x + e)*sin(f*x + e) + 1/2*(-I*f^3*(-d^2/f^4)^(3/4
)*cos(f*x + e)^2 - I*d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(
d*sin(f*x + e)/cos(f*x + e)) + 1/4*(2*d*f^2*cos(f*x + e)^2 - d*f^2)*sqrt(-
d^2/f^4)) + f*(-d^2/f^4)^(1/4)*log(d^2 + 2*(f^3*(-d^2/f^4)^(3/4)*cos(f*x +
 e)*sin(f*x + e) - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e
)/cos(f*x + e))) - f*(-d^2/f^4)^(1/4)*log(d^2 - 2*(f^3*(-d^2/f^4)^(3/4)*co
s(f*x + e)*sin(f*x + e) - d*f*(-d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(
f*x + e)/cos(f*x + e))) + I*f*(-d^2/f^4)^(1/4)*log(d^2 - 2*(I*f^3*(-d^2/f^
4)^(3/4)*cos(f*x + e)*sin(f*x + e) + I*d*f*(-d^2/f^4)^(1/4)*cos(f*x + e...
```

3.230.6 Sympy [F]

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*cos(e + f*x)**2, x)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}\right.}{\sqrt{d}}\right.}{16\,df}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*
x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x
 + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(f*x
+ e))^(3/2)*d^2/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{\frac{8\sqrt{d\tan(fx+e)}\,d^3\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} + \frac{2\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f} - \frac{\sqrt{2}}{}}{16\,d}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/16*(8*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*
f) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f -
sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqr
t(abs(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*
sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^2\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

input
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2), x)
```

3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}} + \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

output
```
4/5*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos
(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+4/5*co
s(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f+2/5*sec(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f
```

3.231.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.49 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\sqrt{d\tan(e+fx)}\left(-4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sec(e+fx)\tan(e+fx)+3\sqrt{\sec^2(e\right.}{15f\sqrt{\sec^2(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sqrt[d*Tan[e + f*x]]*(-4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]
^2]*Sec[e + f*x]*Tan[e + f*x] + 3*Sqrt[Sec[e + f*x]^2]*(2*Sin[e + f*x] + S
ec[e + f*x]*Tan[e + f*x])))/(15*f*Sqrt[Sec[e + f*x]^2])
```

3.231.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3093, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^3\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}\right)+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2}{5}\left(\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}\right)$$

input
```
Int[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (2*((-2*Cos[e + f*x]*Ell
ipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Sin[2*e + 2*f*x]])
 + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)))/5
```

3.231. $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.231.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.231.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(118) = 236$.

Time = 1.46 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.75

method	result
default	$-\frac{\sqrt{d\tan(fx+e)}\left(-4\cot(fx+e)\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-c\ldots}\right.}{}$

input
```
int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/f*(d*tan(f*x+e))^(1/2)*(-4*cot(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)
*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),
1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+2*cot(f*x+e)*(cot(f*x+e)-csc
(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^
(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))-4*csc(f*x+e)
*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((
-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1
/2)+2*csc(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(
1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)
^(1/2),1/2*2^(1/2))+2*2^(1/2)*cot(f*x+e)-csc(f*x+e)*2^(1/2)-sec(f*x+e)^2*c
sc(f*x+e)*2^(1/2))*2^(1/2)
```

3.231.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.72

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\left(i\sqrt{i\,d}\cos\left(fx+e\right)^2 E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)-i\sqrt{-i\,d}\cos\left(fx+e\right)^2 E(\arcsin\left(\right.\right.}{}$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(I*sqrt(I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*
x + e)), -1) - I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_e(arcsin(cos(f*x + e)
- I*sin(f*x + e)), -1) - I*sqrt(I*d)*cos(f*x + e)^2*elliptic_f(arcsin(cos(
f*x + e) + I*sin(f*x + e)), -1) + I*sqrt(-I*d)*cos(f*x + e)^2*elliptic_f(a
rcsin(cos(f*x + e) - I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*
sin(f*x + e)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2)
```

3.231.6 Sympy [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\sec^3(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**3, x)
```

3.231.7 Maxima [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

3.231.8 Giac [F]

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)^3\,dx$$

input
```
integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\cos\left(e+f\,x\right)}^3}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3,x)
```

output
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3, x)
```

3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.232.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}+\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}$$

output
```
2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e
+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+2*cos(f*
x+e)*(d*tan(f*x+e))^(3/2)/d/f
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = -\frac{2\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

input
```
Integrate[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e +
 f*x]^2])*Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

3.232.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3093}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-2\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3095}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3052}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqr
t[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.232.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(94) = 188.

Time = 1.42 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.89

method	result
default	$-\frac{\csc(fx+e)\left(-2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(fx+e)+\csc(}\right.}{}$

input
```
int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*csc(f*x+e)*(-2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)
)^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+
e)+csc(f*x+e)+1)^(1/2)*cos(f*x+e)+(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x
+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x
+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(cot(f*x+e)-csc(f*x+e)+1
)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)
^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+(cot(f*x+e)-csc(f*x+e
)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*
EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)
-2^(1/2))*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

3.232.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.75

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{-i\sqrt{i\,d}E(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)+i\sqrt{-i\,d}E(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)|}{}$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I*d)*elliptic_e(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + I*sq
rt(-I*d)*elliptic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + I*sqrt(I*
d)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) - I*sqrt(-I*d)*el
liptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) + 2*sqrt(d*sin(f*x + e
)/cos(f*x + e))*sin(f*x + e))/f
```

3.232.6 Sympy [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\sec(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*sec(e + f*x), x)
```

3.232.7 Maxima [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

3.232.8 Giac [F]

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\sec(fx+e)\,dx$$

input
```
integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/cos(e + f*x), x)
```

3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.233.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

output
```
-cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+
1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

input
```
Integrate[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2]*
Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

3.233.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3095}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}}$$

$$\Big\downarrow \text{ 3052}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[S
in[2*e + 2*f*x]])
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.233.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(69) = 138$.

Time = 0.95 (sec) , antiderivative size = 377, normalized size of antiderivative = 8.02

method	result
default	$-\frac{\csc(fx+e)\left(2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(fx+e)+\csc(f\ldots}{}$

input
```
int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*csc(f*x+e)*(2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e
))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x
+e)+csc(f*x+e)+1)^(1/2)*cos(f*x+e)-(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*
x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*
x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*(cot(f*x+e)-csc(f*x+e)+
1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1
)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-(cot(f*x+e)-csc(f*x+
e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)
*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e
)^2-2^(1/2)*cos(f*x+e))*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

3.233.5 Fricas [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

3.233.6 Sympy [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(e+fx\right)}\cos\left(e+fx\right)dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))*cos(e + f*x), x)
```

3.233.7 Maxima [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

3.233.8 Giac [F]

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)\,dx$$

input
```
integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos(e+f\,x)\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int(cos(e + f*x)*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)*(d*tan(e + f*x))^(1/2), x)
```

3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.234.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

output
```
-1/2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(co
s(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+1/3*c
os(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\sqrt{d\tan(e+fx)}\left(\sqrt{\sec^2(e+fx)}(\sin(e+fx)+\sin(3(e+fx)))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(e\right.\right.}{12f\sqrt{\sec^2(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Sqrt[d*Tan[e + f*x]]*(Sqrt[Sec[e + f*x]^2]*(Sin[e + f*x] + Sin[3*(e + f*x
)]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sec[e + f*x]*Tan
[e + f*x]))/(12*f*Sqrt[Sec[e + f*x]^2])
```

3.234.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^3}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

input
```
Int[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt
[Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.234.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.234.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(96) = 192.

Time = 1.08 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.81

method	result
default	$-\frac{\csc(fx+e)\left(2\left(\cos^4(fx+e)\right)\sqrt{2}+6\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{}\right.}{}$

input
```
int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/f*csc(f*x+e)*(2*cos(f*x+e)^4*2^(1/2)+6*(cot(f*x+e)-csc(f*x+e)+1)^(1/
2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2
),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*cos(f*x+e)-3*(cot(f*x+e)-c
sc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1
)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*cos(f*x+e)
+6*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE
((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^
(1/2)-3*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-co
t(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/
2*2^(1/2))+2^(1/2)*cos(f*x+e)^2-3*2^(1/2)*cos(f*x+e))*(d*tan(f*x+e))^(1/2)
*2^(1/2)
```

3.234.5 Fricas [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}\cos\left(fx+e\right)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.234.7 Maxima [F]

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^3\,dx$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

3.234.8 Giac [F(-2)]

Exception generated.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeDone
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos\left(e+f\,x\right)^3\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

input
```
int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2), x)
```

3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.235.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

output
```
-7/20*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(c
os(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+7/30
*cos(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f+1/5*cos(f*x+e)^5*(d*tan(f*x+e))^(3/
2)/d/f
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\Big(20\sin(2(e+fx)) + 3\sin(4(e+fx)) + 28\,\text{Hypergeometric2F1}\left(\tfrac{3}{4},\tfrac{3}{2},\tfrac{7}{4},-\tan\right.}{120f}$$

input
```
Integrate[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(20*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x
)] + 28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x
]^2]*Tan[e + f*x]))/(120*f)
```

3.235.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3092, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^5}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{7}{10}\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)^3}dx+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3092}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{d\tan(e+fx)}}{\sec(e+fx)}dx+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3095}$$

$$\frac{7}{10}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\int\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}\right)+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7}{10}\left(\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}\right)$$

input
```
Int[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (7*((Cos[e + f*x]*Ellipt
icE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[Sin[2*e + 2*f*x]])
+ (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)))/10
```

3.235. $\int\cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

3.235.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.235.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(122) = 244$.

Time = 1.38 (sec) , antiderivative size = 404, normalized size of antiderivative = 3.64

method	result
default	$-\frac{\csc(fx+e)\left(12\left(\cos^6(fx+e)\right)\sqrt{2}+2\left(\cos^4(fx+e)\right)\sqrt{2}+42\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,E\left(\sqrt{-\cot(fx+e)+\right.\right.}{}$

input
```
int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/f*csc(f*x+e)*(12*cos(f*x+e)^6*2^(1/2)+2*cos(f*x+e)^4*2^(1/2)+42*(co
t(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticE((-cot
(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*
cos(f*x+e)-21*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2
)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1
/2),1/2*2^(1/2))*cos(f*x+e)+42*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)
-csc(f*x+e))^(1/2)*EllipticE((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))
*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-21*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(
f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(
f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+7*2^(1/2)*cos(f*x+e)^2-21*2^(1/2)*
cos(f*x+e))*(d*tan(f*x+e))^(1/2)*2^(1/2)
```

3.235.5 Fricas [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

3.235.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**5*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.235.7 Maxima [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

3.235.8 Giac [F]

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(fx+e)}\cos(fx+e)^5\,dx$$

input
```
integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \int \cos(e+f\,x)^5\,\sqrt{d\tan(e+f\,x)}\,dx$$

input
```
int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2), x)
```

3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2(d\tan(a+bx))^{5/2}}{5bd} + \frac{4(d\tan(a+bx))^{9/2}}{9bd^3} + \frac{2(d\tan(a+bx))^{13/2}}{13bd^5}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d+4/9*(d*tan(b*x+a))^(9/2)/b/d^3+2/13*(d*tan(b*
x+a))^(13/2)/b/d^5
```

3.236.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\left(-32-8\sec^2(a+bx)-5\sec^4(a+bx)+45\sec^6(a+bx)\right)\sqrt{d\tan(a+bx)}}{585b}$$

input
```
Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*(-32 - 8*Sec[a + b*x]^2 - 5*Sec[a + b*x]^4 + 45*Sec[a + b*x]^6)*Sqrt[
d*Tan[a + b*x]])/(585*b)
```

3.236.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^6(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^{3/2}\left(\tan^2(a+bx)+1\right)^2 d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{11/2}}{d^4}+\frac{2(d\tan(a+bx))^{7/2}}{d^2}+(d\tan(a+bx))^{3/2}\right) d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{13/2}}{13d^5}+\frac{4(d\tan(a+bx))^{9/2}}{9d^3}+\frac{2(d\tan(a+bx))^{5/2}}{5d}}{b}$$

input
```
Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((2*(d*Tan[a + b*x])^(5/2))/(5*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*d^3) + (
2*(d*Tan[a + b*x])^(13/2))/(13*d^5))/b
```

3.236.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.236.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$	52
default	$\frac{\frac{2(d\tan(bx+a))^{\frac{13}{2}}}{13}+\frac{4d^2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^4(d\tan(bx+a))^{\frac{5}{2}}}{5}}{d^5b}$	52

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^5/b*(1/13*(d*tan(b*x+a))^(13/2)+2/9*d^2*(d*tan(b*x+a))^(9/2)+1/5*d^4*(
d*tan(b*x+a))^(5/2))
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(32\,d\cos\left(bx+a\right)^6+8\,d\cos\left(bx+a\right)^4+5\,d\cos\left(bx+a\right)^2-45\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/585*(32*d*cos(b*x + a)^6 + 8*d*cos(b*x + a)^4 + 5*d*cos(b*x + a)^2 - 45
*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x + a)^6)
```

3.236.6 Sympy [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^6\left(a+bx\right)dx$$

input
```
integrate(sec(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**6, x)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\left(d\tan\left(bx+a\right)\right)^{\frac{13}{2}}+130\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}d^2+117\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^4\right)}{585\,bd^5}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/585*(45*(d*tan(b*x + a))^(13/2) + 130*(d*tan(b*x + a))^(9/2)*d^2 + 117*(
d*tan(b*x + a))^(5/2)*d^4)/(b*d^5)
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(45\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^6+130\sqrt{d\tan(bx+a)}d^6\tan(bx+a)^4+117\sqrt{d\tan(b$$

$$585\,bd^5$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/585*(45*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^6 + 130*sqrt(d*tan(b*x + a
))*d^6*tan(b*x + a)^4 + 117*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^2)/(b*d^
5)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 392, normalized size of antiderivative = 5.85

$$\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{64\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b} - \frac{64\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{585\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)} - \frac{32\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{195\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^2} + \frac{1216\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3} - \frac{3488\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{117\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^4} + \frac{384\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^5} - \frac{128\,d\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^6}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^6,x)
```

output
```
(1216*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
)/(117*b*(exp(a*2i + b*x*2i) + 1)^3) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i -
 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b*(exp(a*2i + b*x*2i) + 1)) -
(32*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(195*b*(exp(a*2i + b*x*2i) + 1)^2) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b) - (3488*d*(-(d*(exp(a*2i + b*
x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(117*b*(exp(a*2i + b*x*2i
) + 1)^4) + (384*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
+ 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^5) - (128*d*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i
) + 1)^6)
```

3.236. $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2(d\tan(a+bx))^{5/2}}{5bd} + \frac{2(d\tan(a+bx))^{9/2}}{9bd^3}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d+2/9*(d*tan(b*x+a))^(9/2)/b/d^3
```

3.237.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d(-4-\sec^2(a+bx)+5\sec^4(a+bx))\sqrt{d\tan(a+bx)}}{45b}$$

input
```
Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*(-4 - Sec[a + b*x]^2 + 5*Sec[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(45*b)
```

3.237.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^4(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^{3/2}\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{7/2}}{d^2}+(d\tan(a+bx))^{3/2}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{9/2}}{9d^3}+\frac{2(d\tan(a+bx))^{5/2}}{5d}}{b}$$

input
```
Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((2*(d*Tan[a + b*x])^(5/2))/(5*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*d^3))/b
```

3.237.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.237.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$	37
default	$\frac{\frac{2(d\tan(bx+a))^{\frac{9}{2}}}{9}+\frac{2d^2(d\tan(bx+a))^{\frac{5}{2}}}{5}}{b\,d^3}$	37

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(1/9*(d*tan(b*x+a))^(9/2)+1/5*d^2*(d*tan(b*x+a))^(5/2))
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(4\,d\cos\left(bx+a\right)^4+d\cos\left(bx+a\right)^2-5\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45\,b\cos\left(bx+a\right)^4}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(4*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 5*d)*sqrt(d*sin(b*x + a)/co
s(b*x + a))/(b*cos(b*x + a)^4)
```

3.237.6 Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\sec^4(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**4, x)
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\,(d\tan(bx+a))^{\frac{9}{2}} + 9\,(d\tan(bx+a))^{\frac{5}{2}}\,d^2\right)}{45\,bd^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/45*(5*(d*tan(b*x + a))^(9/2) + 9*(d*tan(b*x + a))^(5/2)*d^2)/(b*d^3)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(5\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^4 + 9\sqrt{d\tan(bx+a)}d^4\tan(bx+a)^2\right)}{45\,bd^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/45*(5*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^4 + 9*sqrt(d*tan(b*x + a))*d
^4*tan(b*x + a)^2)/(b*d^3)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.13

$$\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{8\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b} - \frac{8\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} + \frac{56\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{15\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2} - \frac{64\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^3} + \frac{32\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^4}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^4,x)
```

output
```
(56*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/
(15*b*(exp(a*2i + b*x*2i) + 1)^2) - (8*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)) - (8*d*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b)
 - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
))/(9*b*(exp(a*2i + b*x*2i) + 1)^3) + (32*d*(-(d*(exp(a*2i + b*x*2i)*1i -
1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*(exp(a*2i + b*x*2i) + 1)^4)
```

3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.238.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d
```

3.238.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

input
```
Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

3.238.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(a+bx)^2(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int (d\tan(a+bx))^{3/2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

input
```
Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

3.238.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.238.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19
default	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(d*tan(b*x+a))^(5/2)/b/d
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5\,b\cos\left(bx+a\right)^2}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(d*cos(b*x + a)^2 - d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x +
 a)^2)
```

3.238.6 Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^2\left(a+bx\right)dx$$

input
```
integrate(sec(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**2, x)
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,(d\tan(bx+a))^{\frac{5}{2}}}{5\,bd}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/5*(d*tan(b*x + a))^(5/2)/(b*d)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,\sqrt{d\tan(bx+a)}d\tan(bx+a)^2}{5\,b}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/5*sqrt(d*tan(b*x + a))*d*tan(b*x + a)^2/b
```

3.238.9 Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.55

$$\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}\,(\cos(2\,a+2\,b\,x)-2\,\cos(4\,a+4\,b\,x)-\cos(6\,a+6\,b\,x)+2)}{5\,b\,(15\,\cos(2\,a+2\,b\,x)+6\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)+10)}$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^2,x)
```

output
```
(2*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
 - 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) + 2))/(5*b*(15*cos(2*a + 2*b*x) +
 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

3.239 $\int (d\tan(a+bx))^{3/2}\,dx$

3.239.1 Optimal result

Integrand size = 12, antiderivative size = 210

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} - \frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}b} + \frac{2d\sqrt{d\tan(a+bx)}}{b}$$

output
```
1/2*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/4*d^(3/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-1/
4*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^
(1/2)+2*d*(d*tan(b*x+a))^(1/2)/b
```

3.239.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(\right.}{2}\right.}{b\tan^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(d*Tan[a + b*x])^(3/2),x]
```

output
```
((ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt
[Tan[a + b*x]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x
]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]]/(2*Sqr
t[2]) + 2*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2
))
```

3.239.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow 3957$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{d^3\int\frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)d^2+d^2)}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow 266$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\int\frac{1}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow 755$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3\left(\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int\frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{b}$$

$$\Big\downarrow 1476$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-$$

$$\frac{2d^3\left(\frac{\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int\frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d}}{2d}\right.}{b}$$

$$\Big\downarrow 1082$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-$$

$$\frac{2d^3\left(\frac{\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)}{b}$$

$$\Big\downarrow 217$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-$$

$$\frac{2d^3\left(\frac{\int\frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}\right)}{b}$$

$$\Big\downarrow 1479$$

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-$$

$$\frac{2d^3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}}{2}\right.}{b}$$

3.239. $\int(d\tan(a+bx))^{3/2}\,dx$

$\downarrow$ 25

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arct}}{2d}\right.}{b}$$

$\downarrow$ 27

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\text{arct}}{2d}\right.}{b}$$

$\downarrow$ 1103

$$\frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{2d^3\left(\frac{\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+\right.}{2\sqrt{2}\sqrt{d}}}{2d}\right.}{b}$$

input
```
Int[(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/(2*d) + (-1/2*
Log[d - Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(Sqrt[2]*Sqrt[d
]) + Log[d + Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]
*Sqrt[d]))/(2*d)))/b + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

3.239. $\int (d\tan(a+bx))^{3/2}\,dx$

3.239.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.239.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{2d\Big(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\Big(\ln\Big(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\Big)+2\arctan\Big(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\Big)-2\arctan\Big(}{8}}{b}$
default	$\frac{2d\Big(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\Big(\ln\Big(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\Big)+2\arctan\Big(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\Big)-2\arctan\Big(}{8}}{b}$

input
```
int((d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*d*((d*tan(b*x+a))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b*x+a)+(d^2
)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)-(d^2)^(1/4
)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*
(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)
+1)))
```

3.239.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int (d\tan(a+bx))^{3/2}\,dx =$$

$$-\frac{\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\log\left(\sqrt{d\tan(bx+a)}d+\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\right)+i\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\log\left(\sqrt{d\tan(bx+a)}d+i\left(-\frac{d^6}{b^4}\right)^{\frac{1}{4}} b\right)-i\left(\cdots\right.}{}$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d + (-d^6/b^4)^(1/4)*b)
+ I*(-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d + I*(-d^6/b^4)^(1/4)*b)
- I*(-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d - I*(-d^6/b^4)^(1/4)*b)
- (-d^6/b^4)^(1/4)*b*log(sqrt(d*tan(b*x + a))*d - (-d^6/b^4)^(1/4)*b) - 4*
sqrt(d*tan(b*x + a))*d)/b
```

3.239.6 Sympy [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2), x)
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int (d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d^{\frac{5}{2}}\log\left(d\tan\right.}{}$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^2)/(b*d)
```

3.239.8 Giac [F]

$$\int (d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2), x)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.35

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{2\,d\,\sqrt{d\tan(a+b\,x)}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b} + \frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b}$$

input
```
int((d*tan(a + b*x))^(3/2),x)
```

output
```
(2*d*(d*tan(a + b*x))^(1/2))/b + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*t
an(a + b*x))^(1/2))/d^(1/2))*1i)/b + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)
*(d*tan(a + b*x))^(1/2))/d^(1/2))*1i)/b
```

3.239. $\int (d\tan(a+bx))^{3/2}\,dx$

3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.240.1 Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b}+\frac{d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}b} - \frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

output
```
-1/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/8*
d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/16*d^(3
/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+
1/16*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b
*2^(1/2)-1/2*d*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b
```

3.240.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.49

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\csc(a+bx)\left(\sin(a+bx)+\arcsin(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin(\right.\right.}{8b}$$

input
```
Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/8*(d*Csc[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

3.240.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3087, 252, 266, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(d\tan(a+bx))^{3/2}}{(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{1}{4}d^2\int \frac{1}{\sqrt{d\tan(a+bx)}(\tan^2(a+bx)+1)}d\tan(a+bx)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{1}{2}d\int \frac{1}{\tan^2(a+bx)+1}d\sqrt{d\tan(a+bx)}-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \; 755$$

$$\frac{\frac{1}{2}d\left(\frac{\int \frac{d^2(d-d\tan(a+bx))}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{2d}+\frac{\int \frac{d^2(\tan(a+bx)d+d)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{2d}\right)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\int \frac{\tan(a+bx)d+d}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)-\frac{d\sqrt{d\tan(a+bx)}}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{}{\tan(a+bx)\ldots}\right.\right.}{b}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\ldots\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{b}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}+\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)\right)-\frac{d\sqrt{d\tan(\ldots}}{2(\tan^2(a+\ldots}}{b}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan\left(\ldots\right.}{}\right.\right.}{b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\ldots}{}\right.}{\sqrt{2}\ldots}\right.\right.}{b}$$

3.240. $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}}{}\right.}{\sqrt{2}}\right.\right.}{b}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{1}{2}d\left(\frac{1}{2}d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}d\left(\frac{\log\left(d\tan(a+bx)+\sqrt{2}\sqrt{d}\sqrt{d\tan(a+bx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(\right.}{}\right.\right.}{b}$$

input
```
Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

output
```
((d*((d*(-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[
d])))/2 + (d*(-1/2*Log[d + d*Tan[a + b*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a +
 b*x]]]/(Sqrt[2]*Sqrt[d]) + Log[d + d*Tan[a + b*x] + Sqrt[2]*Sqrt[d]*Sqrt[
d*Tan[a + b*x]]]/(2*Sqrt[2]*Sqrt[d])))/2))/2 - (d*Sqrt[d*Tan[a + b*x]])/(2
*(1 + Tan[a + b*x]^2)))/b
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.240. $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.240.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(169) = 338$.

Time = 2.17 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.33

method	result
default	$-\frac{\cos(bx+a)\left(4\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}\,\sqrt{2}\,(\cos^2(bx+a))+4\cos(bx+a)\sqrt{2}\,\sqrt{-\frac{\cos(bx+a)\sin(bx+a)}{(\cos(bx+a)+1)^2}}-\ln\left(\frac{2\sin(bx+a)\sqrt{-\left(\cot^3(bx+a)\right)+3}}{}\right.\right.}{}$

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b*cos(b*x+a)*(4*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*2^(1
/2)*cos(b*x+a)^2+4*cos(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)-ln((2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*co
t(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)
*cos(b*x+a)+2*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x
+a)))+ln(-(cot(b*x+a)*cos(b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+
3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a
)-csc(b*x+a))^(1/2)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))
+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+2*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+
a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a))))*d*(d
*tan(b*x+a))^(1/2)/(cos(b*x+a)+1)/(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2
)^(1/2)*2^(1/2)
```

3.240.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 927, normalized size of antiderivative = 4.12

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(16*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 - (-d^6/b^4)^
(1/4)*b*log(-2*d^5*cos(b*x + a)^2 + 2*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*
sin(b*x + a) + d^5 + 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) +
 (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) +
 (-d^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a)^2 + 2*sqrt(-d^6/b^4)*b^2*d^2*c
os(b*x + a)*sin(b*x + a) + d^5 - 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*si
n(b*x + a) + (-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(
b*x + a))) + I*(-d^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a)^2 - 2*sqrt(-d^6/
b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) + d^5 - 2*(I*(-d^6/b^4)^(1/4)*b*d^3
*cos(b*x + a)*sin(b*x + a) - I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d
*sin(b*x + a)/cos(b*x + a))) - I*(-d^6/b^4)^(1/4)*b*log(-2*d^5*cos(b*x + a
)^2 - 2*sqrt(-d^6/b^4)*b^2*d^2*cos(b*x + a)*sin(b*x + a) + d^5 - 2*(-I*(-d
^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos
(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (-d^6/b^4)^(1/4)*b*log(-
d^5 + 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^6/b^4)^(3/
4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (-d^6/b^4)^(1/
4)*b*log(-d^5 - 2*((-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b*x + a) - (-d^
6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - I*(-
d^6/b^4)^(1/4)*b*log(-d^5 - 2*(I*(-d^6/b^4)^(1/4)*b*d^3*cos(b*x + a)*sin(b
*x + a) + I*(-d^6/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/co...
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\sqrt{2}d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}d$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x +
a))*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{1}{16}d\left(\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(b}\right.}{2\sqrt{|d|}}\right.}{b}\right.$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
1/16*d*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 2*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b
 + sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*
sqrt(abs(d)) + abs(d))/b - sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) - sqrt(
2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x + a))*
d^2/((d^2*tan(b*x + a)^2 + d^2)*b))
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)^2\,(d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.241.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{4d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}} - \frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}$$

output
```
4/77*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-4
/77*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b-2/77*d*sec(b*x+a)^3*(d*tan(b*x+a))
^(1/2)/b+2/11*d*sec(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.77 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx =$$
$$-\frac{d\sec^5(a+bx)\left(-23+6\cos(2(a+bx))+\cos(4(a+bx))+16\cos^6(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.\right.}{154b}$$

input
```
Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/154*(d*Sec[a + b*x]^5*(-23 + 6*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16
*Cos[a + b*x]^6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec
[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

3.241.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3091, 3042, 3093, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^5(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\int \frac{\sec^5(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\int \frac{\sec(a+bx)^5}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\int\frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\int\frac{\sec(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\downarrow$ 3093

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2}{3}\int\frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd}\right)$$

$\downarrow$ 3094

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+}}{7bd}\right.$$

$\downarrow$ 3042

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\tan(a+}}{7bd}\right.$$

$\downarrow$ 3053

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)+\frac{2\sec^3(a+bx)\sqrt{d\,\mathrm{ta}}}{7bd}\right.$$

$\downarrow$ 3042

3.241. $\int\sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\left(\frac{6}{7}\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}} + \frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right) + \frac{2\sec^3(a+bx)\sqrt{d\ta}}{7bd}\right.$$

$$\Big\downarrow \; 3120$$

$$\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\left(\frac{2\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7bd} + \frac{6}{7}\left(\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd} + \frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}}{3b\sqrt{d\tan(a+bx)}}\right.\right.$$

input
```
Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sec[a + b*x]^5*Sqrt[d*Tan[a + b*x]])/(11*b) - (d^2*((2*Sec[a + b*x]^3
*Sqrt[d*Tan[a + b*x]])/(7*b*d) + (6*((2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a
 + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Sec[a + b*
x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)))/7))/11
```

3.241.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

3.241. $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.241.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.95

method	result
default	$\frac{d\sqrt{d\tan(bx+a)}\left(4\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,F\left(\sqrt{1+\right.\right.}{}$

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/77/b*d*(d*tan(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*(4*sin(b*x+a)*cos(b*x+a)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1
+cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+
4*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2
)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2
),1/2*2^(1/2))+2*sin(b*x+a)*tan(b*x+a)*2^(1/2)+tan(b*x+a)^2*sec(b*x+a)*2^(
1/2)-7*tan(b*x+a)^2*sec(b*x+a)^3*2^(1/2))*2^(1/2)
```

3.241.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(2\sqrt{i\,d}\,d\cos\left(bx+a\right)^5 F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+2\sqrt{-i\,d}\,d\cos\left(bx+a\right)\right.}{77\,b}$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/77*(2*sqrt(I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) + 2*sqrt(-I*d)*d*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1) - (2*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 7*d)
*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^5)
```

3.241.6 Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^5\left(a+bx\right)dx$$

input
```
integrate(sec(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**5, x)
```

3.241.7 Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(bx+a\right)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

3.241.8 Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int (d\tan(bx+a))^{\frac{3}{2}}\sec(bx+a)^5\;dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)^5}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5, x)
```

3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.242.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

output
```
2/21*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-2
/21*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b+2/7*d*sec(b*x+a)^3*(d*tan(b*x+a))^
(1/2)/b
```

3.242.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\sec^3(a+bx)\left(-5+\cos(2(a+bx))+4\cos^4(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2}\right.}{21b}$$

input
```
Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/21*(d*Sec[a + b*x]^3*(-5 + Cos[2*(a + b*x)] + 4*Cos[a + b*x]^4*Hypergeo
metric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan
[a + b*x]])/b
```

3.242.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3091, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^3(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\int \frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\int \frac{\sec(a+bx)^3}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2}{3}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2}{3}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$$\Big\downarrow \text{3094}$$

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\downarrow$ 3042

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\downarrow$ 3053

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\downarrow$ 3042

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}+\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)$$

$\downarrow$ 3120

$$\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{1}{7}d^2\left(\frac{2\sec(a+bx)\sqrt{d\tan(a+bx)}}{3bd}+\frac{2\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b\sqrt{d\tan(a+bx)}}\right)$$

input
```
Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b) - (d^2*((2*EllipticF[a - P
i/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*Sqrt[d*Tan[a + b*x
]]) + (2*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)))/7
```

3.242. $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.242.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.242.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(119) = 238$.

Time = 1.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.26

method	result
default	$\frac{d\sqrt{d\tan(bx+a)}\left(2\sin(bx+a)\cos(bx+a)\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,F\left(\sqrt{1+\right.\right.}{}$

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/b*d*(d*tan(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*(2*sin(b*x+a)*cos(b*x+a)*(1
+csc(b*x+a)-cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1
+cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+
2*sin(b*x+a)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2
)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2
),1/2*2^(1/2))+sin(b*x+a)*tan(b*x+a)*2^(1/2)-3*tan(b*x+a)^2*sec(b*x+a)*2^(
1/2))*2^(1/2)
```

3.242.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2\left(\sqrt{i\,d}d\cos{(bx+a)}^3\,F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+\sqrt{-i\,d}d\cos{(bx+a)}^3\,F\cdots\right.}{21\,b\cos{(bx+a}\cdots}$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*d)*d*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - (d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(
b*x + a)))/(b*cos(b*x + a)^3)
```

3.242.6 Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sec^3{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**3, x)
```

3.242.7 Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

3.242.8 Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \frac{(d\tan(a+b\,x))^{3/2}}{\cos(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3, x)
```

3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.243.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

output
```
1/3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+2/
3*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

3.243.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\cos(a+bx)\left(\sec^2(a+bx)-\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)}{3b}$$

input
```
Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*d*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[
a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.243.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3091, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx$$

$$\Big\downarrow \text{3094}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 3120$$

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3b\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/3*(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]]
)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

3.243.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.243.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(95) = 190.

Time = 1.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.76

method	result
default	$-\frac{\left(-\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)(\cos^2}{}$

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(-(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*
(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/
2*2^(1/2))*cos(b*x+a)^2-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b
*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+sin(b*x+a)*2^(1/2))*tan(b*x+a)*d*(d*t
an(b*x+a))^(1/2)/(cos(b*x+a)^2-1)*2^(1/2)
```

3.243.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\sqrt{i\,d}d\cos(bx+a)\,F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,d}d\cos(bx+a)\,F(\arcs}{3\,b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x +
 a)), -1) + sqrt(-I*d)*d*cos(b*x + a)*elliptic_f(arcsin(cos(b*x + a) - I*s
in(b*x + a)), -1) + 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)
)
```

3.243.6 Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(a+bx)})^{\frac{3}{2}}\sec{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x), x)
```

3.243.7 Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

3.243.8 Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan{(bx+a)})^{\frac{3}{2}}\sec{(bx+a)}\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx=\int \frac{(d\tan(a+b\,x))^{3/2}}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^(3/2)/cos(a + b*x), x)
```

3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.244.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d^2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

output
```
-1/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-d
*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

3.244.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx=\frac{d\cos(a+bx)\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a}{b}$$

input
```
Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d*Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

3.244.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3090, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{2}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3094}$$

$$\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3053}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

input
```
Int[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*
b*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

3.244.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.244.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(95) = 190.

Time = 3.02 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.81

method	result
default	$\frac{\sin(bx+a)\left(-\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right.\right.}{}$

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*sin(b*x+a)*(-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a)
)^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a)
)^(1/2),1/2*2^(1/2))*cos(b*x+a)-(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)
+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sin(b*x+a)*2^(1/2)*cos(b*x+a))*(d*tan(b*
x+a))^(1/2)*d/(cos(b*x+a)^2-1)*2^(1/2)
```

3.244.5 Fricas [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)\;dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)*tan(b*x + a), x)
```

3.244.6 Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(a+bx))^{\frac{3}{2}}\cos(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(3/2)*cos(a + b*x), x)
```

3.244.7 Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a), x)
```

3.244.8 Giac [F(-2)]

Exception generated.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos(a+b\,x)\,(d\tan(a+b\,x))^{3/2}\,dx$$

input
```
int(cos(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.245.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b} - \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

output
```
-1/12*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/
4*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+
1/6*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b-1/3*d*cos(b*x+a)^3*(d*tan(b*x+a))^
(1/2)/b
```

3.245.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{\cos(a+bx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a+bx)}+\cos(2(a+bx))\sqrt{\tan(\cdots}}{6b\tan^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/6*(Cos[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a +
 b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + Cos[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(
d*Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2))
```

3.245.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3090, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{6}d^2\int \frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\int \frac{1}{\sec(a+bx)\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{6}d^2\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}d^2\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$$\Big\downarrow \text{3094}$$

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$\Big\downarrow$ 3053

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}d^2\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

$\Big\downarrow$ 3120

$$\frac{1}{6}d^2\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{d\tan(a+bx)}}\right)-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

input
```
Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/3*(d*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b + (d^2*((EllipticF[a - Pi/4
 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Tan[a + b*x]])
 + (Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(b*d)))/6
```

3.245. $\int\cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.245.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.245.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.34 (sec) , antiderivative size = 1939, normalized size of antiderivative = 17.95

method	result	size
default	Expression too large to display	1939

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*sin(b*x+a)*(6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(
b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b
*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-
csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1
+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+6*I*Ellipt
icPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(1+csc(b*x+a)-
cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*cos(b*x+a)-6*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+8*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)-6*(co
t(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-
cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2
*2^(1/2))*cos(b*x+a)-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*
x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b
*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+8*(cot(b*x+a)-csc(b*x+a))^(
1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*Elli
pticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-6*(cot(b*x+a
)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x
+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2
))-6*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+...
```

3.245.5 Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^3*tan(b*x + a), x)
```

3.245.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.245.7 Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^3, x)
```

3.245.8 Giac [F(-2)]

Exception generated.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos\left(a+b\,x\right)^3\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d^2 \operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}} + \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b} + \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b} - \frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

output
```
-1/24*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/
4*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+
1/12*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b+1/30*d*cos(b*x+a)^3*(d*tan(b*x+a)
)^(1/2)/b-1/5*d*cos(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

3.246.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\cos(2(a+bx))\csc(a+bx)\left(10\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right),-1\right)\sqrt{\sec^2(a-\cdots}\right.}{120b\sqrt{\tan(a+bx)}\,(-1\cdots}$$

input
```
Integrate[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Cos[2*(a + b*x)]*Csc[a + b*x]*(10*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/
4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + (-3 + 10*Cos[2*(a + b*x
)] + 3*Cos[4*(a + b*x)])*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(120*
b*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

3.246.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3090, 3042, 3092, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^{3/2}}{\sec(a+bx)^5}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{1}{10}d^2\int \frac{\cos^3(a+bx)}{\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{10}d^2\int \frac{1}{\sec(a+bx)^3\sqrt{d\tan(a+bx)}}dx-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3092

$$\frac{1}{10}d^2\left(\frac{5}{6}\int \frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}d^2\left(\frac{5}{6}\int \frac{1}{\sec(a+bx)\sqrt{d\tan(a+bx)}}dx+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3092

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3094

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3042

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}\right)-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\Big\downarrow$ 3053

3.246. $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\downarrow$ 3042

$$\frac{1}{10}d^2\left(\frac{5}{6}\left(\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{d\tan(a+bx)}}+\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}\right)+\frac{\cos^3(a+bx)\sqrt{d\tan(a\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

$\downarrow$ 3120

$$\frac{1}{10}d^2\left(\frac{\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3bd}+\frac{5}{6}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}}{bd}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}(a}{2b\sqrt{d\tan(a+bx)}}\right.\right.$$
$$\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}$$

input
```
Int[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

output
```
-1/5*(d*Cos[a + b*x]^5*Sqrt[d*Tan[a + b*x]])/b + (d^2*((Cos[a + b*x]^3*Sqr
t[d*Tan[a + b*x]])/(3*b*d) + (5*((EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x
]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Tan[a + b*x]]) + (Cos[a + b*x]*Sqrt[
d*Tan[a + b*x]])/(b*d)))/6))/10
```

3.246.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.246.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 1958, normalized size of antiderivative = 14.40

method	result	size
default	Expression too large to display	1958

input
```
int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240/b*sin(b*x+a)*(24*2^(1/2)*cos(b*x+a)^5*sin(b*x+a)+30*I*(1+csc(b*x+a)-
cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))
^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3
0*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(
b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*cos(b*x+a)-4*2^(1/2)*cos(b*x+a)^3*sin(b*x+a)+30*I*(1+csc(
b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(
b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(
1/2))*cos(b*x+a)-30*I*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b
*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+50*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+cs
c(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-30*(cot(b*x+a)-csc(b*x+
a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)
*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x
+a)-30*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+c
sc(b*x+a)-cot(b*x+a))^(1/2)*EllipticPi((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*cos(b*x+a)+50*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-30*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-...
```

3.246.5 Fricas [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^5\,dx$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^5*tan(b*x + a), x)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int (d\tan(bx+a))^{\frac{3}{2}}\cos(bx+a)^5\,dx$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^5, x)
```

3.246.8 Giac [F(-2)]

Exception generated.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:The choice was done assuming 0=[0,0
]ext_reduce Error: Bad Argument TypeThe choice was done assuming 0=[0,0]ex
t_reduce
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \int \cos\left(a+b\,x\right)^5\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2), x)
```

3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.247.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f+4/11*(d*tan(f*x+e))^(11/2)/d^3/f+2/15*(d*tan(
f*x+e))^(15/2)/d^5/f
```

3.247.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2(117+44\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{7/2}}{1155df}$$

input
```
Integrate[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(117 + 44*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[
e + f*x])^(7/2))/(1155*d*f)
```

3.247.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^6(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(e+fx))^{5/2}\left(\tan^2(e+fx)+1\right)^2d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{13/2}}{d^4}+\frac{2(d\tan(e+fx))^{9/2}}{d^2}+(d\tan(e+fx))^{5/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{15/2}}{15d^5}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3}+\frac{2(d\tan(e+fx))^{7/2}}{7d}}{f}$$

input
```
Int[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((2*(d*Tan[e + f*x])^(7/2))/(7*d) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3) +
 (2*(d*Tan[e + f*x])^(15/2))/(15*d^5))/f
```

3.247.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.247.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\frac{\frac{2(d\tan(fx+e))^{\frac{15}{2}}}{15}+\frac{4d^2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^4(d\tan(fx+e))^{\frac{7}{2}}}{7}}{d^5 f}$$

input
```
int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x)
```

output
```
2/d^5/f*(1/15*(d*tan(f*x+e))^(15/2)+2/11*d^2*(d*tan(f*x+e))^(11/2)+1/7*d^4
*(d*tan(f*x+e))^(7/2))
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{2\left(32\,d^2\cos\left(fx+e\right)^6+24\,d^2\cos\left(fx+e\right)^4+21\,d^2\cos\left(fx+e\right)^2-77\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{1155\,f\cos\left(fx+e\right)^7}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(32*d^2*cos(f*x + e)^6 + 24*d^2*cos(f*x + e)^4 + 21*d^2*cos(f*x +
e)^2 - 77*d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x +
 e)^7)
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\,(d\tan\left(fx+e\right))^{\frac{15}{2}}+210\,(d\tan\left(fx+e\right))^{\frac{11}{2}}\,d^2+165\,(d\tan\left(fx+e\right))^{\frac{7}{2}}\,d^4\right)}{1155\,d^5 f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/1155*(77*(d*tan(f*x + e))^(15/2) + 210*(d*tan(f*x + e))^(11/2)*d^2 + 165
*(d*tan(f*x + e))^(7/2)*d^4)/(d^5*f)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(77\sqrt{d\tan(fx+e)}d^7\tan(fx+e)^7 + 210\sqrt{d\tan(fx+e)}d^7\tan(fx+e)^5 + 165\sqrt{d\tan(\right.}{1155\,d^5 f}$$

input
```
integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/1155*(77*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^7 + 210*sqrt(d*tan(f*x +
e))*d^7*tan(f*x + e)^5 + 165*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^3)/(d^5
*f)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 474, normalized size of antiderivative = 7.07

$$\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,32i}{385\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,2432i}{231\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,1504i}{33\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,4288i}{55\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,896i}{15\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^6} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,256i}{15\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^7}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^6,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64
i)/(1155*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i)
+ 1))^(1/2)*64i)/(1155*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i +
 f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(385*f*(exp(e*2i +
 f*x*2i) + 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x
*2i) + 1))^(1/2)*2432i)/(231*f*(exp(e*2i + f*x*2i) + 1)^3) + (d^2*(-(d*(ex
p(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*1504i)/(33*f*(e
xp(e*2i + f*x*2i) + 1)^4) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e
*2i + f*x*2i) + 1))^(1/2)*4288i)/(55*f*(exp(e*2i + f*x*2i) + 1)^5) + (d^2*
(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*896i)/(
15*f*(exp(e*2i + f*x*2i) + 1)^6) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))
/(exp(e*2i + f*x*2i) + 1))^(1/2)*256i)/(15*f*(exp(e*2i + f*x*2i) + 1)^7)
```

3.247. $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.248.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df} + \frac{2(d\tan(e+fx))^{11/2}}{11d^3f}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f+2/11*(d*tan(f*x+e))^(11/2)/d^3/f
```

3.248.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(9+2\cos(2(e+fx)))\sec^2(e+fx)(d\tan(e+fx))^{7/2}}{77df}$$

input
```
Integrate[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(9 + 2*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(d*Tan[e + f*x])^(7/2))/(77*d*f
)
```

3.248.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^4(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(e+fx))^{5/2}\left(\tan^2(e+fx)+1\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(e+fx))^{9/2}}{d^2}+(d\tan(e+fx))^{5/2}\right)d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(e+fx))^{11/2}}{11d^3}+\frac{2(d\tan(e+fx))^{7/2}}{7d}}{f}$$

input
```
Int[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((2*(d*Tan[e + f*x])^(7/2))/(7*d) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3))/
f
```

3.248.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.248.4 Maple [A] (verified)

Time = 236.85 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$	37
default	$\frac{\frac{2(d\tan(fx+e))^{\frac{11}{2}}}{11}+\frac{2d^2(d\tan(fx+e))^{\frac{7}{2}}}{7}}{f\,d^3}$	37

input
```
int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/d^3*(1/11*(d*tan(f*x+e))^(11/2)+1/7*d^2*(d*tan(f*x+e))^(7/2))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(4\,d^2\cos\left(fx+e\right)^4+3\,d^2\cos\left(fx+e\right)^2-7\,d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{77\,f\cos\left(fx+e\right)^5}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/77*(4*d^2*cos(f*x + e)^4 + 3*d^2*cos(f*x + e)^2 - 7*d^2)*sqrt(d*sin(f*x
 + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+11\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2\right)}{77\,d^3 f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/77*(7*(d*tan(f*x + e))^(11/2) + 11*(d*tan(f*x + e))^(7/2)*d^2)/(d^3*f)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2\left(7\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^5+11\sqrt{d\tan(fx+e)}d^5\tan(fx+e)^3\right)}{77\,d^3f}$$

input
```
integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/77*(7*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 11*sqrt(d*tan(f*x + e))*
d^5*tan(f*x + e)^3)/(d^3*f)
```

3.248.9 Mupad [B] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 352, normalized size of antiderivative = 7.82

$$\begin{aligned}\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx &= \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f} \\ &+ \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,8i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,296i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,944i}{77\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3} \\ &- \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,160i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^4} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{11\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^5}\end{aligned}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^4,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i
)/(77*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1
))^(1/2)*8i)/(77*f*(exp(e*2i + f*x*2i) + 1)) - (d^2*(-(d*(exp(e*2i + f*x*2
i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*296i)/(77*f*(exp(e*2i + f*x*2
i) + 1)^2) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) +
 1))^(1/2)*944i)/(77*f*(exp(e*2i + f*x*2i) + 1)^3) - (d^2*(-(d*(exp(e*2i +
 f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i +
 f*x*2i) + 1)^4) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x
*2i) + 1))^(1/2)*64i)/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.249.1 Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f
```

3.249.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{2(d\tan(e+fx))^{7/2}}{7df}$$

input
```
Integrate[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

3.249.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(e+fx)^2(d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int (d\tan(e+fx))^{5/2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

input
```
Int[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

3.249.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.249.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19

input
```
int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(d*tan(f*x+e))^(7/2)/d/f
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{2\left(d^2\cos\left(fx+e\right)^2-d^2\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin\left(fx+e\right)}{7\,f\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*(d^2*cos(f*x + e)^2 - d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x
+ e)/(f*cos(f*x + e)^3)
```

3.249.6 Sympy [F]

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\int (d\tan{(e+fx)})^{\frac{5}{2}}\sec^2{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

output
```
Integral((d*tan(e + f*x))**(5/2)*sec(e + f*x)**2, x)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2\,(d\tan{(fx+e)})^{\frac{7}{2}}}{7\,df}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/7*(d*tan(f*x + e))^(7/2)/(d*f)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx=\frac{2\,\sqrt{d\tan{(fx+e)}}d^2\tan{(fx+e)}^3}{7\,f}$$

input
```
integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^3/f
```

3.249.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.45

$$\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,2i}{7\,f} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,12i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)} + \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,24i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2} - \frac{d^2\sqrt{-\frac{d\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,16i}{7\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$

input
```
int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^2,x)
```

output
```
(d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*2i
)/(7*f) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*12i)/(7*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*x*2i
)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7*f*(exp(e*2i + f*x*2i)
+ 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1)
)^(1/2)*16i)/(7*f*(exp(e*2i + f*x*2i) + 1)^3)
```

3.250 $\int (d\tan(e+fx))^{5/2}\,dx$

3.250.1 Optimal result

Integrand size = 12, antiderivative size = 212

$$\int (d\tan(e+fx))^{5/2}\,dx=\frac{d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}+\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f}+\frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

output
```
1/2*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(5/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)+1/
4*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^
(1/2)+2/3*d*(d*tan(f*x+e))^(3/2)/f
```

3.250.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{d(d\tan(e+fx))^{3/2}\left(-3\arctan\left(\sqrt[4]{-\tan^2(e+fx)}\right)\sqrt[4]{-\tan(e+fx)}+3\text{arctanh}\left(\sqrt[4]{-\tan^2\cdots}\right.\right.}{3f\tan^{\frac{7}{4}}(e+fx)}$$

input
```
Integrate[(d*Tan[e + f*x])^(5/2),x]
```

output
```
(d*(d*Tan[e + f*x])^(3/2)*(-3*ArcTan[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*
x])^(1/4) + 3*ArcTanh[(-Tan[e + f*x]^2)^(1/4)]*(-Tan[e + f*x])^(1/4) + 2*T
an[e + f*x]^(7/4)))/(3*f*Tan[e + f*x]^(7/4))
```

3.250.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-d^2\int \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-d^2\int \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{d^3\int\frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)d^2+d^2}d(d\tan(e+fx))}{f}$$

$\downarrow$ 266

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\int\frac{d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}}{f}$$

$\downarrow$ 826

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\int\frac{d^2\tan^2(e+fx)+d}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\downarrow$ 1476

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int\frac{1}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}\right.\right.}{f}$$

$\downarrow$ 1082

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\int\frac{1}{-d^2\tan^2(e+fx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int\frac{1}{-d^2\tan^2(e+fx)-1}d\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan\right.}{f}$$

$\downarrow$ 217

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int\frac{d-d^2\tan^2(e+fx)}{d^4\tan^4(e+fx)+d^2}d\sqrt{d\tan(e+fx)}\right)}{f}$$

$\downarrow$ 1479

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{\right.}{}\right.\right.}{f}$$

$\downarrow$ 25

3.250. $\int(d\tan(e+fx))^{5/2}\,dx$

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{f}$$

↓ 27

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)-\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{d^2\tan^2(e+fx)+\sqrt{2}d^{3/2}\tan(e+fx)+d}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.}{f}$$

↓ 1103

$$\frac{2d(d\tan(e+fx))^{3/2}}{3f}-\frac{2d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(e+fx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(e+fx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(e+fx)+d^2\tan^2(e+fx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d\right.}{}\right.}{f}$$

input
```
Int[(d*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^3*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d])) +
ArcTan[1 + Sqrt[2]*Sqrt[d]*Tan[e + f*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - S
qrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Lo
g[d + Sqrt[2]*d^(3/2)*Tan[e + f*x] + d^2*Tan[e + f*x]^2]/(2*Sqrt[2]*Sqrt[d
]))/2))/f + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```

3.250.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.250. $\int (d\tan(e+fx))^{5/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.250.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$
default	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\frac{\sqrt{}}{}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$

input
```
int((d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*d*(1/3*(d*tan(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x
+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d
^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e
))^(1/2)+1)))
```

3.250.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{4\sqrt{d\tan(fx+e)}d^2\tan(fx+e) - 3\left(-\frac{d^{10}}{f^4}\right)^{\frac{1}{4}} f\log\left(\sqrt{d\tan(fx+e)}d^7 + \left(-\frac{d^{10}}{f^4}\right)^{\frac{3}{4}} f^3\right) + 3}{}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e) - 3*(-d^10/f^4)^(1/4)*f*log(s
qrt(d*tan(f*x + e))*d^7 + (-d^10/f^4)^(3/4)*f^3) + 3*I*(-d^10/f^4)^(1/4)*f
*log(sqrt(d*tan(f*x + e))*d^7 + I*(-d^10/f^4)^(3/4)*f^3) - 3*I*(-d^10/f^4)
^(1/4)*f*log(sqrt(d*tan(f*x + e))*d^7 - I*(-d^10/f^4)^(3/4)*f^3) + 3*(-d^1
0/f^4)^(1/4)*f*log(sqrt(d*tan(f*x + e))*d^7 - (-d^10/f^4)^(3/4)*f^3))/f
```

3.250.6 Sympy [F]

$$\int (d\tan(e+fx))^{5/2}\,dx = \int (d\tan(e+fx))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(5/2),x)
```

output
```
Integral((d*tan(e + f*x))**(5/2), x)
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int (d\tan(e+fx))^{5/2}\,dx = -\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\right.}{\sqrt{d}}\right.}{12\,df}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-1/12*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e
) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f
*x + e))^(3/2)*d^2)/(d*f)
```

3.250.8 Giac [F(-1)]

Timed out.

$$\int (d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.250.9 Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{2\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f} - \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f} + \frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

input
```
int((d*tan(e + f*x))^(5/2),x)
```

output
```
(2*d*(d*tan(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(5/2)*atan(((-1)^(1/4)*
(d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*d^(5/2)*atanh(((-1)^(1/4
)*(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.251.1 Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f}-\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}$$

output
```
-3/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/8*
d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/16*d^(5
/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)-
3/16*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f
*2^(1/2)-1/2*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f
```

3.251.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.48

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{d^2\Big(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+3\csc(e+fx)\log\Big(\cos(e+fx)+\sin(e+fx)+\sqrt{\mathrm{si}}}{8f}$$

input
```
Integrate[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
-1/8*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + 3*Csc[e +
f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] + 2*Sqrt[Si
n[2*(e + f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

3.251.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {3042, 3087, 252, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\tan(e+fx))^{5/2}}{\sec(e+fx)^2}dx$$

$$\Big\downarrow \; 3087$$

$$\frac{\int \frac{(d\tan(e+fx))^{5/2}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)}{f}$$

$$\Big\downarrow \; 252$$

$$\frac{\frac{3}{4}d^2\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{3}{2}d\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 27

$$\frac{\frac{3}{2}d^3\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 826

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{2(\tan^2(e+fx)+1)}}{f}$$

$\Big\downarrow$ 1476

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}\right.\right.}{f}$$

$\Big\downarrow$ 1082

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+\right.}{f}$$

$\Big\downarrow$ 217

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)-\frac{d(d\tan(e+f}{2(\tan^2(e+f}}{f}$$

$\Big\downarrow$ 1479

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}}{}\right.}{}\right.\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{}}\right.\right.}{f}$$

3.251. $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

$\big\downarrow$ 27

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{}\right.}{\sqrt{}}\right.\right.}{f}$$

$\big\downarrow$ 1103

$$\frac{\frac{3}{2}d^3\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(e\right.}{}\right.\right.}{f}$$

input
```
Int[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

output
```
((3*d^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sq
rt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt
[d]))/2 + (Log[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/
(2*Sqrt[2]*Sqrt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[
e + f*x]]]/(2*Sqrt[2]*Sqrt[d]))/2))/2 - (d*(d*Tan[e + f*x])^(3/2))/(2*(1 +
 Tan[e + f*x]^2)))/f
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.251. $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.251.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(169) = 338$.

Time = 2.48 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.45

method	result
default	$\frac{(\sin^2(fx+e))\cos(fx+e)\left(4\sqrt{2}\cos(fx+e)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+4\sqrt{2}\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)-3\ln\left(-\frac{\cot(f\ldots}{}\right.\right.}{}$

input
```
int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/f*sin(f*x+e)^2*cos(f*x+e)*(4*2^(1/2)*cos(f*x+e)*(-sin(f*x+e)*cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+4*2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)
-2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(
f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+c
sc(f*x+e)+2)/(cos(f*x+e)-1))+3*ln(-(cot(f*x+e)*cos(f*x+e)-2*cot(f*x+e)+2*s
in(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+
e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*cos(f*x+e)-sin(f*x+e)+csc(f
*x+e)+2)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*arctan((2^(1/2
)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)
/(cos(f*x+e)-1)))*(d*tan(f*x+e))^(1/2)*d^2/(cos(f*x+e)-1)/(cos(f*x+e)+1)^2
/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/2)
```

3.251.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.31

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(16*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)
- 3*I*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) + 27/4*(2
*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*(I*(-d^10/f^4)^(
1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) + I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e
)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*(-d^10/f^4)^(1/4)*f*log(27/2
*d^8*cos(f*x + e)*sin(f*x + e) + 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)
*sqrt(-d^10/f^4) - 27/2*(-I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x +
 e) - I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x
+ e))) + 3*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) - 27
/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) + 27/2*((-d^10/f^4
)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) - (-d^10/f^4)^(3/4)*f^3*cos(f*x +
e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*(-d^10/f^4)^(1/4)*f*log(27/2*
d^8*cos(f*x + e)*sin(f*x + e) - 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*
sqrt(-d^10/f^4) - 27/2*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e)
- (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e)))
 + 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 + 54*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x
+ e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x +
 e)/cos(f*x + e))) - 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 - 54*((-d^10/f^4)^(1
/4)*d^5*f*cos(f*x + e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*sin(f*x + e)
)*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*I*(-d^10/f^4)^(1/4)*f*log(27*d...
```

3.251.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.86

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{\right.}}{\sqrt{}}\right.}{16\,df}$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e)
 + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f
*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*
x + e))^(3/2)*d^4/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.03

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$

$$-\frac{1}{16}\left(\frac{8\sqrt{d\tan(fx+e)}d^2\tan(fx+e)}{\left(d^2\tan(fx+e)^2+d^2\right)f}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\right.}{}\right.$$

input
```
integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/16*(8*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)
*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))
/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*
x + e))*sqrt(abs(d)) + abs(d))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*
x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f))*d^2
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \int \cos\left(e+f\,x\right)^2\,\left(d\tan(e+f\,x)\right)^{5/2}\,dx$$

input
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2),x)
```

output
```
int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2), x)
```

3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.252.1 Optimal result

Integrand size = 21, antiderivative size = 253

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = -\frac{3d^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f}+\frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f}-\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}$$

output
```
-3/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/6
4*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/128*d
^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/
2)-3/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e
))/f*2^(1/2)+3/16*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f-1/4*d*cos(f*x+e)^4
*(d*tan(f*x+e))^(3/2)/f
```

3.252.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.49

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx =$$
$$-\frac{d^2\Big(3\arcsin(\cos(e+fx)-\sin(e+fx))\csc(e+fx)\sqrt{\sin(2(e+fx))}+3\csc(e+fx)\log\Big(\cos(e+fx)+\text{s}}{64}$$

input
```
Integrate[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
-1/64*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x]*Sqrt[Sin[2*
(e + f*x)]] + 3*Csc[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*
(e + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - 2*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*
x)])*Sqrt[d*Tan[e + f*x]])/f
```

3.252.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3087, 252, 253, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\tan(e+fx))^{5/2}}{\sec(e+fx)^4}dx$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int \frac{(d\tan(e+fx))^{5/2}}{(\tan^2(e+fx)+1)^3}d\tan(e+fx)}{f}$$
$$\Big\downarrow \text{252}$$
$$\frac{\frac{3}{8}d^2\int \frac{\sqrt{d\tan(e+fx)}}{(\tan^2(e+fx)+1)^2}d\tan(e+fx)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow 253$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{4}\int \frac{\sqrt{d\tan(e+fx)}}{\tan^2(e+fx)+1}d\tan(e+fx)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow 266$$

$$\frac{\frac{3}{8}d^2\left(\frac{\int \frac{d^3\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}}{2d}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow 27$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\int \frac{d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan(e+fx))^{3/2}}{4(\tan^2(e+fx)+1)^2}}{f}$$

$$\Big\downarrow 826$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\int \frac{\tan(e+fx)d+d}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan(e+fx))^{3/2}}{2d(\tan^2(e+fx)+1)}\right)-\frac{d(d\tan}{4(\tan^2}}{f}$$

$$\Big\downarrow 1476$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}+\frac{1}{2}\int \frac{1}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+\right.\right.\right.}{f}$$

$$\Big\downarrow 1082$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan\right.\right.}{f}$$

$$\Big\downarrow 217$$

$$\frac{\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(e+fx)}{\tan^2(e+fx)d^2+d^2}d\sqrt{d\tan(e+fx)}\right)+\frac{(d\tan}{2d(\tan}\right.}{f}$$

$$\Big\downarrow 1479$$

3.252. $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

$$\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.$$
$$f$$

$\downarrow$ 25

$$\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan}{}\right.\right.\right.$$
$$f$$

$\downarrow$ 27

$$\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d-\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(e+fx)}}{\tan(e+fx)d+d+\sqrt{2}\sqrt{d\tan(e+fx)}\sqrt{d}}d\sqrt{d\tan(e+fx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan(}{}\right.\right.\right.$$
$$f$$

$\downarrow$ 1103

$$\frac{3}{8}d^2\left(\frac{1}{2}d\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(e+fx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(e+fx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\right.}{}\right.\right.\right.$$
$$f$$

input
```
Int[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

output
```
(-1/4*(d*(d*Tan[e + f*x])^(3/2))/(1 + Tan[e + f*x]^2)^2 + (3*d^2*((d*((-(A
rcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])) + Arc
Tan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]))/2 + (Lo
g[d + d*Tan[e + f*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2*Sqrt[2]*Sq
rt[d]) - Log[d + d*Tan[e + f*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[e + f*x]]]/(2
*Sqrt[2]*Sqrt[d]))/2))/2 + (d*Tan[e + f*x])^(3/2)/(2*d*(1 + Tan[e + f*x]^2
))))/8)/f
```

3.252. $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.252.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.252. $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

3.252.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(193) = 386$.

Time = 59.81 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.53

method	result
default	$\frac{\cos(fx+e)\left(\sin^2(fx+e)\right)\left(16\sqrt{2}\left(\cos^3(fx+e)\right)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+16\sqrt{2}\left(\cos^2(fx+e)\right)\sqrt{-\frac{\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(f\right.}{}$

input
```
int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/f*cos(f*x+e)*sin(f*x+e)^2*(16*2^(1/2)*cos(f*x+e)^3*(-sin(f*x+e)*cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+16*2^(1/2)*cos(f*x+e)^2*(-sin(f*
x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-12*2^(1/2)*cos(f*x+e)*(
-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-12*2^(1/2)*(-sin
(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+3*ln(-(cot(f*x+e)*co
s(f*x+e)-2*cot(f*x+e)+2*sin(f*x+e)*(-cot(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e
)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3+cot(f*x+e)-csc(f*x+e))^(1/2)-2*co
s(f*x+e)-sin(f*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*ln((2*sin(f*x+e)*(-cot
(f*x+e)^3+3*cot(f*x+e)^2*csc(f*x+e)-3*csc(f*x+e)^2*cot(f*x+e)+csc(f*x+e)^3
+cot(f*x+e)-csc(f*x+e))^(1/2)-cot(f*x+e)*cos(f*x+e)+sin(f*x+e)+2*cos(f*x+e
)+2*cot(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))-6*arctan((2^(1/2)*(-sin(f*x+e
)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-
1))-6*arctan((2^(1/2)*(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)+cos(f*x+e)-1)/(cos(f*x+e)-1)))*(d*tan(f*x+e))^(1/2)*d^2/(cos(f*x+e)
-1)/(cos(f*x+e)+1)^2/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*2^(1/
2)
```

3.252.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 985, normalized size of antiderivative = 3.89

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/256*(16*(4*d^2*cos(f*x + e)^3 - 3*d^2*cos(f*x + e))*sqrt(d*sin(f*x + e)
/cos(f*x + e))*sin(f*x + e) - 3*I*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x
 + e)*sin(f*x + e) + 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/
f^4) - 27/2*(I*(-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) + I*(-d^1
0/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*I*
(-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) + 27/4*(2*d^3*f
^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*(-I*(-d^10/f^4)^(1/4)*
d^5*f*cos(f*x + e)*sin(f*x + e) - I*(-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*
sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*(-d^10/f^4)^(1/4)*f*log(27/2*d^8*co
s(f*x + e)*sin(f*x + e) - 27/4*(2*d^3*f^2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-
d^10/f^4) + 27/2*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) - (-d^
10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*(
-d^10/f^4)^(1/4)*f*log(27/2*d^8*cos(f*x + e)*sin(f*x + e) - 27/4*(2*d^3*f^
2*cos(f*x + e)^2 - d^3*f^2)*sqrt(-d^10/f^4) - 27/2*((-d^10/f^4)^(1/4)*d^5*
f*cos(f*x + e)*sin(f*x + e) - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d
*sin(f*x + e)/cos(f*x + e))) + 3*(-d^10/f^4)^(1/4)*f*log(27*d^8 + 54*((-d^
10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 - (-d^10/f^4)^(3/4)*f^3*cos(f*x + e)*si
n(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*(-d^10/f^4)^(1/4)*f*log
(27*d^8 - 54*((-d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)^2 - (-d^10/f^4)^(3/4)*f
^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*I*...
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.89

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{\right.}}{\sqrt{}}\right.}{128}$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(d) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e
) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(
f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*ta
n(f*x + e))^(7/2)*d^4 - (d*tan(f*x + e))^(3/2)*d^6)/(d^4*tan(f*x + e)^4 +
2*d^4*tan(f*x + e)^2 + d^4))/(d*f)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{1}{128}\,d^2\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(\right.}}{2\sqrt{|d|}}\right.}{df}\right.$$

input
```
integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/128*d^2*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
 + 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt(2)*abs(d)^(3/2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs
(d)))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*t
an(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*t
an(f*x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) +
8*(3*sqrt(d*tan(f*x + e))*d^4*tan(f*x + e)^3 - sqrt(d*tan(f*x + e))*d^4*ta
n(f*x + e))/((d^2*tan(f*x + e)^2 + d^2)^2*f))
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \int \cos\left(e+f\,x\right)^4\,{\left(d\tan(e+f\,x)\right)}^{5/2}\,dx$$

input
```
int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2),x)
```

output
```
int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2), x)
```

3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.253.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}} + \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df} + \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

output
```
-4/7*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+
f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+4/7*s
ec(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+2/7*sec(f*x+e)^3*(d*tan(f*x+e))^(1/2)/d
/f
```

3.253.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.68 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\Big((2+\cos(2(e+fx)))\sec^4(e+fx)+4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\Big)\,\text{si}}{7f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*((2 + Cos[2*(e + f*x)])*Sec[e + f*x]^4 + 4*Hypergeometric2F1[1/4, 1/2,
5/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])*Sin[e + f*x])/(7*f*Sqrt[d*Tan[
e + f*x]])
```

3.253.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3093, 3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^5}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{6}{7}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sec(e+fx)^3}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3093}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$\Big\downarrow$ 3053

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}\right)+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

$\Big\downarrow$ 3120

$$\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{6}{7}\left(\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{3f\sqrt{d\tan(e+fx)}}\right)$$

input
```
Int[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (6*((2*EllipticF[e - Pi/
4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*Sqrt[d*Tan[e + f*x]]
) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)))/7
```

3.253. $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.253.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.01

method	result
default	$\frac{\left(4\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+4\,\mathrm{se}\right.}{}$

input
```
int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/f/(d*tan(f*x+e))^(1/2)*(4*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-
csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+
csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+4*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(
1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*Ellipti
cF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*tan(f*x+e)*sec(f*x+e)*2
^(1/2)+tan(f*x+e)*sec(f*x+e)^3*2^(1/2))*2^(1/2)
```

3.253.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\left(2\sqrt{i\,d}\cos\left(fx+e\right)^3F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+2\sqrt{-i\,d}\cos\left(fx+e\right)^3F(\arcsin}{7\,df\,\cos\left(fx+e\right)^3}$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(2*sqrt(I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*
x + e)), -1) + 2*sqrt(-I*d)*cos(f*x + e)^3*elliptic_f(arcsin(cos(f*x + e)
- I*sin(f*x + e)), -1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*sin(f*x + e)/cos(f*
x + e)))/(d*f*cos(f*x + e)^3)
```

3.253.6 Sympy [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec^5\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)**5/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**5/sqrt(d*tan(e + f*x)), x)
```

3.253.7 Maxima [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec(fx+e)^5}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

3.253.8 Giac [F]

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec(fx+e)^5}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^5\,\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)), x)
```

3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.254.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}} + \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

output
```
-2/3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+
f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+2/3*s
ec(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

3.254.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\left(\sec^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{3f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Sec[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[e + f*x]^2]*S
qrt[Sec[e + f*x]^2])*Sin[e + f*x])/(3*f*Sqrt[d*Tan[e + f*x]])
```

3.254.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3093, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^3}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)\operatorname{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{3f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*
Sqrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

3.254.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.254.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(94) = 188$.

Time = 1.44 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.53

method	result
default	$\frac{\left(2\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+2\,\mathrm{se}\right.}{}$

input
```
int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(d*tan(f*x+e))^(1/2)*(2*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-
csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+
csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(
1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*Ellipti
cF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+tan(f*x+e)*sec(f*x+e)*2^(
1/2))*2^(1/2)
```

3.254.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\left(\sqrt{i\,d}\cos(fx+e)\,F(\arcsin(\cos(fx+e)+i\,\sin(fx+e))\,|-1)+\sqrt{-i\,d}\cos(fx+e)\,F(\arcsin(\cos(\right.}{3\,df\cos(fx+e)}$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x +
e)), -1) + sqrt(-I*d)*cos(f*x + e)*elliptic_f(arcsin(cos(f*x + e) - I*sin(
f*x + e)), -1) - sqrt(d*sin(f*x + e)/cos(f*x + e)))/(d*f*cos(f*x + e))
```

3.254.6 Sympy [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec^3{(e+fx)}}{\sqrt{d\tan{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)**3/sqrt(d*tan(e + f*x)), x)
```

3.254.7 Maxima [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^3}{\sqrt{d\tan{(fx+e)}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.254.8 Giac [F]

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^3}{\sqrt{d\tan{(fx+e)}}}\,dx$$

input
```
integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^3\,\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.255.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}$$

output
```
-(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x)
,2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)
```

3.255.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^3(e+fx)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}\left(1+\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sec
[e + f*x]^3*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x]^
2)^(3/2))
```

3.255.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3094}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt
[d*Tan[e + f*x]])
```

3.255. $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx$

3.255.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.255.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.19

method	result
default	$\frac{F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,(1+\sec(}{f\sqrt{d\tan(fx+e)}}$

input
```
int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))*(cot(f*x+e)-cs
c(f*x+e))^(1/2)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)
^(1/2)/(d*tan(f*x+e))^(1/2)*(1+sec(f*x+e))*2^(1/2)
```

3.255.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{\sqrt{i\,d}F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)|-1)+\sqrt{-i\,d}F(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)|-}{df}$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I*d)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + sqrt(-
I*d)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1))/(d*f)
```

3.255.6 Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sec(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

3.255.7 Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.255.8 Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)),x)
```

output
```
int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)), x)
```

3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.256.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}} + \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

output
```
-1/2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+
f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+cos(f
*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

3.256.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos(2(e+fx))\sec(e+fx)\left(\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sec^2(e+fx)-\sqrt{\sec^2(}\right.}{f\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}\left(-1+\tan^2(e+fx)\right)}$$

input
```
Integrate[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[2*(e + f*x)]*Sec[e + f*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*
Sqrt[Tan[e + f*x]]], -1]*Sec[e + f*x]^2 - Sqrt[Sec[e + f*x]^2]*Sqrt[Tan[e
+ f*x]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*Tan[e + f*x]]*
(-1 + Tan[e + f*x]^2))
```

3.256.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{2f\sqrt{d\tan(e+fx)}}$$

input
```
Int[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sq
rt[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

3.256.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

3.256. $\int\frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.256.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(93) = 186.

Time = 3.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.53

method	result
default	$\frac{\left(\sqrt{\cot(fx+e)-\csc(fx+e)+1}\,\sqrt{\cot(fx+e)-\csc(fx+e)}\,\sqrt{-\cot(fx+e)+\csc(fx+e)+1}\,F\left(\sqrt{-\cot(fx+e)+\csc(fx+e)+1},\frac{\sqrt{2}}{2}\right)+\sec(\ldots\right.}{}$

input
```
int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(d*tan(f*x+e))^(1/2)*((cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-cs
c(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+cs
c(f*x+e)+1)^(1/2),1/2*2^(1/2))+sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)
*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticF((
-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+2^(1/2)*sin(f*x+e))*2^(1/2)
```

3.256.5 Fricas [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)/(d*tan(f*x + e)), x)
```

3.256.6 Sympy [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos{\left(e+fx \right)}}{\sqrt{d\tan{\left(e+fx \right)}}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral(cos(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

3.256.7 Maxima [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.256.8 Giac [F]

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(e+f\,x)}{\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int(cos(e + f*x)/(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)/(d*tan(e + f*x))^(1/2), x)
```

3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.257.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{5\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

output
```
-5/12*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi
+f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+5/6*
cos(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+1/3*cos(f*x+e)^3*(d*tan(f*x+e))^(1/2)/
d/f
```

3.257.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{11\sin(e+fx)+\sin(3(e+fx))-10\sqrt[4]{-1}\cos(e+fx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right),-1\right)\sqrt{}}{12f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(11*Sin[e + f*x] + Sin[3*(e + f*x)] - 10*(-1)^(1/4)*Cos[e + f*x]*EllipticF
[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sqrt[Sec[e + f*x]^2]*Sqrt[T
an[e + f*x]])/(12*f*Sqrt[d*Tan[e + f*x]])
```

3.257.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3092, 3042, 3092, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(e+fx)^3\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{5}{6}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \frac{1}{\sec(e+fx)\sqrt{d\tan(e+fx)}}dx+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3092}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}dx+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3094}$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3053}$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\right)+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{5}{6}\left(\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)}{2f\sqrt{d\tan(e+fx)}}\right)$$

input
```
Int[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f) + (5*((EllipticF[e - Pi/4 +
f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[d*Tan[e + f*x]]) +
(Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)))/6
```

3.257. $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

3.257.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.257.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 1906, normalized size of antiderivative = 17.49

method	result	size
default	Expression too large to display	1906

input
```
int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48/f/(d*tan(f*x+e))^(1/2)*(6*I*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x
+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/2*
I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-6*I*sec(f*x+e)*(cot(f*x+e)
-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(f*x+e)
+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2
))-6*I*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot
(f*x+e)+csc(f*x+e)+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/
2+1/2*I,1/2*2^(1/2))+6*I*sec(f*x+e)*(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f
*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2-1/
2*I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)+6*(cot(f*x+e)-csc(f*x+e)
+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)
+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)-32*(cot(
f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)+csc(
f*x+e)+1)^(1/2)*EllipticF((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2*2^(1/2))+6*
(cot(f*x+e)-csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*(-cot(f*x+e)
+csc(f*x+e)+1)^(1/2)*EllipticPi((-cot(f*x+e)+csc(f*x+e)+1)^(1/2),1/2+1/2*I
,1/2*2^(1/2))-8*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)+6*sec(f*x+e)*(cot(f*x+e)-c
sc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e))^(1/2)*EllipticPi((-cot(f*x+e)+c
sc(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)
-32*sec(f*x+e)*(-cot(f*x+e)+csc(f*x+e)+1)^(1/2)*(cot(f*x+e)-csc(f*x+e)+...
```

3.257.5 Fricas [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(fx+e\right)^3}{\sqrt{d\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3/(d*tan(f*x + e)), x)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.257.8 Giac [F]

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos(fx+e)^3}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

input
```
int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2),x)
```

output
```
int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2), x)
```

3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.258.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}+\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)+4/3*(d*tan(b*x+a))^(3/2)/b/d^3+2/7*(d*tan(b*x+
a))^(7/2)/b/d^5
```

3.258.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-42+(22+6\sec^2(a+bx))\tan^2(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-42 + (22 + 6*Sec[a + b*x]^2)*Tan[a + b*x]^2)/(21*b*d*Sqrt[d*Tan[a + b*x]
])
```

3.258.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^6}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(\tan^2(a+bx)+1)^2}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{(d\tan(a+bx))^{5/2}}{d^4}+\frac{2\sqrt{d\tan(a+bx)}}{d^2}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{7/2}}{7d^5}+\frac{4(d\tan(a+bx))^{3/2}}{3d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}}{b}$$

input
```
Int[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2/(d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*d^3) + (2*(d*
Tan[a + b*x])^(7/2))/(7*d^5))/b
```

3.258.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.258.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

method	result	size
default	$-\frac{2\left(32-8\left(\sec^{2}\left(bx+a\right)\right)-3\left(\sec^{4}\left(bx+a\right)\right)\right)}{21b\sqrt{d\tan\left(bx+a\right)}\, d}$	41

input
```
int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/b/(d*tan(b*x+a))^(1/2)/d*(32-8*sec(b*x+a)^2-3*sec(b*x+a)^4)
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(32\cos(bx+a)^4 - 8\cos(bx+a)^2 - 3\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{21\,bd^2\cos(bx+a)^3\sin(bx+a)}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/21*(32*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 3)*sqrt(d*sin(b*x + a)/cos(b
*x + a))/(b*d^2*cos(b*x + a)^3*sin(b*x + a))
```

3.258.6 Sympy [F]

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}} - \frac{3\,(d\tan(bx+a))^{\frac{7}{2}} + 14\,(d\tan(bx+a))^{\frac{3}{2}}d^2}{d^4}\right)}{21\,bd}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/21*(21/sqrt(d*tan(b*x + a)) - (3*(d*tan(b*x + a))^(7/2) + 14*(d*tan(b*x
 + a))^(3/2)*d^2)/d^4)/(b*d)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{21}{\sqrt{d\tan(bx+a)}b} - \frac{3\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)^3+14\sqrt{d\tan(bx+a)}b^6d^{27}\tan(bx+a)}{b^7d^{28}}\right)}{21\,d}$$

input
```
integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/21*(21/(sqrt(d*tan(b*x + a))*b) - (3*sqrt(d*tan(b*x + a))*b^6*d^27*tan(
b*x + a)^3 + 14*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b*x + a))/(b^7*d^28))/d
```

3.258.9 Mupad [B] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.12

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\left(\frac{20\mathrm{i}}{21\,b\,d^2} + \frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,64\mathrm{i}}{21\,b\,d^2}\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,20\mathrm{i}}{21\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)} + \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

input
```
int(1/(cos(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

output
```
((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20i)/(
21*b*d^2*(exp(a*2i + b*x*2i) + 1)) - ((20i/(21*b*d^2) + (exp(a*2i + b*x*2i
)*64i)/(21*b*d^2))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i)
+ 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) + ((-(d*(exp(a*2i + b*x*2i)*1i - 1i)
)/(exp(a*2i + b*x*2i) + 1))^(1/2)*24i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^2
) - ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*16
i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^3)
```

3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.259.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)+2/3*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.259.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\frac{2(-3+\tan^2(a+bx))}{3bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*(-3 + Tan[a + b*x]^2))/(3*b*d*Sqrt[d*Tan[a + b*x]])
```

3.259.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^4}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{\tan^2(a+bx)+1}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{\sqrt{d\tan(a+bx)}}{d^2}+\frac{1}{(d\tan(a+bx))^{3/2}}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(d\tan(a+bx))^{3/2}}{3d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}}{b}$$

input
```
Int[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2/(d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*d^3))/b
```

3.259.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.259.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2\left(4-\left(\sec^{2}\left(b x+a\right)\right)\right)}{3 b \sqrt{d \tan\left(b x+a\right)}\, d}$	31

input
```
int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b/(d*tan(b*x+a))^(1/2)/d*(4-sec(b*x+a)^2)
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(4\cos\left(bx+a\right)^2-1\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,bd^2\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(4*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*d^2*cos(b
*x + a)*sin(b*x + a))
```

3.259.6 Sympy [F]

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^4\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

3.259.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/(b*d)
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{d\tan(bx+a)}\tan(bx+a)}{bd} - \frac{3}{\sqrt{d\tan(bx+a)}b}\right)}{3\,d}$$

input
```
integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
2/3*(sqrt(d*tan(b*x + a))*tan(b*x + a)/(b*d) - 3/(sqrt(d*tan(b*x + a))*b))
/d
```

3.259.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{4\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{3\,b\,d^2\,{\sin\left(2\,a+2\,b\,x\right)}^2}$$

input
```
int(1/(cos(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

output
```
-(4*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*a +
 2*b*x) + 1))^(1/2))/(3*b*d^2*sin(2*a + 2*b*x)^2)
```

3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.260.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-2/(b*d*Sqrt[d*Tan[a + b*x]])
```

3.260.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(a+bx)^2}{(d\tan(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int \frac{1}{(d\tan(a+bx))^{3/2}}d\tan(a+bx)}{b}$$
$$\Big\downarrow \text{17}$$
$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
-2/(b*d*Sqrt[d*Tan[a + b*x]])
```

3.260.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.260.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2}{bd\sqrt{d\tan(bx+a)}}$	19
default	$-\frac{2}{bd\sqrt{d\tan(bx+a)}}$	19

input
```
int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/d/(d*tan(b*x+a))^(1/2)
```

3.260.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{bd^2\sin(bx+a)}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*d^2*sin(b*x + a))
```

3.260.6 Sympy [F]

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-2/(sqrt(d*tan(b*x + a))*b*d)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{\sqrt{d\tan(bx+a)}bd}$$

input
```
integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-2/(sqrt(d*tan(b*x + a))*b*d)
```

3.260.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{b\,d^2\,\sin\left(a+b\,x\right)^2}$$

input
```
int(1/(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

output
```
-(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b
*d^2*sin(a + b*x)^2)
```

3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

3.261.1 Optimal result

Integrand size = 12, antiderivative size = 212

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}bd^{3/2}} - \frac{2}{bd\sqrt{d\tan(a+bx)}}$$

output
```
1/2*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/2*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/4*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+1/
4*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^
(1/2)-2/b/d/(d*tan(b*x+a))^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.39

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(a+bx)}\right)\sqrt[4]{-\tan^2(a+bx)}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(a+bx}\right.}{bd\sqrt{d\tan(a+bx)}}$$

input

```
Integrate[(d*Tan[a + b*x])^(-3/2),x]
```

output

```
(-2 - ArcTan[(-Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4) + ArcTanh[(-
Tan[a + b*x]^2)^(1/4)]*(-Tan[a + b*x]^2)^(1/4))/(b*d*Sqrt[d*Tan[a + b*x]])
```

3.261.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3955$$

$$-\frac{\int \sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \ 3957$$

$$-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 266$$

$$-\frac{2\int \frac{d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 826$$

$$-\frac{2\left(\frac{1}{2}\int \frac{d^2\tan^2(a+bx)+d}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)}{bd}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 1476$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}\right)\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d^2\tan^2(a+bx)-1}d\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(}\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 217$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d^2\tan^2(a+bx)}{d^4\tan^4(a+bx)+d^2}d\sqrt{d\tan(a+bx)}\right)}{bd}-$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.}{bd}$$

$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 25$$

3.261. $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\ldots\right.}{\ldots}\right.\ldots}{bd}\ldots$$
$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)-\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{d^2\tan^2(a+bx)+\sqrt{2}d^{3/2}\tan(a+bx)+d}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\ldots\right.}{\ldots}\right.\ldots}{bd}\ldots$$
$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{d}\tan(a+bx)+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{d}\tan(a+bx)\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}d^{3/2}\tan(a+bx)+d^2\tan^2(a+bx)+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(\sqrt{2}d^{3/2}\ldots\right.}{\ldots}\right.\ldots}{bd}\ldots$$
$$\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[(d*Tan[a + b*x])^(-3/2),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d])) + ArcT
an[1 + Sqrt[2]*Sqrt[d]*Tan[a + b*x]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d - Sqrt[
2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]) - Log[d
+ Sqrt[2]*d^(3/2)*Tan[a + b*x] + d^2*Tan[a + b*x]^2]/(2*Sqrt[2]*Sqrt[d]))/
2))/(b*d) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```

3.261.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.261. $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.261.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\sqrt{\ }\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{b}$
default	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(bx+a)}}{(d^2)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\sqrt{\ }\right.}{8d^2(d^2)^{\frac{1}{4}}}\right.}{b}$

input
```
int(1/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*d*(-1/d^2/(d*tan(b*x+a))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(
b*x+a)-(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)
+(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(
d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*
x+a))^(1/2)+1)))
```

3.261.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{bd^2\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{4}}\log\left(b^3d^5\left(-\frac{1}{b^4d^6}\right)^{\frac{3}{4}}+\sqrt{d\tan(bx+a)}\right)\tan(bx+a)-i\,bd^2\left(-\frac{1}{b^4d^6}\right)^{\frac{1}{4}}\log\left(i\,b^3d^5\left(-\frac{1}{b^4d^6}\right)^{\frac{3}{4}}+\sqrt{}\right.}{}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(b*d^2*(-1/(b^4*d^6))^(1/4)*log(b^3*d^5*(-1/(b^4*d^6))^(3/4) + sqrt(d
*tan(b*x + a)))*tan(b*x + a) - I*b*d^2*(-1/(b^4*d^6))^(1/4)*log(I*b^3*d^5*
(-1/(b^4*d^6))^(3/4) + sqrt(d*tan(b*x + a)))*tan(b*x + a) + I*b*d^2*(-1/(b
^4*d^6))^(1/4)*log(-I*b^3*d^5*(-1/(b^4*d^6))^(3/4) + sqrt(d*tan(b*x + a)))
*tan(b*x + a) - b*d^2*(-1/(b^4*d^6))^(1/4)*log(-b^3*d^5*(-1/(b^4*d^6))^(3/
4) + sqrt(d*tan(b*x + a)))*tan(b*x + a) + 4*sqrt(d*tan(b*x + a)))/(b*d^2*t
an(b*x + a))
```

3.261.6 Sympy [F]

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral((d*tan(a + b*x))**(-3/2), x)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}} +}{4\,bd}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x + a
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sqrt
(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a)
 - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8/sqrt(d*tan(b*x +
a)))/(b*d)
```

3.261.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.261.9 Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.36

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = \frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)}{b\,d^{3/2}} - \frac{2}{b\,d\,\sqrt{d\tan(a+b\,x)}}$$

input
```
int(1/(d*tan(a + b*x))^(3/2),x)
```

output
```
((-1)^(1/4)*atanh(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2)
) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3
/2)) - 2/(b*d*(d*tan(a + b*x))^(1/2))
```

3.261. $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.262.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

output
```
5/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/16*ln(d^
(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+5
/16*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*
2^(1/2)-5/2/b/d/(d*tan(b*x+a))^(1/2)+1/2*cos(b*x+a)^2/b/d/(d*tan(b*x+a))^(
1/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.46

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\csc(a+bx)\left(-17\cos(a+bx)+\cos(3(a+bx))+5\arcsin(\cos(a+bx)-\sin(a+\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Csc[a + b*x]*(-17*Cos[a + b*x] + Cos[3*(a + b*x)] + 5*ArcSin[Cos[a + b*x]
 - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*Tan[a + b*x]])/
(8*b*d^2)
```

3.262.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3087, 253, 264, 266, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sec(a+bx)^2(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int \frac{1}{(d\tan(a+bx))^{3/2}(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\frac{5}{4}\int \frac{1}{(d\tan(a+bx))^{3/2}(\tan^2(a+bx)+1)}d\tan(a+bx)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{5}{4}\left(-\frac{\int \frac{\sqrt{d\tan(a+bx)}}{\tan^2(a+bx)+1}d\tan(a+bx)}{d^2}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$\Big\downarrow$ 266

$$\frac{\frac{5}{4}\left(-\frac{2\int \frac{d^3\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{d^3}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$\Big\downarrow$ 27

$$\frac{\frac{5}{4}\left(-\frac{2\int \frac{d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx)}}}{b}$$

$\Big\downarrow$ 826

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\int \frac{\tan(a+bx)d+d}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{1}{2d(\tan^2(a+bx)+1)\sqrt{d\tan(a+bx}}$$

$\Big\downarrow$ 1476

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}+\frac{1}{2}\int \frac{1}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}\right)-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}}{d}}{b}$$

$\Big\downarrow$ 1082

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}-\frac{\int \frac{1}{-d\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\frac{}{d\sqrt{}}\right.}{b}$$

$\Big\downarrow$ 217

$$\frac{\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)-\frac{1}{2}\int \frac{d-d\tan(a+bx)}{\tan^2(a+bx)d^2+d^2}d\sqrt{d\tan(a+bx)}\right)}{d}-\frac{2}{d\sqrt{d\tan(a+bx)}}\right)+\frac{}{2d(ta}}{b}$$

$\Big\downarrow$ 1479

3.262. $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

$$\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+b}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{d}\right.}{b}$$

$\downarrow$ 25

$$\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{d}\right.}{b}$$

$\downarrow$ 27

$$\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d-\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{2}\sqrt{d}}-\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt{d\tan(a+bx)}}{\tan(a+bx)d+d+\sqrt{2}\sqrt{d\tan(a+bx)}\sqrt{d}}d\sqrt{d\tan(a+bx)}}{2\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx}}{\sqrt{d}}\right.}{\sqrt{2}\sqrt{d}}\right.\right.}{d}\right.}{b}$$

$\downarrow$ 1103

$$\frac{5}{4}\left(-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}}\right)+\frac{1}{2}\left(\frac{\log\left(d\tan(a+bx)-\sqrt{2}\sqrt{d}\sqrt{d\tan(a+bx)}+d\right)}{2\sqrt{2}\sqrt{d}}-\frac{\log\left(d\tan(a+bx)+\sqrt{2}\sqrt{d}\sqrt{d\tan}\right.}{2\sqrt{2}\sqrt{d}}\right.\right.}{d}\right.}{b}$$

input
```
Int[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(1/(2*d*Sqrt[d*Tan[a + b*x]]*(1 + Tan[a + b*x]^2)) + (5*((-2*((-(ArcTan[1
- (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d])) + ArcTan[1 +
(Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]))/2 + (Log[d + d*
Tan[a + b*x] - Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*Sqrt[d]) -
 Log[d + d*Tan[a + b*x] + Sqrt[2]*Sqrt[d]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]
*Sqrt[d]))/2))/d - 2/(d*Sqrt[d*Tan[a + b*x]])))/4)/b
```

3.262. $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.262. $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.262.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(189) = 378$.

Time = 19.77 (sec) , antiderivative size = 937, normalized size of antiderivative = 3.76

method	result	size
default	Expression too large to display	937

input
```
int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/b*csc(b*x+a)*(4*cos(b*x+a)^2*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)-20*sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)+10*cos(b*x+a)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+
a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/(-1+cos(b*x+a)))+10*co
s(b*x+a)*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))-5*cos(b*x+a)*ln(-(cot(b*x+a)*cos(
b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-
3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(
b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))+5*cos(b*x+a)*ln((2*sin(b*
x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+cs
c(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)
+2*cos(b*x+a)+sin(b*x+a)-csc(b*x+a)-2)/(-1+cos(b*x+a)))-10*arctan((sin(b*x
+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)-cos(b*x+a)+1)/
(-1+cos(b*x+a)))-10*arctan((sin(b*x+a)*2^(1/2)*(-cos(b*x+a)*sin(b*x+a)/(co
s(b*x+a)+1)^2)^(1/2)+cos(b*x+a)-1)/(-1+cos(b*x+a)))+5*ln(-(cot(b*x+a)*cos(
b*x+a)-2*cot(b*x+a)+2*sin(b*x+a)*(-cot(b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-
3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+cot(b*x+a)-csc(b*x+a))^(1/2)-2*cos(
b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(-1+cos(b*x+a)))-5*ln((2*sin(b*x+a)*(-cot(
b*x+a)^3+3*cot(b*x+a)^2*csc(b*x+a)-3*cot(b*x+a)*csc(b*x+a)^2+csc(b*x+a)^3+
cot(b*x+a)-csc(b*x+a))^(1/2)-cot(b*x+a)*cos(b*x+a)+2*cot(b*x+a)+2*cos(b...
```

3.262.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 1034, normalized size of antiderivative = 4.15

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/32*(5*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-1/2*cos(b*x + a)*sin(b*x + a) + 1/
2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*
cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*
d^3*cos(b*x + a)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6)))*sin(b*x + a) - 5*b*d^2*(
-1/(b^4*d^6))^(1/4)*log(-1/2*cos(b*x + a)*sin(b*x + a) - 1/2*(b^3*d^4*(-1/
(b^4*d^6))^(3/4)*cos(b*x + a)^2 - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*si
n(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/4*(2*b^2*d^3*cos(b*x + a
)^2 - b^2*d^3)*sqrt(-1/(b^4*d^6)))*sin(b*x + a) - 5*I*b*d^2*(-1/(b^4*d^6))
^(1/4)*log(-1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(I*b^3*d^4*(-1/(b^4*d^6))^
(3/4)*cos(b*x + a)^2 + I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a
))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2
*d^3)*sqrt(-1/(b^4*d^6)))*sin(b*x + a) + 5*I*b*d^2*(-1/(b^4*d^6))^(1/4)*lo
g(-1/2*cos(b*x + a)*sin(b*x + a) + 1/2*(-I*b^3*d^4*(-1/(b^4*d^6))^(3/4)*co
s(b*x + a)^2 - I*b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(
d*sin(b*x + a)/cos(b*x + a)) - 1/4*(2*b^2*d^3*cos(b*x + a)^2 - b^2*d^3)*sq
rt(-1/(b^4*d^6)))*sin(b*x + a) - 5*b*d^2*(-1/(b^4*d^6))^(1/4)*log(2*(b^3*d
^4*(-1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^(1/
4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sin(b*x + a) + 5
*b*d^2*(-1/(b^4*d^6))^(1/4)*log(-2*(b^3*d^4*(-1/(b^4*d^6))^(3/4)*cos(b*x +
 a)*sin(b*x + a) - b*d*(-1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(...
```

3.262.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos^2{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(cos(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{10\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} + \frac{10\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}} - \frac{5\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right.}{\sqrt{d}}}{16\,bd}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x +
 a)))/sqrt(d))/sqrt(d) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - 5*sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 5*sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8*(5*d^2*tan
(b*x + a)^2 + 4*d^2)/((d*tan(b*x + a))^(5/2) + sqrt(d*tan(b*x + a))*d^2))/
(b*d)
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{\frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} + \frac{10\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2} - \frac{5\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\right.}{bd^2}}{16\,d}$$

input
```
integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
-1/16*(10*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 10*sqrt(2)*abs(d)^(3/2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs
(d)))/(b*d^2) - 5*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d
*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 5*sqrt(2)*abs(d)^(3/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d
^2) + 8*(5*d^2*tan(b*x + a)^2 + 4*d^2)/((sqrt(d*tan(b*x + a))*d^2*tan(b*x
+ a)^2 + sqrt(d*tan(b*x + a))*d^2)*b))/d
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.263.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

output
```
-2*sec(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+24/5*cos(b*x+a)*(sin(a+1/4*Pi+b*x
)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b
*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+24/5*cos(b*x+a)*(d*tan(b*x+a))^(3/
2)/b/d^3+12/5*sec(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.263.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.75

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\tan\ldots\right.}{5bd^2\sqrt{\sec^2(a+\ldots}}$$

input
```
Integrate[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(-8*Hypergeometric2F1[3/4, 3/2, 7/4,
-Tan[a + b*x]^2]*Tan[a + b*x]^2 + Sqrt[Sec[a + b*x]^2]*(-5 + 12*Sin[a + b*
x]^2 + Tan[a + b*x]^2)))/(5*b*d^2*Sqrt[Sec[a + b*x]^2])
```

3.263.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3088, 3042, 3093, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^5}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{6\int \sec^3(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \sec(a+bx)^3\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3093}$$

$$\frac{6\left(\frac{2}{5}\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2}{5}\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3093}$$

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int\cos(a+bx)\sqrt{d\tan(a+bx)}dx\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int\frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3095

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3052

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3042

$$\frac{6\left(\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)+\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$\downarrow$ 3119

$$\frac{6\left(\frac{2\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd}+\frac{2}{5}\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\right)\right)}{d^2}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

3.263. $\int\frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}dx$

input
```
Int[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) + (6*((2*Sec[a + b*x]*(d*Ta
n[a + b*x])^(3/2))/(5*b*d) + (2*((-2*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x
, 2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]]) + (2*Cos[a + b*x]*(d
*Tan[a + b*x])^(3/2))/(b*d)))/5))/d^2
```

3.263.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

3.263. $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.263.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(147) = 294$.

Time = 1.62 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.75

method	result
default	$-\frac{\left(12\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-24\sqrt{}\right.}{}$

input
```
int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/(d*tan(b*x+a))^(1/2)/d*(12*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-24*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-c
sc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+c
sc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+12*sec(b*x+a)*(1+csc(b*x+a)-cot(b
*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-24*sec(b*x+a)*(1+
csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+1
2*2^(1/2)-6*sec(b*x+a)*2^(1/2)-sec(b*x+a)^3*2^(1/2))*2^(1/2)
```

3.263.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.61

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(6i\sqrt{i\,d}\cos{(bx+a)}^2\,E(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)\sin{(bx+a)}-6i\sqrt{-i\,d}\cos{(bx+a)}^2\right.}{}$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(6*I*sqrt(I*d)*cos(b*x + a)^2*elliptic_e(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_e(arc
sin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - 6*I*sqrt(I*d)*cos(b
*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x +
a) + 6*I*sqrt(-I*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) - I*sin(
b*x + a)), -1)*sin(b*x + a) + (12*cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 1)*s
qrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*cos(b*x + a)^2*sin(b*x + a))
```

3.263.6 Sympy [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^5{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**5/(d*tan(a + b*x))**(3/2), x)
```

3.263.7 Maxima [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec{(bx+a)}^5}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.263.8 Giac [F]

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^5\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)), x)
```

3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.264.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}$$

output
```
-2*sec(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+4*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^
(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a)
)^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+4*cos(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.264.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=-\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(3\cos(2(a+bx))\sqrt{\sec^2(a+bx)}+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+\right.\right.}{3bd^2\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(3*Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x
]^2] + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Tan[a + b*x]^2)
)/(3*b*d^2*Sqrt[Sec[a + b*x]^2])
```

3.264.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3088, 3042, 3093, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^3}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3093}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{2\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd}-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{b\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*((-2*Cos[a + b*x]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sin[2*a + 2*b*x]]) +
(2*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d)))/d^2
```

3.264.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.264.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(121) = 242$.

Time = 1.48 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.53

method	result
default	$-\frac{\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+2\sqrt{}\right.}{}$

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(d*tan(b*x+a))^(1/2)/d*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a
))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+csc(b
*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b
*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*2^(1
/2)-sec(b*x+a)*2^(1/2))*2^(1/2)
```

3.264.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{i\,d}E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)\sin\left(bx+a\right)-i\sqrt{-i\,d}E(\arcsin\left(\cos\left(bx+a\right)-i\,\sin\right.\right.}{}$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(
b*x + a) - I*sqrt(-I*d)*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)),
-1)*sin(b*x + a) - I*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x
+ a)), -1)*sin(b*x + a) + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x +
a)/cos(b*x + a)))/(b*d^2*sin(b*x + a))
```

3.264.6 Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

3.264.7 Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.264.8 Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.265.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
-2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^
(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a)
)^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

3.265.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sin(a+bx)\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sin[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*(d*Tan[a + b*x])^(3/2))
```

3.265.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3088, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$-\frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
 - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

3.265.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.265. $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.265.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(97) = 194$.

Time = 1.11 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.51

method	result
default	$-\frac{\left(-2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+\sqrt{\cdots}\right.}{\cdots}$

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/d/(d*tan(b*x+a))^(1/2)*(-2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*
x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-2*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))
^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+sec(b*x+a)*(1+csc(b*x+a
)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2))*2
^(1/2)
```

3.265.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.17

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)^2 + i\sqrt{i\,d}\,E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a) - i\sqrt{-i\,d}\,E(\cdots}{\cdots}$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + I*sqrt(I*d)*ellipti
c_e(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a) - I*sqrt(-I*d)
*elliptic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) - I*sq
rt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1)*sin(b*x + a)
 + I*sqrt(-I*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(
b*x + a))/(b*d^2*sin(b*x + a))
```

3.265.6 Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec{(a+bx)}}{(d\tan{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(sec(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

3.265.7 Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.265.8 Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\sec{(bx+a)}}{(d\tan{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

output
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.266.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
-2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+3*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^
(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a)
)^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

3.266.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sin(a+bx)\left(1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Sin[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*(d*Tan[a + b*x])^(3/2))
```

3.266.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3089, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{3\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{3\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
 - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

3.266.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.266. $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(97) = 194$.

Time = 1.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 4.69

method	result
default	$\left(6\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{\cot(}\right.$

input
```
int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(d*tan(b*x+a))^(1/2)/d*(6*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+
a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+
a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*
x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+6*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a
))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-3*sec(b*x+a)*(1+csc(b
*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b
*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2^(1/2
)*cos(b*x+a)-3*2^(1/2))*2^(1/2)
```

3.266.5 Fricas [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

3.266.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

output
```
Integral(cos(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

3.266.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.266.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.267.1 Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}$$

output
```
-2*cos(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+7/2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*
x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-7/3*cos(b*x+a)^3*(d*tan(b*x+a))^(3/
2)/b/d^3
```

3.267.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-13+\cos(2(a+bx))-14\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+\right.\right.}{6b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Sin[a + b*x]*(-13 + Cos[2*(a + b*x)] - 14*Hypergeometric2F1[3/4, 3/2, 7/4
, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(6*b*(d*Tan[a + b
*x])^(3/2))
```

3.267.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3089, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)^3(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{7\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)^3}dx}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{7\left(\frac{1}{2}\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{7\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{7\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{7\left(\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}+\frac{\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}\right)}{d^2}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*((Cos[a + b*x]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*Sqrt[Sin[2*a + 2*b*x]]) +
 (Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d)))/d^2
```

3.267.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.267.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(125) = 250$.

Time = 1.49 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.39

method	result
default	$\frac{\left(2(\cos^3(bx+a))\sqrt{2}-21\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,F\left(\sqrt{1+\csc(bx+a)-\cot(\right.\right.}{}$

input
```
int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b/(d*tan(b*x+a))^(1/2)/d*(2*cos(b*x+a)^3*2^(1/2)-21*(cot(b*x+a)-csc(b
*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1
/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+42*(1+csc(b*x+a
)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-21*sec(b*x
+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(
b*x+a)-csc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(
1/2))+42*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x
+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a
))^(1/2),1/2*2^(1/2))+7*2^(1/2)*cos(b*x+a)-21*2^(1/2))*2^(1/2)
```

3.267.5 Fricas [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos(bx+a)^3}{(d\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.267.7 Maxima [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.267.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

3.268.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

output

```
-2*cos(b*x+a)^5/b/d/(d*tan(b*x+a))^(1/2)+77/20*cos(b*x+a)*(sin(a+1/4*Pi+b*
x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(
b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-77/30*cos(b*x+a)^3*(d*tan(b*x+a))
^(3/2)/b/d^3-11/5*cos(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b/d^3
```

3.268.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.91 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin(a+bx)\left(-277+34\cos(2(a+bx))+3\cos(4(a+bx))-308\,\text{Hypergeometric}\right.}{120b(d\tan(a+bx))^{3/2}}$$

input
```
Integrate[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(Sin[a + b*x]*(-277 + 34*Cos[2*(a + b*x)] + 3*Cos[4*(a + b*x)] - 308*Hyper
geometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a +
b*x]^2))/(120*b*(d*Tan[a + b*x])^(3/2))
```

3.268.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3089, 3042, 3092, 3042, 3092, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(a+bx)^5(d\tan(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{11\int \cos^5(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)^5}dx}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{11\left(\frac{7}{10}\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\left(\frac{7}{10}\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)^3}dx+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{1}{2}\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3095}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{2\sqrt{\sin(a+bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{11\left(\frac{7}{10}\left(\frac{\cos(a+bx)\sqrt{d\tan(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}\right)+\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

3.268. $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

$$-\frac{11\left(\frac{\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd}+\frac{7}{10}\left(\frac{\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd}+\frac{\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}\right)\right)}{d^2}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

input
```
Int[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

output
```
(-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (11*((Cos[a + b*x]^5*(d*T
an[a + b*x])^(3/2))/(5*b*d) + (7*((Cos[a + b*x]*EllipticE[a - Pi/4 + b*x,
2]*Sqrt[d*Tan[a + b*x]])/(2*b*Sqrt[Sin[2*a + 2*b*x]]) + (Cos[a + b*x]^3*(d
*Tan[a + b*x])^(3/2))/(3*b*d)))/10))/d^2
```

3.268.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

3.268. $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.268.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(151) = 302$.

Time = 1.39 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.77

method	result
default	$\frac{\left(12\left(\cos^{5}(bx+a)\right)\sqrt{2}+22\left(\cos^{3}(bx+a)\right)\sqrt{2}+462\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E}{}$

input
```
int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/120/b/(d*tan(b*x+a))^(1/2)/d*(12*cos(b*x+a)^5*2^(1/2)+22*cos(b*x+a)^3*2^
(1/2)+462*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)
*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1
/2*2^(1/2))-231*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(
1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(
1/2),1/2*2^(1/2))+462*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x
+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(b*x
+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-231*sec(b*x+a)*(1+csc(b*x+a)-cot(b*x+a)
)^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ell
ipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+77*2^(1/2)*cos(b*x+a)-
231*2^(1/2))*2^(1/2)
```

3.268.5 Fricas [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

output
```
Timed out
```

3.268.7 Maxima [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.268.8 Giac [F]

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\cos\left(bx+a\right)^5}{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx=\int \frac{\cos\left(a+b\,x\right)^5}{\left(d\tan\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2),x)
```

output
```
int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2), x)
```

3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.269.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}} - \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}$$

output
```
1/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*tan(b*x+a))^(1/2)-2/
3*sec(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

3.269.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.38

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{2\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,\text{EllipticF}\left(ia\right.\right.}{3bd^2\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a+\right.}$$

input
```
Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(2*Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2
] - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan
[a + b*x]^(3/2)))/(3*b*d^2*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

3.269.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3089, 3042, 3094, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}dx}{3d^2}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3094}$$

$$-\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3053}$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

$$\Big\downarrow \ 3120$$

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

input
```
Int[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

output
```
(-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b
*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

3.269.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3094
```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Simp[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])   Int[
1/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.269. $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

3.269.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(97) = 194$.

Time = 1.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.65

method	result
default	$\frac{(\csc^2(bx+a))(1-\cos(bx+a))^2\left(2\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{2-2\csc(bx+a)+2\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,F\left(\sqrt{1+\csc(bx}\right.\right.}{6b\sqrt{(\csc^3(bx+a))(1-\cos(bx+a))^3-\csc(bx+a)+\cot(bx+a)}\,\sqrt{\csc(bx+a)(1-\cos(bx+a))\left((\csc^2(bx+a))(1-\cos(bx+a))^2-1\right.}}$

input
```
int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/b*csc(b*x+a)^2*(1-cos(b*x+a))^2*(2*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(2-
2*csc(b*x+a)+2*cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((
1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))*(csc(b*x+a)-cot(b*x+a))-csc(b*
x+a)^4*(1-cos(b*x+a))^4+1)/(csc(b*x+a)^3*(1-cos(b*x+a))^3-csc(b*x+a)+cot(b
*x+a))^(1/2)/(csc(b*x+a)*(1-cos(b*x+a))*(csc(b*x+a)^2*(1-cos(b*x+a))^2-1))
^(1/2)/(csc(b*x+a)^2*(1-cos(b*x+a))^2-1)^2/(-d/(csc(b*x+a)^2*(1-cos(b*x+a)
)^2-1)*(csc(b*x+a)-cot(b*x+a)))^(5/2)*2^(1/2)
```

3.269.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\left(\cos(bx+a)^2-1\right)\sqrt{i\,d}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\mid -1)+\left(\cos(bx\right.}{3\left(bd^3\,\mathrm{co}\right.}$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin
(b*x + a)), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*elliptic_f(arcsin(cos(b*
x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*
x + a))/(b*d^3*cos(b*x + a)^2 - b*d^3)
```

3.269.6 Sympy [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

output
```
Integral(sec(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

3.269.7 Maxima [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.269.8 Giac [F]

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{\sec(bx+a)}{(d\tan(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,{\left(d\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

output
```
int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$

3.270.1 Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}} - \frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}} - \frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

output
```
-4/5*cos(b*x+a)/b/d^3/(d*tan(b*x+a))^(1/2)+4/5*cos(b*x+a)*(sin(a+1/4*Pi+b*
x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(
b*x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)-2/5*sec(b*x+a)/b/d/(d*tan(b*x+a))
^(5/2)
```

3.270.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = -\frac{2\left(4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\tan^2(a+bx)\right)\sec^2(a+bx)+3(-2+\csc^2(a+bx)+\csc^4(a+bx))\sqrt{\text{se}}\right.}{15bd^4\sqrt{\sec^2(a+bx)}}$$

input
```
Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

output
```
(-2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 +
3*(-2 + Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x
]*Sqrt[d*Tan[a + b*x]])/(15*b*d^4*Sqrt[Sec[a + b*x]^2])
```

3.270.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3088, 3042, 3088, 3042, 3095, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(a+bx)^3}{(d\tan(a+bx))^{7/2}}dx$$
$$\Big\downarrow \text{3088}$$
$$\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}dx}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$
$$\Big\downarrow \text{3088}$$
$$\frac{2\left(-\frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\left(-\frac{2\int \frac{\sqrt{d\tan(a+bx)}}{\sec(a+bx)}dx}{d^2}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$
$$\Big\downarrow \text{3095}$$

$$\frac{2\left(-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(-\frac{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\int\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}dx}{d^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3052

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{d\tan(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

$\Big\downarrow$ 3119

$$\frac{2\left(-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}\right)}{5d^2}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

input
```
Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

output
```
(-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) + (2*((-2*Cos[a + b*x])/(
b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*S
qrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])))/(5*d^2)
```

3.270.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3095
```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]])   Int[
Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.270.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(121) = 242$.

Time = 1.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.38

method	result
default	$-\frac{\left(-4\sqrt{1+\csc(bx+a)-\cot(bx+a)}\,\sqrt{-\csc(bx+a)+1+\cot(bx+a)}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,E\left(\sqrt{1+\csc(bx+a)-\cot(bx+a)},\frac{\sqrt{2}}{2}\right)+2\sqrt{}\right.}{}$

input
```
int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b/d^3/(d*tan(b*x+a))^(1/2)*(-4*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(
b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((1+csc(
b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cs
c(b*x+a)+1+cot(b*x+a))^(1/2)*(1+csc(b*x+a)-cot(b*x+a))^(1/2)*EllipticF((1+
csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))-4*sec(b*x+a)*(1+csc(b*x+a)-cot(b
*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticE((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*sec(b*x+a)*(1+c
sc(b*x+a)-cot(b*x+a))^(1/2)*(-csc(b*x+a)+1+cot(b*x+a))^(1/2)*(cot(b*x+a)-c
sc(b*x+a))^(1/2)*EllipticF((1+csc(b*x+a)-cot(b*x+a))^(1/2),1/2*2^(1/2))+2*
2^(1/2)+cot(b*x+a)*csc(b*x+a)*2^(1/2))*2^(1/2)
```

3.270.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.19

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx =$$

$$-\frac{2\left(\left(i\cos(bx+a)^2-i\right)\sqrt{i\,d}E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)\sin(bx+a)+\left(-i\cos(bx+a)^2\right.\right.}{}$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*((I*cos(b*x + a)^2 - I)*sqrt(I*d)*elliptic_e(arcsin(cos(b*x + a) + I*
sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*x + a)^2 + I)*sqrt(-I*d)*ellip
tic_e(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (-I*cos(b*
x + a)^2 + I)*sqrt(I*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)),
-1)*sin(b*x + a) + (I*cos(b*x + a)^2 - I)*sqrt(-I*d)*elliptic_f(arcsin(cos
(b*x + a) - I*sin(b*x + a)), -1)*sin(b*x + a) + (2*cos(b*x + a)^4 - 3*cos(
b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((b*d^4*cos(b*x + a)^2 - b*
d^4)*sin(b*x + a))
```

3.270.6 Sympy [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(7/2),x)
```

output
```
Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(7/2), x)
```

3.270.7 Maxima [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

3.270.8 Giac [F]

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{\sec\left(bx+a\right)^3}{\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\left(d\tan\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)),x)
```

output
```
int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)), x)
```

3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$

3.271.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

output
```
3/7*hypergeom([-7/6, -1/2],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\left(-3\sin(e+fx)+2\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx\right.}{7f}$$

input
```
Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e
 + f*x]))/(7*f)
```

3.271.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{10}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{10/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{10/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*
Sin[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

3.271.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.271.4 Maple [F]

$$\int \left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^{2}(fx+e)\right)dx$$

input
```
int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

3.271.5 Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec\left(fx+e\right)^{\frac{4}{3}}\tan\left(fx+e\right)^{2}\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

3.271.6 Sympy [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan^2(e+fx)\sec^{\frac{4}{3}}(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**2,x)
```

output
```
Integral(tan(e + f*x)**2*sec(e + f*x)**(4/3), x)
```

3.271.7 Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

3.271.8 Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3), x)
```

3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$

3.272.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

output
```
3/5*hypergeom([-5/6, -1/2],[1/6],cos(f*x+e)^2)*sec(f*x+e)^(5/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3(-1+\cos^2(e+fx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right))\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f}$$

input
```
Integrate[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(5/3)*Sin[e + f*x])/(5*f)
```

3.272.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{8}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{8/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^2(e+fx)}{\cos^{\frac{8}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{8/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*S
in[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

3.272.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.272.4 Maple [F]

$$\int \left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)dx$$

input
```
int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

3.272.5 Fricas [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

3.272.6 Sympy [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \tan^2{(e+fx)}\sec^{\frac{2}{3}}{(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**2,x)
```

output
```
Integral(tan(e + f*x)**2*sec(e + f*x)**(2/3), x)
```

3.272.7 Maxima [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec{(fx+e)}^{\frac{2}{3}}\tan{(fx+e)}^2\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

3.272.8 Giac [F]

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \sec{(fx+e)}^{\frac{2}{3}}\tan{(fx+e)}^2\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx=\int \tan(e+f\,x)^2\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{2/3}\,dx$$

input
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3), x)
```

3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$

3.273.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

output
```
3/4*hypergeom([-2/3, -1/2],[1/3],cos(f*x+e)^2)*sec(f*x+e)^(4/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3(-1+\cos^2(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(4*f)
```

3.273.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{7}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{7/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{7}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{7/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*S
in[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

3.273.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.273.4 Maple [F]

$$\int \left(\sec^{\frac{1}{3}}\left(fx+e\right)\right)\left(\tan^{2}\left(fx+e\right)\right)dx$$

input
```
int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

output
```
int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

3.273.5 Fricas [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec\left(fx+e\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

3.273.6 Sympy [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan^2(e+fx)\sqrt[3]{\sec(e+fx)}\,dx$$

input
```
integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**2,x)
```

output
```
Integral(tan(e + f*x)**2*sec(e + f*x)**(1/3), x)
```

3.273.7 Maxima [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

3.273.8 Giac [F]

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}dx$$

input
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3), x)
```

3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$

3.274.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

output
```
3/2*hypergeom([-1/2, -1/3],[2/3],cos(f*x+e)^2)*sec(f*x+e)^(2/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

3.274.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f}$$

input
```
Integrate[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(2/3)*Sin[e + f*x])/(2*f)
```

3.274.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{5}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^{5/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^2(e+fx)}{\cos^{\frac{5}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{5/3}}dx$$

$$\Big\downarrow \text{ 3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*S
in[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

3.274.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.274.4 Maple [F]

$$\int \frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

output
```
int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

3.274.5 Fricas [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

3.274.6 Sympy [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan^2(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/sec(f*x+e)**(1/3),x)
```

output
```
Integral(tan(e + f*x)**2/sec(e + f*x)**(1/3), x)
```

3.274.7 Maxima [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

3.274.8 Giac [F]

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3), x)
```

3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$

3.275.1 Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

output
```
3*hypergeom([-1/2, -1/6],[5/6],cos(f*x+e)^2)*sec(f*x+e)^(1/3)*sin(f*x+e)/f
/(sin(f*x+e)^2)^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = -\frac{3\left(-1+\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\right)\sqrt[3]{\sec(e+fx)}\sin(e+fx)}{f}$$

input
```
Integrate[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

output
```
(-3*(-1 + (Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e +
f*x]^2])*Sec[e + f*x]^(1/3)*Sin[e + f*x])/f
```

3.275.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sec^{\frac{4}{3}}(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)^{4/3}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \; 3112$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^2(e+fx)}{\cos^{\frac{4}{3}}(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^2}{\cos(e+fx)^{4/3}}dx$$

$$\Big\downarrow \; 3056$$

$$\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*S
in[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

3.275.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.275.4 Maple [F]

$$\int \frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

output
```
int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

3.275.5 Fricas [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

3.275.6 Sympy [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan^2(e+fx)}{\sec^{\frac{2}{3}}(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**2/sec(f*x+e)**(2/3),x)
```

output
```
Integral(tan(e + f*x)**2/sec(e + f*x)**(2/3), x)
```

3.275.7 Maxima [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

3.275.8 Giac [F]

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(fx+e)^2}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3), x)
```

3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$

3.276.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

output
```
3/13*hypergeom([-13/6, -3/2],[-7/6],cos(f*x+e)^2)*sec(f*x+e)^(13/3)*sin(f*
x+e)/f/(sin(f*x+e)^2)^(1/2)
```

3.276.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\Big(27\sin(e+fx)-18\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx)}{91f}$$

input
```
Integrate[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hyperge
ometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*(-16
 + 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

3.276.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{16}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(e+fx)^{16/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{ 3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{16}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{16/3}}dx$$

$$\Big\downarrow \text{ 3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{3}{2},-\frac{7}{6},\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3
)*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```

3.276.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.276.4 Maple [F]

$$\int \left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^{4}(fx+e)\right)dx$$

input
```
int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

3.276.5 Fricas [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

3.276.6 Sympy [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4(e+fx)\sec^{\frac{4}{3}}(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**4,x)
```

output
```
Integral(tan(e + f*x)**4*sec(e + f*x)**(4/3), x)
```

3.276.7 Maxima [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

3.276.8 Giac [F]

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3), x)
```

3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$

3.277.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

output
```
3/11*hypergeom([-11/6, -3/2],[-5/6],cos(f*x+e)^2)*sec(f*x+e)^(11/3)*sin(f*
x+e)/f/(sin(f*x+e)^2)^(1/2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[6]{\cos^2(e+fx)}}-(2+7\cos(2(e+fx)))\sec^4(e+fx)\right)\sin(e+fx)}{55f\sqrt[3]{\sec(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

output
```
(3*((9*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^
(1/6) - (2 + 7*Cos[2*(e + f*x)])*Sec[e + f*x]^4)*Sin[e + f*x])/(55*f*Sec[e
 + f*x]^(1/3))
```

3.277.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{14}{3}}(e+fx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(e+fx)^{14/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \ 3112$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^4(e+fx)}{\cos^{\frac{14}{3}}(e+fx)}dx$$

$$\Big\downarrow \ 3042$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{14/3}}dx$$

$$\Big\downarrow \ 3056$$

$$\frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{3}{2},-\frac{5}{6},\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3
)*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

3.277.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.277.4 Maple [F]

$$\int \left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^{4}(fx+e)\right)dx$$

input
```
int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

3.277.5 Fricas [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

3.277.6 Sympy [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4(e+fx)\sec^{\frac{2}{3}}(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**4,x)
```

output
```
Integral(tan(e + f*x)**4*sec(e + f*x)**(2/3), x)
```

3.277.7 Maxima [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

3.277.8 Giac [F]

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec(fx+e)^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{2/3}\,dx$$

input
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3), x)
```

3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$

3.278.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

output
```
3/10*hypergeom([-5/3, -3/2],[-2/3],cos(f*x+e)^2)*sec(f*x+e)^(10/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

3.278.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\left(\frac{9\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)}{\sqrt[3]{\cos^2(e+fx)}}+\sec^2(e+fx)\left(-13+4\sec^2(e+fx)\right)\right)\sin(e+fx)}{40f\sec^{\frac{2}{3}}(e+fx)}$$

input
```
Integrate[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

output
```
(3*((9*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^
(1/3) + Sec[e + f*x]^2*(-13 + 4*Sec[e + f*x]^2))*Sin[e + f*x])/(40*f*Sec[e
 + f*x]^(2/3))
```

3.278.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{13}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{13/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{13}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{13/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)
*Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

3.278.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.278.4 Maple [F]

$$\int \left(\sec^{\frac{1}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)dx$$

input
```
int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

output
```
int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

3.278.5 Fricas [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \sec(fx+e)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

3.278.6 Sympy [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan^4{(e+fx)}\sqrt[3]{\sec{(e+fx)}}\,dx$$

input
```
integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**4,x)
```

output
```
Integral(tan(e + f*x)**4*sec(e + f*x)**(1/3), x)
```

3.278.7 Maxima [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec{(fx+e)}^{\frac{1}{3}}\tan{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

3.278.8 Giac [F]

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \sec{(fx+e)}^{\frac{1}{3}}\tan{(fx+e)}^4\,dx$$

input
```
integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

input
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3), x)
```

3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$

3.279.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

output
```
3/8*hypergeom([-3/2, -4/3],[-1/3],cos(f*x+e)^2)*sec(f*x+e)^(8/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sec^{\frac{2}{3}}(e+fx)\left(-11\sin(e+fx)+9\sqrt[3]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+fx\right.}{16f}$$

input
```
Integrate[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(2/3)*(-11*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/3)*Hyperge
ometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 2*Sec[e + f*x]*Ta
n[e + f*x]))/(16*f)
```

3.279.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{11}{3}}(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(e+fx)^{11/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin^4(e+fx)}{\cos^{\frac{11}{3}}(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{11/3}}dx$$

$$\Big\downarrow \text{3056}$$

$$\frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*
Sin[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```

3.279.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.279.4 Maple [F]

$$\int \frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

output
```
int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

3.279.5 Fricas [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

3.279.6 Sympy [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan^4(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/sec(f*x+e)**(1/3),x)
```

output
```
Integral(tan(e + f*x)**4/sec(e + f*x)**(1/3), x)
```

3.279.7 Maxima [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

3.279.8 Giac [F]

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3), x)
```

3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$

3.280.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

output
```
3/7*hypergeom([-3/2, -7/6],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e
)/f/(sin(f*x+e)^2)^(1/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \frac{3\sqrt[3]{\sec(e+fx)}\Big(-10\sin(e+fx)+9\sqrt[6]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)\sin(e+f}{7f}$$

input
```
Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Sec[e + f*x]^(1/3)*(-10*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/6)*Hyperge
ometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[
e + f*x]))/(7*f)
```

3.280.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sec^{\frac{10}{3}}(e+fx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(e+fx)^{10/3}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \; 3112$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin^4(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\sqrt[3]{\cos(e+fx)}\sqrt[3]{\sec(e+fx)}\int \frac{\sin(e+fx)^4}{\cos(e+fx)^{10/3}}dx$$

$$\Big\downarrow \; 3056$$

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{6},\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

input
```
Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

output
```
(3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*
Sin[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

3.280.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

3.280.4 Maple [F]

$$\int \frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

output
```
int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

3.280.5 Fricas [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

3.280.6 Sympy [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan^4(e+fx)}{\sec^{\frac{2}{3}}(e+fx)}\,dx$$

input
```
integrate(tan(f*x+e)**4/sec(f*x+e)**(2/3),x)
```

output
```
Integral(tan(e + f*x)**4/sec(e + f*x)**(2/3), x)
```

3.280.7 Maxima [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

3.280.8 Giac [F]

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \int \frac{\tan(fx+e)^4}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx=\int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3), x)
```

3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$

3.281.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(13/6)*hypergeom([3/2, 13/6],[5/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(4/3)*tan(f*x+e)^3/f
```

3.281.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan(e+fx)}{4f\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

output
```
(3*Hypergeometric2F1[-1/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec[e + f*x])^(4/3
)*Tan[e + f*x])/(4*f*Sqrt[-Tan[e + f*x]^2])
```

3.281.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\sec(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(d\sec(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{13}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

3.281.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.281.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}} \left(\tan^2(fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

3.281.5 Fricas [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^2, x)
```

3.281.6 Sympy [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{4}{3}}\tan^2(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**2, x)
```

3.281.7 Maxima [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

3.281.8 Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3), x)
```

3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$

3.282.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{11/6}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(11/6)*hypergeom([3/2, 11/6],[5/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(2/3)*tan(f*x+e)^3/f
```

3.282.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}{2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

output
```
(-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[
e + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

3.282.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{6},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

3.282.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.282.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} \left(\tan^2 (fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)
```

3.282.5 Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec (fx+e))^{\frac{2}{3}}\tan (fx+e)^2 \,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

3.282.6 Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec (e+fx))^{\frac{2}{3}}\tan^2 (e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**2, x)
```

3.282.7 Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

3.282.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3), x)
```

3.283 $\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$

3.283.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{5/3}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(5/3)*hypergeom([3/2, 5/3],[5/2],sin(f*x+e)^2)*(d*sec(f
*x+e))^(1/3)*tan(f*x+e)^3/f
```

3.283.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = -\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
(-3*Cot[e + f*x]*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[
e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

3.283.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{3},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(
d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

3.283.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.283.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} \left(\tan^2(fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

3.283.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

3.283.6 Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

3.283.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

3.283.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

input
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3), x)
```

3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

3.284.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

output
```
1/3*(cos(f*x+e)^2)^(4/3)*hypergeom([4/3, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+
e)^3/f/(d*sec(f*x+e))^(1/3)
```

3.284.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x])/(f*(d
*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

3.284.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*T
an[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

3.284.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.284.4 Maple [F]

$$\int \frac{\tan^2(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

3.284.5 Fricas [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

3.284.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(1/3), x)
```

3.284.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

3.284.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3), x)
```

3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

3.285.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx=\frac{\cos^2(e+fx)^{7/6}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

output
```
1/3*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+
e)^3/f/(d*sec(f*x+e))^(2/3)
```

3.285.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx=-\frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[-1/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x])/(2*f*
(d*Sec[e + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])
```

3.285.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(e+fx)^2}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

input
```
Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

output
```
((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*T
an[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

3.285.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.285.4 Maple [F]

$$\int \frac{\tan^2(fx+e)}{(d\sec(fx+e))^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

3.285.5 Fricas [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

3.285.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(2/3),x)
```

output
```
Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(2/3), x)
```

3.285.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

3.285.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^2}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3), x)
```

3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$

3.286.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{19/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(19/6)*hypergeom([5/2, 19/6],[7/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(4/3)*tan(f*x+e)^5/f
```

3.286.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{3d\csc(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{4f}$$

input
```
Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

output
```
(3*d*Csc[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sec[e + f*x]^2]*(d*Sec
[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/(4*f)
```

3.286.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\sec(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(d\sec(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{19}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

3.286.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.286.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{4}{3}} \left(\tan^4(fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

3.286.5 Fricas [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^4, x)
```

3.286.6 Sympy [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(e+fx))^{\frac{4}{3}}\tan^4(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**4, x)
```

3.286.7 Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.286.8 Giac [F]

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{4}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3), x)
```

3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$

3.287.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx=\frac{\cos^2(e+fx)^{17/6}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(17/6)*hypergeom([5/2, 17/6],[7/2],sin(f*x+e)^2)*(d*sec
(f*x+e))^(2/3)*tan(f*x+e)^5/f
```

3.287.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx=\frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sec^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}{2f}$$

input
```
Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

output
```
(3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(2/3)*Sqrt[-Tan[e + f*x]^2])/(2*f)
```

3.287.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(d\sec(e+fx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(d\sec(e+fx))^{2/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{17}{6},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]
*(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

3.287.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.287.4 Maple [F]

$$\int (d\sec (fx+e))^{\frac{2}{3}} \left(\tan^4 (fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

3.287.5 Fricas [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec (fx+e))^{\frac{2}{3}} \tan (fx+e)^4 \,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

3.287.6 Sympy [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec (e+fx))^{\frac{2}{3}} \tan^4 (e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**4, x)
```

3.287.7 Maxima [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

3.287.8 Giac [F]

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{2}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3), x)
```

3.288 $\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$

3.288.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{8/3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(8/3)*hypergeom([5/2, 8/3],[7/2],sin(f*x+e)^2)*(d*sec(f
*x+e))^(1/3)*tan(f*x+e)^5/f
```

3.288.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \frac{3\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{f}$$

input
```
Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
(3*Cot[e + f*x]*Hypergeometric2F1[-3/2, 1/6, 7/6, Sec[e + f*x]^2]*(d*Sec[e
 + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])/f
```

3.288.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt[3]{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt[3]{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{8}{3},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(
d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

3.288.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.288.4 Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} \left(\tan^4(fx+e)\right) dx$$

input
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

output
```
int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

3.288.5 Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

3.288.6 Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

output
```
Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

3.288.7 Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

3.288.8 Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

input
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3), x)
```

3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

3.289.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

output
```
1/5*(cos(f*x+e)^2)^(7/3)*hypergeom([7/3, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x+
e)^5/f/(d*sec(f*x+e))^(1/3)
```

3.289.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan^3(e+fx)}{f\sqrt[3]{d\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*Hypergeometric2F1[-3/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(f*
(d*Sec[e + f*x])^(1/3)*(-Tan[e + f*x]^2)^(3/2))
```

3.289.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\sqrt[3]{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\text{Hypergeometric2F1}\left(\frac{7}{3},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

input
```
Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

output
```
((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*T
an[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

3.289.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.289.4 Maple [F]

$$\int \frac{\tan^4(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

3.289.5 Fricas [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

3.289.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(1/3), x)
```

3.289.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

3.289.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3), x)
```

3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

3.290.1 Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

output
```
1/5*(cos(f*x+e)^2)^(13/6)*hypergeom([13/6, 5/2],[7/2],sin(f*x+e)^2)*tan(f*
x+e)^5/f/(d*sec(f*x+e))^(2/3)
```

3.290.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sec^2(e+fx)\right)\tan^3(e+fx)}{2f(d\sec(e+fx))^{2/3}\left(-\tan^2(e+fx)\right)^{3/2}}$$

input
```
Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

output
```
(-3*Hypergeometric2F1[-3/2, -1/3, 2/3, Sec[e + f*x]^2]*Tan[e + f*x]^3)/(2*
f*(d*Sec[e + f*x])^(2/3)*(-Tan[e + f*x]^2)^(3/2))
```

3.290.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(d\sec(e+fx))^{2/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{13}{6},\frac{5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

input
```
Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

output
```
((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]
*Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```

3.290.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.290.4 Maple [F]

$$\int \frac{\tan^4(fx+e)}{(d\sec(fx+e))^{\frac{2}{3}}}dx$$

input
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

output
```
int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

3.290.5 Fricas [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

output
```
integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

3.290.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(2/3),x)
```

output
```
Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(2/3), x)
```

3.290.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

3.290.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(fx+e)^4}{(d\sec(fx+e))^{\frac{2}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3),x)
```

output
```
int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3), x)
```

3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

3.291.1 Optimal result

Integrand size = 25, antiderivative size = 178

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{\sqrt{b}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2bf}$$

output
```
-1/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)
/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+1/4*d^3*arctanh((b*sin(f*x+e)
)^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*si
n(f*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/b/f
```

3.291.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\cdots\right.\right.}{4f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+\cdots}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(-ArcTan[Sqrt[Tan[e + f*x]]
/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/
4)] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)^
(1/4)*Sqrt[Tan[e + f*x]])
```

3.291.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{1}{4}d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{266}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{827}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{216}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

$$\Big\downarrow \text{219}$$

$$\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(b*d^3*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
 + f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b*
f)
```

3.291. $\int(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

3.291.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.291.4 Maple [A] (verified)

Time = 17.76 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,d^2\left(\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)+\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*d^2/(cos(f*x+e)+1)/(sin(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*(arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*sin(f*x+e)+2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(144) = 288$.

Time = 0.40 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.43

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \left[-\frac{2\sqrt{-bd}d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-\cdots}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+b\cdots\right.}\right.}{\cdots}\right.$$

$$-\frac{2\sqrt{bd}d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cdots}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{\cdots}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(2*sqrt(-b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f
*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - s
qrt(-b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*
x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x
+ e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16
*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/
(f*cos(f*x + e)), -1/32*(2*sqrt(b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(
f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
 e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*c
os(f*x + e) - sqrt(b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*
cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*s
in(f*x + e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x +
e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x +
e) + 8)) - 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)...
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.291.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

3.291.8 Giac [F]

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx=\int \sqrt{b\tan\left(e+f\,x\right)}\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2), x)
```

3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

3.292.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}$$

output
```
d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))
^(1/2)/sin(f*x+e)^(1/2)+d^2*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(1/2)
```

3.292.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{d\sqrt{d\sec(e+fx)}\left(-3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{b\tan}}{3f}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-1/3*(d*Sqrt[d*Sec[e + f*x]]*(-3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e
 + f*x]^2]*(Sec[e + f*x]^2)^(1/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/f
```

3.292.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3093, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

output
```
-((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[
d*Sec[e + f*x]])
```

3.292.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.292.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.88

method	result
default	$-\frac{\csc(fx+e)\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*csc(f*x+e)*((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+
csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot
(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2-2*(-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc
(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2)
)*cos(f*x+e)^2+(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(
f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(-I*(I-cot(f*x+e)+csc(f*x
+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e
)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(
f*x+e)+2^(1/2)*cos(f*x+e)-2^(1/2))*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/
2)*d*2^(1/2)
```

3.292.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)-i\,\sqrt{-2i\,bd}d\text{weierstrassZeta}(4,0,\text{weier}}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x +
e) - I*sqrt(-2*I*b*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, co
s(f*x + e) + I*sin(f*x + e))) + I*sqrt(2*I*b*d)*d*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

3.292.6 Sympy [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan{(e+fx)}}(d\sec{(e+fx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2), x)
```

3.292.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec{(fx+e)})^{\frac{3}{2}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

3.292.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int (d\sec{(fx+e)})^{\frac{3}{2}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)}\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2), x)
```

3.293 $\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$

3.293.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{\sqrt{b}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

output
```
-d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*
sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1
/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1
/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}\sqrt{\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
((-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e
+ f*x]]/(Sec[e + f*x]^2)^(1/4)])*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]
)/(f*(Sec[e + f*x]^2)^(1/4)*Sqrt[Tan[e + f*x]])
```

3.293.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d\sqrt{b\tan(e+fx)}\int \frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\int \frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2bd\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*b*d*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
+ f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*
Sin[e + f*x]])
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.293.4 Maple [A] (verified)

Time = 17.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\left(\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(f\right.\right.}{f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*(arctanh((sin(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))+arctan((sin(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e))))*cos(f*x+e)/(cos(f*x+e)+1)/(sin(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(108) = 216$.

Time = 0.38 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.95

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx$$

$$= \left[-\frac{2\sqrt{-bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{}{\cos}}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{}\right.$$

$$-\frac{2\sqrt{bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+}}}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*sqrt(-b*d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f
*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b
*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e))) - sqrt(-b*d)*log((b*d
*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x +
 e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x
+ e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(
f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f, -1/8*(2*sqrt(b*d)*arctan(1/4*(cos(f
*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f
*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*
sin(f*x + e))) - sqrt(b*d)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

3.293.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan{(e+fx)}}\sqrt{d\sec{(e+fx)}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x)), x)
```

3.293.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec{(fx+e)}}\sqrt{b\tan{(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

3.293.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx = \int \sqrt{b\tan\left(e+f\,x\right)}\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2), x)
```

3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$

3.294.1 Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^
(1/2)/sin(f*x+e)^(1/2)
```

3.294.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{3/2}}{3bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4
)*(b*Tan[e + f*x])^(3/2))/(3*b*f*Sqrt[d*Sec[e + f*x]])
```

3.294.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e
+ f*x]]*Sqrt[Sin[e + f*x]])
```

3.294.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.294.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.74 (sec) , antiderivative size = 342, normalized size of antiderivative = 6.22

method	result
default	$-\frac{\sqrt{-\frac{b(\csc(fx+e)-\cot(fx+e))}{\left(\csc^2(fx+e)\right)(1-\cos(fx+e))^2-1}}\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{2}\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\sqrt{i(\csc(fx+e)-\cot(fx}\right.}{}$
risch	$-\frac{i\sqrt{2}\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}+\frac{i\left(\frac{2i\left(-ibd\,e^{2i(fx+e)}+idb\right)}{bd\sqrt{e^{i(fx+e)}\left(-ibd\,e^{2i(fx+e)}+idb\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\sqrt{-2\,e^{i(fx+e)}+2}\sqrt{-e^{i(fx+e)}}\left(-2E\left(\sqrt{e^{i(fx+e)}+1},\right.\right.}{\sqrt{-ibd\,e^{3i(fx+e)}+idb\,e^{i(fx+e)}}}\right.}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\left(e^{2i(f\right.}$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(1/2)*
(2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)
+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2
)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*E
llipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2*csc(f*x+e)^2*
(1-cos(f*x+e))^2)/(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-c
os(f*x+e))^2-1))^(1/2)/(1-cos(f*x+e))*sin(f*x+e)*2^(1/2)
```

3.294.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{i\sqrt{-2i\,bd}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))-i\sqrt{2i\,bd}\text{weierstr}}{df}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x
+ e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

3.294.6 Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan{(e+fx)}}}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

3.294.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

3.294.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2), x)
```

3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$

3.295.1 Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

output
```
2/3*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(3/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

3.295.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3085}$$

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

3.295.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.295.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\sin(fx+e)\sqrt{b\tan(fx+e)}}{3fd\sqrt{d\sec(fx+e)}}$	35

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)/d/(d*sec(f*x+e))^(1/2)
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{3\,d^2f}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*si
n(f*x + e)/(d^2*f)
```

3.295.6 Sympy [A] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \begin{cases} \frac{2\sqrt{b\tan(e+fx)}\tan(e+fx)}{3f(d\sec(e+fx))^{\frac{3}{2}}} & \text{for } f\neq 0 \\ \frac{x\sqrt{b\tan(e)}}{(d\sec(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Piecewise((2*sqrt(b*tan(e + f*x))*tan(e + f*x)/(3*f*(d*sec(e + f*x))**(3/2
)), Ne(f, 0)), (x*sqrt(b*tan(e))/(d*sec(e))**(3/2), True))
```

3.295.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

3.295.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sin(2e+2fx)\sqrt{\frac{d}{\cos(e+fx)}}\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{3d^2f}$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
(sin(2*e + 2*f*x)*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e +
2*f*x) + 1))^(1/2))/(3*d^2*f)
```

3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$

3.296.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

output
```
-4/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*
x+e))^(1/2)/sin(f*x+e)^(1/2)+2/5*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(
5/2)
```

3.296.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\right)(b\tan(e+}{15bf(d\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(5/4))*(b*Tan[e + f*x])^(3/2))/(15*b*f*(d*Sec[e + f*x])^(5/2))
```

3.296.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3092}$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*S
ec[e + f*x])^(5/2))
```

3.296.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.296.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.78

method	result
default	$-\frac{\csc(fx+e)\left(-2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx\right.\right.}{}$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/f*csc(f*x+e)*(-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)
-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-
cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+4*(-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-cs
c(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2
))*cos(f*x+e)+2^(1/2)*cos(f*x+e)^3-2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*
(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*El
lipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+4*(-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e
)-csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^
(1/2))+2^(1/2)*cos(f*x+e)-2*2^(1/2))*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(
1/2)/d^2*2^(1/2)
```

3.296.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e)+i\sqrt{-2i\,bd}\,\mathrm{weierstrassZeta}(4\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
*sin(f*x + e) + I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^3*f)
```

3.296.6 Sympy [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan{(e+fx)}}}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

3.296.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan{(fx+e)}}}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

3.296.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\tan{(fx+e)}}}{(d\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2), x)
```

3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$

3.297.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}+\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}$$

output
```
2/7*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(7/2)+8/21*(b*tan(f*x+e))^(3/2
)/b/d^2/f/(d*sec(f*x+e))^(3/2)
```

3.297.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx=\frac{(19\sin(e+fx)+3\sin(3(e+fx)))\sqrt{b\tan(e+fx)}}{42d^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

output
```
((19*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(42*d^3*f*Sq
rt[d*Sec[e + f*x]])
```

3.297.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}dx$$
$$\Big\downarrow \text{3092}$$
$$\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{7d^2}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$
$$\Big\downarrow \text{3085}$$
$$\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e +
f*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

3.297.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

3.297.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{2 \sin(fx+e)\sqrt{b\tan(fx+e)}\,\left(3(\cos^2(fx+e))+4\right)}{21 f\sqrt{d\sec(fx+e)}\,d^3}$	47

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)*(3*cos(f*x+e)^2+4)/(d*sec(f*x+e))^(
1/2)/d^3
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^3+4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)}{21\,d^4 f}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
2/21*(3*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.297.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

3.297.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\tan\left(fx+e\right)}}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(22\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{84\,d^4\,f}$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
((d/cos(e + f*x))^(1/2)*(22*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(84*d^4*f)
```

3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$

3.298.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}$$

output
```
-8/15*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipt
icE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f
*x+e))^(1/2)/sin(f*x+e)^(1/2)+2/9*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^
(9/2)+4/15*(b*tan(f*x+e))^(3/2)/b/d^2/f/(d*sec(f*x+e))^(5/2)
```

3.298.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\left(17+5\cos(2(e+fx))+8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+\right.}{45d^3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

output
```
((17 + 5*Cos[2*(e + f*x)] + 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*
x]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]])/(45*d^3*f
*(d*Sec[e + f*x])^(3/2))
```

3.298.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3092, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \ 3092$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3092$$

$$\frac{2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \ 3096$$

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3121

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3119

$$\frac{2\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

input
```
Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Sec[e + f*x])^(9/2)) + (2*((4*Ellipti
cE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*
x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e + f*x
])^(5/2))))/(3*d^2)
```

3.298. $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$

3.298.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.298.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.55

method	result
default	$-\frac{\csc(fx+e)\left(5\left(\cos^5(fx+e)\right)\sqrt{2}-12\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\right.}{}$

input
```
int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45/f*csc(f*x+e)*(5*cos(f*x+e)^5*2^(1/2)-12*(I*(-I+cot(f*x+e)-csc(f*x+e)
))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))
^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x
+e)+24*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))
^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(
f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos(f*x+e)^3-12*(I*(-I+cot(
f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*
x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2
*2^(1/2))+24*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+
e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e
)-csc(f*x+e)))^(1/2),1/2*2^(1/2))+6*2^(1/2)*cos(f*x+e)-12*2^(1/2))*(b*tan(
f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d^4*2^(1/2)
```

3.298.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2\left(\left(5\,\cos\left(fx+e\right)^4+6\,\cos\left(fx+e\right)^2\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)+6i\,\sqrt{-}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
2/45*((5*cos(f*x + e)^4 + 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e) + 6*I*sqrt(-2*I*b*d)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqr
t(2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) -
I*sin(f*x + e))))/(d^5*f)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.298.7 Maxima [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

3.298.8 Giac [F]

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan(fx+e)}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\tan(e+f\,x)}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2), x)
```

3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

3.299.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - \frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f}$$

output
```
1/6*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+
e)^(1/2)/f/(b*tan(f*x+e))^(1/2)+1/3*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^
(1/2)/f-1/6*b*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

3.299.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.62 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b(d\sec(e+fx))^{5/2}\left(-2+\cos^2(e+fx)+\cos^4(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(\right.}{6f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-1/6*(b*(d*Sec[e + f*x])^(5/2)*(-2 + Cos[e + f*x]^2 + Cos[e + f*x]^4*Hyper
geometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[
b*Tan[e + f*x]])/f
```

3.299.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3096}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3121}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}{3f}-\frac{1}{6}b^2\left(\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}\right)$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f) - (b^2*((d^2*Ellipti
cF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(f*Sqrt
[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b*f))
)/6
```

3.299. $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

3.299.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.299. $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

3.299.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.25

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,b\,d^2\left(i(\cos^2(fx+e))\sin(fx+e)\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,F\left(\sqrt{-i(i\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b*d^2/(cos(f*x+e)-1)/(cos
(f*x+e)+1)*(I*cos(f*x+e)^2*sin(f*x+e)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)
*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e))
)^(1/2),1/2*2^(1/2))*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)+I*cos(f*x+e)*sin
(f*x+e)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(
1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*(-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2)+sin(f*x+e)^2*2^(1/2)-2*tan(f*x+e)^2*2^(1/2))*2
^(1/2)
```

3.299.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{-2i\,bd}bd^2\cos(fx+e)^2\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2i\,bd}bd^2\cos(fx+e)}{12\,f\,\text{co}}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(sqrt(-2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPInverse(4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrass
PInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(b*d^2*cos(f*x + e)^2 -
2*b*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*
x + e)^2)
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.299.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

3.299.8 Giac [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2), x)
```

3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

3.300.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} - \frac{b^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f}$$

output
```
-1/4*b^(3/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(
b*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)-1/4*b^(3/2)*d*arctanh((b*sin(f*
x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/(b*tan(f*
x+e))^(1/2)+1/2*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f
```

3.300.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{(d\sec(e+fx))^{3/2}\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right.\right.}{4f\sec^2(e+fx)^{3/4}\tan^{\frac{3}{2}}(e}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
((d*Sec[e + f*x])^(3/2)*(-ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)
] - ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e + f*x]^2
)^(3/4)*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(3/2))/(4*f*(Sec[e + f*x]^2)^
(3/4)*Tan[e + f*x]^(3/2))
```

3.300.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3091, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{1}{4}b^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{1}{4}b^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \; 3096$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^2d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^2d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 3044$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 266$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 756$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{2f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 216$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}-\frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}$$

3.300. $\int(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

$$\Big\downarrow 219$$

$$\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f} - \frac{b^3d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-1/2*(b^3*d*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Si
n[e + f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(f*Sqr
t[b*Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

3.300.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.300.4 Maple [A] (verified)

Time = 20.76 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
default	$\frac{\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,bd\left(\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)-\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/f*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2)*b*d/(cos(f*x+e)+1)/(sin(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*(arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(
cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)-arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)+2*sec(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(135) = 270$.

Time = 0.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 4.55

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \left[\frac{2\sqrt{-bd}bd\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cdots}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd\cdots\right.}\right.}{\cdots}\right.$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(2*sqrt(-b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*
x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + sq
rt(-b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
- 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8
*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*
b*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e
)), -1/32*(2*sqrt(b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 +
 (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(
f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) -
sqrt(b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
 + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x +
 e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*
b*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x ...
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.300.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

3.300.8 Giac [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2), x)
```

3.301 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx$

3.301.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{f}$$

output
```
b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)
/f/(b*tan(f*x+e))^(1/2)+b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

3.301.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = -\frac{b\sqrt{d\sec(e+fx)}\left(-1+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\tan(e+fx)}}{f}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-((b*Sqrt[d*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e +
f*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/f)
```

3.301.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3091, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{b\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

output
```
-((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e +
f*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e +
f*x]])/f
```

3.301.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.301.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.09

method	result
default	$\frac{\sin(fx+e)\left(i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\cdots\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*sin(f*x+e)*(I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-c
sc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2+I*(-I*(I-cot(f*x+e)+
csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-cs
c(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2
))*cos(f*x+e)-2^(1/2)*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*(d*sec(f*x+e))^(1/2
)*b/(cos(f*x+e)-1)/(cos(f*x+e)+1)*2^(1/2)
```

3.301.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx =$$
$$-\frac{\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2i\,bd}b\text{weierstrassPInverse}(4,0,\cos(\cdots}{2\,f}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)) - 2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/f
```

3.301.6 Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+fx))^{\frac{3}{2}}\sqrt{d\sec(e+fx)}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x)), x)
```

3.301.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

3.301.8 Giac [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}\,dx = \int (b\tan(e+f\,x))^{3/2}\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2), x)
```

3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

3.302.1 Optimal result

Integrand size = 25, antiderivative size = 167

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{b^{3/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}} + \frac{b^{3/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

output
```
-2*d*csc(f*x+e)*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)+b^(3/2)*d*arct
an((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/
2)/(b*sin(f*x+e))^(3/2)+b^(3/2)*d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b
*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)/(b*sin(f*x+e))^(3/2)
```

3.302.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.70 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}(b\tan(e+fx))^{5/2}}{5bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(2*Hypergeometric2F1[5/4, 5/4, 9/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/4
)*(b*Tan[e + f*x])^(5/2))/(5*b*f*Sqrt[d*Sec[e + f*x]])
```

3.302.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3096, 3042, 3044, 27, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \sec(e+fx)(b\sin(e+fx))^{3/2}dx}{(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{3/2}}{\cos(e+fx)}dx}{(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d(b\tan(e+fx))^{3/2}\int \frac{b^2(b\sin(e+fx))^{3/2}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\int \frac{(b\sin(e+fx))^{3/2}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(b^2\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{bd(b\tan(e+fx))^{3/2}\left(2b^2\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sin(e+fx)}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(b*d*(2*b^2*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Si
n[e + f*x]]/(2*b^(3/2))) - 2*Sqrt[b*Sin[e + f*x]])*(b*Tan[e + f*x])^(3/2))
/(f*(d*Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2))
```

3.302.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.302. $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

3.302.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(139) = 278.

Time = 19.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.75

method	result
default	$-\frac{\sin(fx+e)\left(\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\cos(fx+e)-\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\arctan\left(\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*sin(f*x+e)*(arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+c
sc(f*x+e)))*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-(sin(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e
)+csc(f*x+e)))*cos(f*x+e)+(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh((sin
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-(sin(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+
csc(f*x+e)))-2*sin(f*x+e))*(b*tan(f*x+e))^(1/2)*b/(cos(f*x+e)-1)/(d*sec(f*
x+e))^(1/2)/(cos(f*x+e)+1)
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(139) = 278.

Time = 0.55 (sec) , antiderivative size = 741, normalized size of antiderivative = 4.44

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \left[-\frac{2\,bd\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\,\cos(fx+e)^2-\left(\cos(fx+e)^2+6\,\cos(fx+e)+4\right)\sin(fx+e)-2\,\cos\cdots\right.}{4\left(b\cos(fx+e)^2-(b\cos(fx+e)+b)\sin(fx+\cdots\right.}\right.}{\cdots}\right.$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*b*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x
+ e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - b*d*sqrt(-b/d)*log((b*c
os(f*x + e)^4 - 72*b*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^
3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos
(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x + e)^2 - 2*b)*s
in(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2
 - 2)*sin(f*x + e) + 8)) + 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/c
os(f*x + e))*cos(f*x + e))/(d*f), 1/8*(2*b*d*sqrt(b/d)*arctan(1/4*(cos(f*x
 + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x
 + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sq
rt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) -
 b)) + b*d*sqrt(b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*co
s(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x
+ e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 2
8*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*
x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f)]
```

3.302.6 Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(e+fx)})^{\frac{3}{2}}}{\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)/sqrt(d*sec(e + f*x)), x)
```

3.302.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{\sqrt{d\sec{(fx+e)}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

3.302.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2), x)
```

3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

3.303.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

output
```
-2/3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^
(1/2)/d^2/f/(b*tan(f*x+e))^(1/2)-2/3*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e
))^(3/2)
```

3.303.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2b\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b*(-1 + Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*f*(d*Sec[e + f*x])^(3/2))
```

3.303.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3090, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e +
f*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d
*Sec[e + f*x])^(3/2))
```

3.303.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.303. $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.303.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.83

method	result
default	$\frac{\sin(fx+e)\left(-i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)\right.\right.}{}$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*sin(f*x+e)*(-I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-
csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-I*(-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc
(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2)
)+2^(1/2)*cos(f*x+e)*sin(f*x+e))*b*(b*tan(f*x+e))^(1/2)/(cos(f*x+e)-1)/(d*
sec(f*x+e))^(1/2)/d/(cos(f*x+e)+1)*2^(1/2)
```

3.303.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\sqrt{-2i\,bd}b\,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))}{3\,d^2f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
 e)^2 - sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*
x + e)) - sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f
*x + e)))/(d^2*f)
```

3.303.6 Sympy [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(e+fx)})^{\frac{3}{2}}}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*tan(e + f*x))**(3/2)/(d*sec(e + f*x))**(3/2), x)
```

3.303.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

3.303.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)})^{\frac{3}{2}}}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2), x)
```

3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$

3.304.1 Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

output
```
2/5*(b*tan(f*x+e))^(5/2)/b/f/(d*sec(f*x+e))^(5/2)
```

3.304.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2b\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

3.304.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3085}$$

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

3.304.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.304.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2(\sin^2(fx+e))\sqrt{b\tan(fx+e)}\,b}{5f\,d^2\sqrt{d\sec(fx+e)}}$	38

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*sin(f*x+e)^2*(b*tan(f*x+e))^(1/2)*b/d^2/(d*sec(f*x+e))^(1/2)
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(b\cos\left(fx+e\right)^3 - b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{5\,d^3f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(b*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))/(d^3*f)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 45.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \begin{cases} \frac{2(b\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)}{5f(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f \neq 0 \\ \frac{x(b\tan(e))^{\frac{3}{2}}}{(d\sec(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Piecewise((2*(b*tan(e + f*x))**(3/2)*tan(e + f*x)/(5*f*(d*sec(e + f*x))**(
5/2)), Ne(f, 0)), (x*(b*tan(e))**(3/2)/(d*sec(e))**(5/2), True))
```

3.304.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

3.304.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

3.304.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,(\cos(e+f\,x)-\cos(3\,e+3\,f\,x))\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{10\,d^3\,f}$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
(b*(d/cos(e + f*x))^(1/2)*(cos(e + f*x) - cos(3*e + 3*f*x))*((b*sin(2*e +
2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*d^3*f)
```

3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

3.305.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{4b^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

output
```
-4/21*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)
^(1/2)/d^4/f/(b*tan(f*x+e))^(1/2)-2/7*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+
e))^(7/2)+2/21*b*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+e))^(3/2)
```

3.305.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.62

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{b\left(1+3\cos(2(e+fx))-4\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
-1/21*(b*(1 + 3*Cos[2*(e + f*x)] - 4*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan
[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(d^2*f*(d*Sec[e
 + f*x])^(3/2))
```

3.305.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3090, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3096$$

$$\frac{b^2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3121$$

$$\frac{b^2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{b^2\left(\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{7d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(-2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*Sec[e + f*x])^(7/2)) + (b^2*((4*Ellipt
icF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*d^2
*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x]
)^(3/2))))/(7*d^2)
```

3.305. $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

3.305.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.305. $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

3.305.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.23

method	result
default	$\frac{\sin(fx+e)\left(-2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e}\right.\right.}{}$

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/f*sin(f*x+e)*(-2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+
e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(
I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+3*2^(1/2)*cos(f*x+
e)^3*sin(f*x+e)-2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-c
sc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)*sin(f*x+e))*(b
*tan(f*x+e))^(1/2)*b/(cos(f*x+e)-1)/d^3/(d*sec(f*x+e))^(1/2)/(cos(f*x+e)+1
)*2^(1/2)
```

3.305.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.89

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2\left(\sqrt{-2i\,bd}b\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}b\text{we}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)) - (3*b*cos(f*x + e)^4 - b*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f
*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.305.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

3.305.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2), x)
```

3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$

3.306.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}} + \frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

output
```
-2/9*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(9/2)+2/45*b*(b*tan(f*x+e))^(
1/2)/d^2/f/(d*sec(f*x+e))^(5/2)+8/45*b*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f
*x+e))^(1/2)
```

3.306.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.53

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{b(13+5\cos(2(e+fx)))\sin^2(e+fx)\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(b*(13 + 5*Cos[2*(e + f*x)])*Sin[e + f*x]^2*Sqrt[b*Tan[e + f*x]])/(45*d^4*
f*Sqrt[d*Sec[e + f*x]])
```

3.306.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3090, 3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3085}$$

$$\frac{b^2\left(\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{9d^2}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (b^2*((2*Sqrt[b
*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e + f*x]])/
(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])))/(9*d^2)
```

3.306.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

3.306.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2\left(\sin^{2}\left(fx+e\right)\right) b\sqrt{b\tan\left(fx+e\right)}\,\left(5\left(\cos^{2}\left(fx+e\right)\right)+4\right)}{45f\sqrt{d\sec\left(fx+e\right)}\,d^{4}}$	50

input
```
int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f*sin(f*x+e)^2*b*(b*tan(f*x+e))^(1/2)*(5*cos(f*x+e)^2+4)/(d*sec(f*x+e
))^(1/2)/d^4
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2\left(5\,b\cos\left(fx+e\right)^{5}-b\cos\left(fx+e\right)^{3}-4\,b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{45\,d^{5}f}$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-2/45*(5*b*cos(f*x + e)^5 - b*cos(f*x + e)^3 - 4*b*cos(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(d^5*f)
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.306.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

3.306.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{3}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

3.306.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{b\sqrt{\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}\,(21\,\cos(3\,e+3\,f\,x)-26\,\cos(e+f\,x)+5\,\cos(5\,e+5\,f\,x))}{360\,d^5\,f}$$

input
```
int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
-(b*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(21*cos(3*e + 3*f*x) - 26*cos(e + f*x) + 5*cos(5*e + 5*f*x)))/(360*d^
5*f)
```

3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

3.307.1 Optimal result

Integrand size = 25, antiderivative size = 208

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3bd^2\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f}$$

output
```
3/32*b^(5/2)*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)
/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/32*b^(5/2)*d^3*arctanh((b*s
in(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*s
in(f*x+e))^(1/2)+1/4*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2)/f-3/16*b*
d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

3.307.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{d^4(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\sec^2(e+fx)^{3/4} - 3\mathrm{arctan}\cdots\right.}{32f(d\sec\cdots}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(d^4*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^
(1/4)]*(Sec[e + f*x]^2)^(3/4) - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]
^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*Sec[e + f*x]^2*(-3 + 4*Sec[e + f*x]^
2)*Tan[e + f*x]^(3/2)))/(32*f*(d*Sec[e + f*x])^(3/2)*Tan[e + f*x]^(5/2))
```

3.307.3 Rubi [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.82, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \; 3091$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f} - \frac{3}{8}b^2\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f} - \frac{3}{8}b^2\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \; 3093$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{3044}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{d^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{266}$$

3.307. $\int(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+d^2(b\tan(e+\right.$$

$$\Big\downarrow \text{216}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\,\mathrm{se}}}{2bf}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}-\frac{3}{8}b^2\left(\frac{bd^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}\right)$$

input
```
Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f) - (3*b^2*((b*d^3*(
-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e + f*x]]/
(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e
+ f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*b*f)))/8
```

3.307. $\int(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

3.307.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.307.4 Maple [A] (verified)

Time = 169.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.44

method	result
default	$\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2d^2\left(3\cos(fx+e)\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)(\sin^2(fx+e))+3\cos(fx+e)\,a\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*b^2*d^2/(cos(f*x+e)-1)/(c
os(f*x+e)+1)^2/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(3*cos(f*x+e)*arctanh((
sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2+3
*cos(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x
+e)))*sin(f*x+e)^2+6*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)^3+6*si
n(f*x+e)^2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*sin(f*x+e)*tan
(f*x+e)^2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-8*tan(f*x+e)^3*(sin(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2))
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(168) = 336$.

Time = 0.45 (sec) , antiderivative size = 852, normalized size of antiderivative = 4.10

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/256*(6*sqrt(-b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2
 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*c
os(f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)
^3 + 3*sqrt(-b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*
cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*s
in(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x +
 e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x +
 e) + 8)) - 16*(3*b^2*d^2*cos(f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3), 1/256
*(6*sqrt(b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos
(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*s
qrt(b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x +
 e)^2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x +
e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/co
s(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(
f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8...
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.307.7 Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

3.307.8 Giac [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2), x)
```

3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

3.308.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}$$

output
```
-1/2*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*se
c(f*x+e))^(1/2)/sin(f*x+e)^(1/2)+1/3*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))
^(3/2)/f-1/2*b*d^2*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

3.308.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{bd^2\left(-3+2\sec^2(e+fx)+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\cdots}\right.}{6f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*d^2*(-3 + 2*Sec[e + f*x]^2 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
f*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(6*f*Sqrt[d*Sec[e
+ f*x]])
```

3.308.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3091, 3042, 3093, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{1}{2}d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx\right)$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3121}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}-\frac{1}{2}b^2\left(\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}\right)$$

input
```
Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*f) - (b^2*(-((d^2*Ell
ipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]
]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*Sec[e +
f*x]])))/2
```

3.308. $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

3.308.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.308.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.72

method	result
default	$\frac{(\sec^2(fx+e))\csc(fx+e)\left(-6\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\sqrt{i(\csc(fx+e)-\cot(fx+e))}E\left(\sqrt{-i(\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f*sec(f*x+e)^2*csc(f*x+e)*(-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-
I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellip
ticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^4+3*(-I*
(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(
csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/
2),1/2*2^(1/2))*cos(f*x+e)^4-6*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(c
ot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE
((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)^3+3*(-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(
f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1
/2*2^(1/2))*cos(f*x+e)^3+3*2^(1/2)*cos(f*x+e)^3-5*2^(1/2)*cos(f*x+e)^2+2*2
^(1/2))*d*(d*sec(f*x+e))^(1/2)*b^2*(b*tan(f*x+e))^(1/2)*2^(1/2)
```

3.308.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \frac{3i\sqrt{-2i\,bd}b^2d\cos(fx+e)^2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\mathrm{co}}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*I*sqrt(-2*I*b*d)*b^2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2*I*b*d)*b^
2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x
 + e) - I*sin(f*x + e))) - 2*(3*b^2*d*cos(f*x + e)^2 - 2*b^2*d)*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)
^2)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.308.7 Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

3.308.8 Giac [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2), x)
```

3.309 $\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$

3.309.1 Optimal result

Integrand size = 25, antiderivative size = 169

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{3b^{5/2}d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}{2f}$$

output
```
3/4*b^(5/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/
(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/4*b^(5/2)*d*arctanh((b*sin(f*x
+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x
+e))^(1/2)+1/2*b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

3.309.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \frac{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) - 3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right.\right.}{4f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/
(Sec[e + f*x]^2)^(1/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1
/4)] + 2*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(3/2)))/(4*f*(Sec[e + f*x]^2)
^(1/4)*Tan[e + f*x]^(5/2))
```

3.309.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3091, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3}{4}b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3}{4}b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^2d\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^2d\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{4\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3bd\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{4f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}-\frac{3b^3d\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{2f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-3*b^3*d*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin
[e + f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d*Sec[e + f*x]]*Sq
rt[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f
)
```

3.309. $\int\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx$

3.309.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.309.4 Maple [A] (verified)

Time = 12.08 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2\left(2\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\sin^3(fx+e))-3\cos(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e}{4f(\cos(fx+e)-1}$

input
```
int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)*b^2/(cos(f*x+e)-1)/(cos(f
*x+e)+1)^2/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(2*(sin(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*sin(f*x+e)^3-3*cos(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2-3*cos(f*x+e)*arctanh((sin(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*sin(f*x+e)^2+2*sin(f*
x+e)^2*tan(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(135) = 270$.

Time = 0.39 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.66

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \left[\frac{6\sqrt{-bd}b^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right.}{}\right.$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/32*(6*sqrt(-b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqr
t(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*
x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + 3*
sqrt(-b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^
2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f
*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x
 + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 1
6*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))
/(f*cos(f*x + e)), 1/32*(6*sqrt(b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*co
s(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(
f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
 e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*c
os(f*x + e) + 3*sqrt(b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*
d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))
*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*sqrt(d/cos(f*x + e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x
+ e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x
+ e) + 8)) + 16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + ...
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{d \sec(e+fx)}(b \tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.309.7 Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \int \sqrt{d\sec(fx+e)}(b\tan(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

3.309.8 Giac [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}\,dx = \int (b\tan(e+f\,x))^{5/2}\sqrt{\frac{d}{\cos(e+f\,x)}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2), x)
```

3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$

3.310.1 Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx=-\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

output
```
3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipt
icE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e
))^(1/2)/sin(f*x+e)^(1/2)+b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

3.310.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{b\left(-1+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
-((b*(-1 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[e + f*x]]))
```

3.310.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3091, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3091}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3}{2}b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3}{2}b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

output
```
(-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Se
c[e + f*x]])
```

3.310.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.310.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 471, normalized size of antiderivative = 5.35

method	result
default	$\frac{b^2\sqrt{b\tan(fx+e)}\left(6\cot(fx+e)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\left(\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*b^2*(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)*(6*cot(f*x+e)*(-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot
(f*x+e)-csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),
1/2*2^(1/2))-3*cot(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*
x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I
*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+6*csc(f*x+e)*(-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e
)-csc(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^
(1/2))-3*csc(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-c
sc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cot(f*x+e)-3*csc(f*x+e)
*2^(1/2)+sec(f*x+e)*csc(f*x+e)*2^(1/2))*2^(1/2)
```

3.310.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin\left(fx+e\right)-3i\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weiers}\cdots}{\cdots}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x
+ e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZet
a(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.310.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

3.310.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{\sqrt{d\sec(fx+e)}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2), x)
```

3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$

3.311.1 Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

output
```
-b^(5/2)*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*
sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+b^(5/2)*arctanh((b*sin(f*x+e))^(1/2
)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1
/2)-2/3*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
```

3.311.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.83

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\sec(e+fx)}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) - 3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) + \sqrt[4]{\sec^2(e+fx)}\sin(2(e+ \ldots}{3d^2f\sqrt[4]{\sec^2(e+fx)}\tan^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*(Sqrt[d*Sec[e + f*x]]*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(
1/4)] - 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + (Sec[e + f*
x]^2)^(1/4)*Sin[2*(e + f*x)]*Sqrt[Tan[e + f*x]])*(b*Tan[e + f*x])^(5/2))/(
d^2*f*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^(5/2))
```

3.311.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3090, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\sqrt{b\tan(e+fx)}\int\frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^3\sqrt{b\tan(e+fx)}\int\frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\int\frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b^3*(-1/2*ArcTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e
+ f*x]]/(2*Sqrt[b]))*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[
b*Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2
))
```

3.311. $\int\frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}dx$

3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.311.4 Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sin(fx+e)\sqrt{b\tan(fx+e)}\left(3\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+3\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\operatorname{arctanh}(}{3f(\cos(fx+e)-1)\sqrt{d\sec(fx+e)}\,d}$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*sin(f*x+e)*(b*tan(f*x+e))^(1/2)*(3*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))+3*
(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)*(cot(f*x+e)+csc(f*x+e)))+2*cos(f*x+e)-2)*b^2/(cos(f*x+e)-1)/(d*sec(f
*x+e))^(1/2)/d
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(138) = 276$.

Time = 0.59 (sec) , antiderivative size = 766, normalized size of antiderivative = 4.56

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \left[-\frac{16\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+6\,b^2d\sqrt{-\frac{b}{d}}\arctan\left(\frac{(\mathrm{co}}{}\right.}{}\right.$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*(16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(
f*x + e)*sin(f*x + e) + 6*b^2*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*
cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*co
s(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*
x + e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^
2*d*sqrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x
+ e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*
cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e
)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(d^2*f), -1/24*(16*b^2*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) + 6*b^2*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (
cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x
+ e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*sqrt(b/d)*log((
b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x +
e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)
*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)
^2 - 2)*sin(f*x + e) + 8)))/(d^2*f)]
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.311.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

3.311.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2), x)
```

3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$

3.312.1 Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=\frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

output
```
-6/5*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*se
c(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/5*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e
))^(5/2)
```

3.312.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx=$$
$$-\frac{b\left(1+\cos(2(e+fx))-2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)(b\tan(e+fx))^{3/}}{5d^2f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
-1/5*(b*(1 + Cos[2*(e + f*x)] - 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e
+ f*x]^2]*(Sec[e + f*x]^2)^(1/4))*(b*Tan[e + f*x])^(3/2))/(d^2*f*Sqrt[d*Se
c[e + f*x]])
```

3.312.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3090, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{3b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

output
```
(6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*
(d*Sec[e + f*x])^(5/2))
```

3.312.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.312. $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.312.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.77

method	result
default	$\frac{\csc(fx+e)b^2\sqrt{b\tan(fx+e)}\left(-6\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(\csc(fx+e)-\cot(fx+e))}E\left(\sqrt{\cdot}\right.\right.}{}$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*csc(f*x+e)*b^2*(b*tan(f*x+e))^(1/2)*(-6*(-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^
(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+
e)+3*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(
1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x
+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos(f*x+e)^3-6*(-I*(I-cot(f*x+
e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-
cot(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1
/2))+3*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))
^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f
*x+e)))^(1/2),1/2*2^(1/2))-4*2^(1/2)*cos(f*x+e)+3*2^(1/2))/(d*sec(f*x+e))^
(1/2)/d^2*2^(1/2)
```

3.312.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)-3i\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weierstrassPInve}}{}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x
 + e)^2*sin(f*x + e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^
2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e))))/(d^3*f)
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.312.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

3.312.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2), x)
```

3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$

3.313.1 Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

output
```
2/7*(b*tan(f*x+e))^(7/2)/b/f/(d*sec(f*x+e))^(7/2)
```

3.313.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2b^2\sin^3(e+fx)\sqrt{b\tan(e+fx)}}{7d^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*b^2*Sin[e + f*x]^3*Sqrt[b*Tan[e + f*x]])/(7*d^3*f*Sqrt[d*Sec[e + f*x]])
```

3.313.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3085}$$

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

output
```
(2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

3.313.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.313.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{2(\sin^3(fx+e))\sqrt{b\tan(fx+e)}\,b^2}{7f\,d^3\sqrt{d\sec(fx+e)}}$	40

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/f*sin(f*x+e)^3*(b*tan(f*x+e))^(1/2)*b^2/d^3/(d*sec(f*x+e))^(1/2)
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2\left(b^2\cos{(fx+e)}^3-b^2\cos{(fx+e)}\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin{(fx+e)}}{7\,d^4f}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
-2/7*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.313.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

3.313.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{b^2\sqrt{\frac{d}{\cos(e+f\,x)}}\,(2\,\sin(2\,e+2\,f\,x)-\sin(4\,e+4\,f\,x))\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}{28\,d^4\,f}$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(7/2),x)
```

output
```
(b^2*(d/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(28*d^4*f)
```

3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

3.314.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}} + \frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}$$

output
```
-4/15*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*El
lipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*s
ec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/9*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+
e))^(9/2)+2/15*b*(b*tan(f*x+e))^(3/2)/d^2/f/(d*sec(f*x+e))^(5/2)
```

3.314.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{b^3\left(1-5\cos(2(e+fx))+4\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e\right.}{45d^4f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(b^3*(1 - 5*Cos[2*(e + f*x)] + 4*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e +
 f*x]^2]*(Sec[e + f*x]^2)^(5/4))*Sin[e + f*x]^2)/(45*d^4*f*Sqrt[d*Sec[e +
f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.314.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3090, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3090}$$

$$\frac{b^2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3092}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3121

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3042

$$\frac{b^2\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

$\Big\downarrow$ 3119

$$\frac{b^2\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3d^2}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

input
```
Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

output
```
(-2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(d*Sec[e + f*x])^(9/2)) + (b^2*((4*Elli
pticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e +
f*x])^(5/2))))/(3*d^2)
```

3.314. $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3090
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Simp[b^2*((n - 1)/(a^2*m))   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1
] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.314. $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

3.314.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.60

method	result
default	$\frac{\csc(fx+e)\left(5(\cos^5(fx+e))\sqrt{2}-12\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,E\right.}{}$

input
```
int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/45/f*csc(f*x+e)*(5*cos(f*x+e)^5*2^(1/2)-12*(I*(-I+cot(f*x+e)-csc(f*x+e))
)^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^
(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+
e)+6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(
1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*
x+e)))^(1/2),1/2*2^(1/2))*cos(f*x+e)-8*2^(1/2)*cos(f*x+e)^3-12*(I*(-I+cot(
f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*
x+e)-csc(f*x+e)))^(1/2)*EllipticE((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2
*2^(1/2))+6*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e
)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)
-csc(f*x+e)))^(1/2),1/2*2^(1/2))-3*2^(1/2)*cos(f*x+e)+6*2^(1/2))*(b*tan(f*
x+e))^(1/2)*b^2/(d*sec(f*x+e))^(1/2)/d^4*2^(1/2)
```

3.314.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx =$$
$$-\frac{2\left(-3i\sqrt{-2i\,bd}b^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))+3i\sqrt{2i\,b}\right.}{}$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-2/45*(-3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + (5*b^
2*cos(f*x + e)^4 - 3*b^2*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))*sin(f*x + e))/(d^5*f)
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(9/2),x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

3.314.8 Giac [F]

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e))^{\frac{5}{2}}}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

input
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2),x)
```

output
```
int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2), x)
```

3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.315.1 Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{3d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{3d^3\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf}$$

output
```
3/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f
*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+3/4*d^3*arctanh((b*sin(f*x+e))
^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan
(f*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/b/f
```

3.315.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.75

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2(d\sec(e+fx))^{3/2}\left(3\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+3\text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right.}{4f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+f}}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*(d*Sec[e + f*x])^(3/2)*(3*ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^
(1/4)] + 3*ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + 2*(Sec[e +
 f*x]^2)^(3/4)*Sqrt[Tan[e + f*x]])*Sqrt[Tan[e + f*x]])/(4*f*(Sec[e + f*x]^
2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

3.315.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3093, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3093}$$

$$\frac{3}{4}d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{3096}$$

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{4\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{3044}$$

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4bf\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{27}$$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{4f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{266}$$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{756}$$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{216}$$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

$$\Big\downarrow \text{219}$$

3.315. $\int\frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}dx$

$$\frac{3bd^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{2f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2bf}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(3*b*d^3*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e
 + f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(2*f*Sqrt
[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*b
*f)
```

3.315.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.315.4 Maple [A] (verified)

Time = 12.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\, d^3 \left(3\sin(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)-3\sin(fx+e)\,\mathrm{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\mathrm{cot}\right.\right.}{4f(\cos(fx+e)+1)\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\,\mathrm{t}}}$

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*(d*sec(f*x+e))^(1/2)*d^3/(cos(f*x+e)+1)/(sin(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)/(b*tan(f*x+e))^(1/2)*(3*sin(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-3*sin(f*x+e)*arctanh((sin(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-2*tan(f*x+e)*(sin(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-2*tan(f*x+e)*sec(f*x+e)*(sin(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2))
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(144) = 288$.

Time = 0.42 (sec) , antiderivative size = 782, normalized size of antiderivative = 4.39

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \left[-\frac{6\,bd^3\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\,\cos(fx+e)^2-\left(\cos(fx+e)^2+6\,\cos(fx+e)+4\right)\sin(fx+e)-2\,\mathrm{co}}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx\right.}\right.}{}\right.$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(6*b*d^3*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
 (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f
*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) - 3*b*d^3
*sqrt(-d/b)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(
7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(
f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)
) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^3*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e)), 1/32*
(6*b*d^3*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*
x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2
 + (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) + 3*b*d^3*sqrt(d/b
)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x
+ e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*c
os(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)
^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*d^3*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e))]
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.315.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.315.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2), x)
```

3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.316.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{d^2\operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}$$

output
```
-d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2
)/f/(b*tan(f*x+e))^(1/2)+d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/b/f
```

3.316.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx=\frac{d^2\sqrt{d\sec(e+fx)}\left(\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2\right.}{f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*Sqrt[d*Sec[e + f*x]]*(Cos[e + f*x]*Hypergeometric2F1[1/4, 3/4, 5/4, -
Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x] + Tan[e + f*x]))/(f*Sq
rt[b*Tan[e + f*x]])
```

3.316.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3093, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3093}$$

$$\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \ 3042$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{2\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}$$

$$\Big\downarrow \ 3120$$

$$\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*
x]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f
*x]])/(b*f)
```

3.316.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3093
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Simp[a^2*((m - 2)/(m + n - 1))   Int[(a*Sec[e +
 f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (
GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.316. $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.316.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.60

method	result
default	$\frac{\sqrt{d \sec(fx+e)}\, d^2 \left(i \sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\, \sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\, \sqrt{i(\csc(fx+e)-\cot(fx+e))}\, F\left(\sqrt{-i(i-\cot(fx\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(d*sec(f*x+e))^(1/2)*d^2/(b*tan(f*x+e))^(1/2)*(I*(-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(
f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))
*cos(f*x+e)+I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x
+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e
)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+tan(f*x+e)*2^(1/2))*2^(1/2)
```

3.316.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}d^2\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2i\,bd}d^2\text{wei}}{2\,b}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
 + e)) + sqrt(2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(
f*x + e)) + 2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/
(b*f)
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.316.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.316.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

3.317.1 Optimal result

Integrand size = 25, antiderivative size = 131

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}f\sqrt{b\tan(e+fx)}}$$

output
```
d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))
^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/
2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/
2)
```

3.317.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right) + \text{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)(d\sec(e+fx))^{3/2}\sqrt{\cdots}}{f\sec^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e +
 f*x]]/(Sec[e + f*x]^2)^(1/4)])*(d*Sec[e + f*x])^(3/2)*Sqrt[Tan[e + f*x]])
/(f*(Sec[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

3.317.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3096}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{b^2}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}(b^2-b^2\sin^2(e+fx))}d(b\sin(e+fx))}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 756$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 216$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{f\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \; 219$$

$$\frac{2bd\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{f\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*b*d*(ArcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e +
 f*x]]/(2*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(f*Sqrt[b*T
an[e + f*x]])
```

3.317.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.317. $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.317.4 Maple [A] (verified)

Time = 10.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

method	result
default	$-\frac{\sin(fx+e)\left(\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)-\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\right)\sqrt{d \sec}}{f(\cos(fx+e)+1)\sqrt{b\tan(fx+e)}\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}}$

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*sin(f*x+e)*(arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+cs
c(f*x+e)))-arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x
+e))))*(d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)/(b*tan(f*x+e))^(1/2)/(sin(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(107) = 214$.

Time = 0.39 (sec) , antiderivative size = 653, normalized size of antiderivative = 4.98

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \left[-\frac{2d\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(\ldots}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx+e\ldots}\right.}{\ldots}\right.$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*d*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos
(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x +
e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) - d*sqrt(-d/b)*log((d*cos(f
*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 -
8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f
*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2
)*sin(f*x + e) + 8)))/f, 1/8*(2*d*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5
*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*c
os(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*
x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + d*sq
rt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
+ (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x
+ e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*
(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

3.317.6 Sympy [F]

$$\int \frac{(d \sec(e+fx))^{3/2}}{\sqrt{b \tan(e+fx)}} \, dx = \int \frac{(d \sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{b \tan{(e+fx)}}} \, dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/sqrt(b*tan(e + f*x)), x)
```

3.317.7 Maxima [F]

$$\int \frac{(d \sec(e+fx))^{3/2}}{\sqrt{b \tan(e+fx)}} \, dx = \int \frac{(d \sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b \tan{(fx+e)}}} \, dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.317.8 Giac [F]

$$\int \frac{(d \sec(e+fx))^{3/2}}{\sqrt{b \tan(e+fx)}} \, dx = \int \frac{(d \sec{(fx+e)})^{\frac{3}{2}}}{\sqrt{b \tan{(fx+e)}}} \, dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2), x)
```

3.318 $\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

3.318.1 Optimal result

Integrand size = 25, antiderivative size = 55

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

output
```
-2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
f/(b*tan(f*x+e))^(1/2)
```

3.318.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\sin(e+fx)}{f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3
/4)*Sin[e + f*x])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.318.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d \sec(e+fx)}}{\sqrt{b \tan(e+fx)}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d \sec(e+fx)}}{\sqrt{b \tan(e+fx)}}dx$$

$$\Big\downarrow \text{ 3096}$$

$$\frac{\sqrt{b \sin(e+fx)}\sqrt{d \sec(e+fx)} \int \frac{1}{\sqrt{b \sin(e+fx)}}dx}{\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b \sin(e+fx)}\sqrt{d \sec(e+fx)} \int \frac{1}{\sqrt{b \sin(e+fx)}}dx}{\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d \sec(e+fx)} \int \frac{1}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d \sec(e+fx)} \int \frac{1}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b \tan(e+fx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\sin(e+fx)} \text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right) \sqrt{d \sec(e+fx)}}{f\sqrt{b \tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

output
```
(2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]
])/(f*Sqrt[b*Tan[e + f*x]])
```

3.318.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.318.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.36

method	result
default	$\frac{i(\cos(fx+e)+1)F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))},\frac{\sqrt{2}}{2}\right)\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(i-\cot}{f\sqrt{b\tan(fx+e)}}$

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/f*(cos(f*x+e)+1)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1
/2))*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/
2)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(d*sec(f*x+e))^(1/2)*2^(1/2)/(b*ta
n(f*x+e))^(1/2)
```

3.318.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{-2i\,bd}\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2i\,bd}\text{weierstrassPInverse}(4,0,\cos{(fx}}{bf}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(
b*f)
```

3.318.6 Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{\sqrt{b\tan{(e+fx)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

3.318.7 Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.318.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

3.319.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

output
```
2*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.319.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3085}$$

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

3.319.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.319.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\tan(fx+e)}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}}$	32
risch	$-\frac{i\sqrt{2}\,(\mathrm{e}^{2i(fx+e)}-1)}{\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,(\mathrm{e}^{2i(fx+e)}+1)\sqrt{-\frac{ib\left(\mathrm{e}^{2i(fx+e)}-1\right)}{\mathrm{e}^{2i(fx+e)}+1}}\,f}$	90

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f*tan(f*x+e)/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{bdf}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b*d
*f)
```

3.319.6 Sympy [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}} & \text{for } f \neq 0 \\ \frac{x}{\sqrt{b\tan(e)}\sqrt{d\sec(e)}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Piecewise((2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x))), N
e(f, 0)), (x/(sqrt(b*tan(e))*sqrt(d*sec(e))), True))
```

3.319.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.319.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\sqrt{b\tan\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\,\sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{d\,f\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
(2*sin(e + f*x)*(d/cos(e + f*x))^(1/2))/(d*f*((b*sin(2*e + 2*f*x))/(cos(2*
e + 2*f*x) + 1))^(1/2))
```

3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

3.320.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

output
```
-4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2
)/d^2/f/(b*tan(f*x+e))^(1/2)+2/3*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(
3/2)
```

3.320.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.74 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{2\left(1+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx\right.}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*(1 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]^2]*(Sec[e + f*x]^
2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

3.320.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3092}$$

$$\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]
])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec
[e + f*x])^(3/2))
```

3.320.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.320.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.52

method	result
default	$\frac{\left(2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,F\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e)}\right.\right.}{}$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2)/d*(2*I*(-I*(I-cot(f*x+e)+c
sc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(
f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))
+2*I*sec(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc
(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x
+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2)*sin(f*x+e))*2^(1/2)
```

3.320.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2+\sqrt{-2i\,bd}\text{weierstrassPIn}\right.}{}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 + sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))
 + sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))
/(b*d^2*f)
```

3.320.6 Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan{(e+fx)}}\,(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2)), x)
```

3.320.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.320.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\sqrt{b\tan{(fx+e)}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)}\,\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

3.321.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}$$

output
```
2/5*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(5/2)+8/5*(b*tan(f*x+e))^(1/2)
/b/d^2/f/(d*sec(f*x+e))^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx=\frac{(9+\cos(2(e+fx)))\sqrt{d\sec(e+fx)}\sin(e+fx)}{5d^3f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
((9 + Cos[2*(e + f*x)])*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*d^3*f*Sqrt[b
*Tan[e + f*x]])
```

3.321.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \ 3092$$

$$\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \ 3085$$

$$\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

output
```
(2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e
+ f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

3.321.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

3.321.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2\tan(fx+e)\left(\cos^2(fx+e)+4\right)}{5f\sqrt{b\tan(fx+e)}\,\sqrt{d\sec(fx+e)}\,d^2}$	45

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*tan(f*x+e)*(cos(f*x+e)^2+4)/(b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2
)/d^2
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\left(\cos\left(fx+e\right)^3+4\,\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{5\,bd^3f}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
2/5*(cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sq
rt(d/cos(f*x + e))/(b*d^3*f)
```

3.321.6 Sympy [A] (verification not implemented)

Time = 57.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \begin{cases} \frac{8\tan^3(e+fx)}{5f\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{5}{2}}} + \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f \neq 0 \\ \frac{x}{\sqrt{b\tan(e)}(d\sec(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

output
```
Piecewise((8*tan(e + f*x)**3/(5*f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(
5/2)) + 2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5/2)), N
e(f, 0)), (x/(sqrt(b*tan(e))*(d*sec(e))**(5/2)), True))
```

3.321.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.321.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\sqrt{b\tan(fx+e)}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{(17\sin(e+fx)+\sin(3e+3fx))\sqrt{\frac{d}{\cos(e+fx)}}}{10\,d^3 f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
((17*sin(e + f*x) + sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(10*d^3*f*((
b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.322.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^3\text{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

output
```
-2*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-d^3*arctan((b*sin(f*x
+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b
*sin(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e)
)^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

3.322.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.69

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{d(d\sec(e+fx))^{3/2}\left(-2\sin(e+fx) + \frac{\left(-\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\text{arctanh}\left(\frac{}{\sqrt[4]{\text{s}}}\right.\right.}{\sqrt[4]{\sec^2(e+}}\right.}{f(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(d*(d*Sec[e + f*x])^(3/2)*(-2*Sin[e + f*x] + ((-ArcTan[Sqrt[Tan[e + f*x]]/
(Sec[e + f*x]^2)^(1/4)] + ArcTanh[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4
)])*Cos[e + f*x]*Tan[e + f*x]^(3/2))/(Sec[e + f*x]^2)^(1/4)))/(f*(b*Tan[e
+ f*x])^(3/2))
```

3.322.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.75, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3042, 3088, 3042, 3096, 3042, 3044, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}dx}{b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \sec(e+fx)\sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{\cos(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{b^3f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d^3\sqrt{b\tan(e+fx)}\int \frac{\sqrt{b\sin(e+fx)}}{b^2-b^2\sin^2(e+fx)}d(b\sin(e+fx))}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d^3\sqrt{b\tan(e+fx)}\int \frac{b^2\sin^2(e+fx)}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{1}{2}\int \frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2d^3\sqrt{b\tan(e+fx)}\left(\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2\sqrt{b}}\right)}{bf\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) + (2*d^3*(-1/2*Ar
cTan[Sqrt[b]*Sin[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sin[e + f*x]]/(2*Sqrt
[b]))*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]
)
```

3.322. $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.322.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.322. $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.322.4 Maple [A] (verified)

Time = 12.63 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.40

method	result
default	$-\frac{d^2\sqrt{d\sec(fx+e)}\left(\cot(fx+e)\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)+\cot(fx+e)\operatorname{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(f\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*d^2*(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/(sin(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)/b*(cot(f*x+e)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot
(f*x+e)+csc(f*x+e)))+cot(f*x+e)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*(cot(f*x+e)+csc(f*x+e)))+2*(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-csc(f*x+e
)*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))-csc(
f*x+e)*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e))
))
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(143) = 286$.

Time = 0.41 (sec) , antiderivative size = 794, normalized size of antiderivative = 4.64

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \left[-\frac{2\,bd^2\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos\cdots\right.}{4\left(d\cos(fx+e)^2-(d\cos(fx+e)+d)\sin(fx\cdots\right.}\right.}{\cdots}\right.$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*b*d^2*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
(cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*s
qrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*
x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sq
rt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*
x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 -
4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x
+ e)), -1/8*(2*b*d^2*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)
^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) +
 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*co
s(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^
2*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e
)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(
f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2
+ 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin...
```

3.322.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.322.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

3.322.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan\left(e+f\,x\right))^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2), x)
```

3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

3.323.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*d^2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+2*d^2*(sin(1/2*e+1/4*
Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+
1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+
e)^(1/2)
```

3.323.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\left(3+\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\tan^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2*(3 + Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(1/4)*Tan[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])
```

3.323.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3088, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$-\frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*Elliptic
E[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]
*Sqrt[Sin[e + f*x]])
```

3.323.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.323. $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.323.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.76

method	result
default	$\frac{\csc(fx+e)\left(2\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{2}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e}\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*csc(f*x+e)*(2*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*
x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((-I*
(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-(-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f
*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-
csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(cs
c(f*x+e)^2*(1-cos(f*x+e))^2-1))^(3/2)*(1-cos(f*x+e))/(csc(f*x+e)^2*(1-cos(
f*x+e))^2+1)^2/(-b/(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e
)))^(3/2)*2^(1/2)
```

3.323.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}+i\sqrt{-2i\,bd}d\sin\left(fx+e\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}}{}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^
2 + I*sqrt(-2*I*b*d)*d*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*d*sin(f*x + e)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
+ e))))/(b^2*f*sin(f*x + e))
```

3.323.6 Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{(b\tan(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral((d*sec(e + f*x))**(3/2)/(b*tan(e + f*x))**(3/2), x)
```

3.323.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

3.323.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(b\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

3.324.1 Optimal result

Integrand size = 25, antiderivative size = 32

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

output
```
-2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

3.324.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

3.324.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3085}$$

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

3.324.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.324.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{2\sqrt{d\sec(fx+e)}}{bf\sqrt{b\tan(fx+e)}}$	29

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{b^2f\sin(fx+e)}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b^
2*f*sin(f*x + e))
```

3.324.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{2\sqrt{d\sec(e+fx)}\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}} & \text{for } f\neq 0 \\ \frac{x\sqrt{d\sec(e)}}{(b\tan(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Piecewise((-2*sqrt(d*sec(e + f*x))*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)
), Ne(f, 0)), (x*sqrt(d*sec(e))/(b*tan(e))**(3/2), True))
```

3.324.7 Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.324.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d}{\cos(e+f\,x)}}}{b\,f\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
```

output
```
-(2*(d/cos(e + f*x))^(1/2))/(b*f*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) +
 1))^(1/2))
```

3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

3.325.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+4*(sin(1/2*e+1/4*Pi+1/2*f
*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x)
,2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

3.325.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\left(3+2\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/4}\sin^2(e+fx)\right)}{3bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-2*(3 + 2*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]^2]*(Sec[e + f*x]
^2)^(5/4)*Sin[e + f*x]^2))/(3*b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

3.325.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3089, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{b\sin(e+fx)}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int \sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\downarrow \text{3042}$$

$$-\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{b^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$\downarrow \text{3119}$$

$$-\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/
2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[
e + f*x]])
```

3.325.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.325. $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.325.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.01

method	result
default	$\frac{\csc(fx+e)(1-\cos(fx+e))\left(4\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{2}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{\cdot}\right.\right.}{f\left(-\frac{b(\csc(}{\left(\csc^2(fx+\right.}\right.}$

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*csc(f*x+e)*(1-cos(f*x+e))*(4*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*2^(1
/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)
*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))-2*(-I*(I-cot(
f*x+e)+csc(f*x+e)))^(1/2)*2^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(I*
(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1
/2),1/2*2^(1/2))-3*csc(f*x+e)^2*(1-cos(f*x+e))^2-1)/(-b/(csc(f*x+e)^2*(1-c
os(f*x+e))^2-1)*(csc(f*x+e)-cot(f*x+e)))^(3/2)/(csc(f*x+e)^2*(1-cos(f*x+e)
)^2-1)^2/(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-cos(f*x+e)
)^2-1))^(1/2)*2^(1/2)
```

3.325.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2+i\sqrt{-2i\,bd}\sin(fx+e)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(\right.}{}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-2*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
+ I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*sin(f*x + e)*weier
strassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))
)/(b^2*d*f*sin(f*x + e))
```

3.325.6 Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan{(e+fx)})^{\frac{3}{2}}\sqrt{d\sec{(e+fx)}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Integral(1/((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x))), x)
```

3.325.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\sec{(fx+e)}}\,(b\tan{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.325.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}\,\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)), x)
```

3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

3.326.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{8\sqrt{d\sec(e+fx)}}{3bd^2f\sqrt{b\tan(e+fx)}}$$

output
```
2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)-8/3*(d*sec(f*x+e))^(1/2)
/b/d^2/f/(b*tan(f*x+e))^(1/2)
```

3.326.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{(-7+\cos(2(e+fx)))\sec^2(e+fx)}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
((-7 + Cos[2*(e + f*x)])*Sec[e + f*x]^2)/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqr
t[b*Tan[e + f*x]])
```

3.326.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3089, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3089$$

$$-\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3085$$

$$-\frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*(b*Tan[e + f*x])
^(3/2))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

3.326.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

3.326.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\frac{2\cos(fx+e)}{3}-\frac{8\sec(fx+e)}{3}}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,bd}$	47

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b/d*(cos(f*x+e)-4*sec(f*x+
e))
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{2\left(\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\,b^2d^2f\sin\left(fx+e\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
2/3*(cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sq
rt(d/cos(f*x + e))/(b^2*d^2*f*sin(f*x + e))
```

3.326.6 Sympy [A] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \begin{cases} -\frac{8\tan^3(e+fx)}{3f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}} - \frac{2\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}} & \text{for} \\ \frac{x}{(b\tan(e))^{\frac{3}{2}}(d\sec(e))^{\frac{3}{2}}} & \text{oth} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x)
)**(3/2)) - 2*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))**(3
/2)), Ne(f, 0)), (x/((b*tan(e))**(3/2)*(d*sec(e))**(3/2)), True))
```

3.326.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.326.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{(\cos(2e+2fx)-7)\sqrt{\frac{d}{\cos(e+fx)}}}{3bd^2f\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
((cos(2*e + 2*f*x) - 7)*(d/cos(e + f*x))^(1/2))/(3*b*d^2*f*((b*sin(2*e + 2
*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

3.327.1 Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}$$

output
```
-2/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+24/5*(sin(1/2*e+1/4*Pi+1/
2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f
*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/d^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e
)^(1/2)-12/5*(b*tan(f*x+e))^(3/2)/b^3/f/(d*sec(f*x+e))^(5/2)
```

3.327.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.89 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \frac{-11+\cos(2(e+fx))-8\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\tan^2(\right.}{5bd^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
(-11 + Cos[2*(e + f*x)] - 8*Hypergeometric2F1[3/4, 5/4, 7/4, -Tan[e + f*x]
^2]*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x]^2)/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]]
*Sqrt[b*Tan[e + f*x]])
```

3.327.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3089, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}dx}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{6\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6\left(\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}dx}{5d^2}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3042

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}dx}{5d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3121

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3042

$$-\frac{6\left(\frac{2\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3119

$$-\frac{6\left(\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}\right)}{b^2}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

output
```
-2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (6*((4*EllipticE[(e
 - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec[e + f*x])^(5
/2))))/b^2
```

3.327. $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

3.327.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.327. $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

3.327.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.45 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.48

method	result
default	$\frac{\left(24\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,E\left(\sqrt{-i(i-\cot(fx+e)+\csc(fx+e)}\right.\right.}{}$

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b/d^2*(24*(-I*(I-cot(f*x+e
)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-c
ot(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/
2))-12*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))
^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f
*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)^2+24*sec(f*x+e)*(-I*(I-cot(f
*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+
e)-cot(f*x+e)))^(1/2)*EllipticE((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2
^(1/2))-12*sec(f*x+e)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+
e)+csc(f*x+e)))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-c
ot(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+6*2^(1/2)-12*sec(f*x+e)*2^(1/2))
*2^(1/2)
```

3.327.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(6i\sqrt{-2i\,bd}\sin(fx+e)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))-\right.}{}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
-2/5*(6*I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2*I*b*d)*sin(f*x +
e)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*
x + e))) - (cos(f*x + e)^4 - 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x
 + e))*sqrt(d/cos(f*x + e)))/(b^2*d^3*f*sin(f*x + e))
```

3.327.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.327.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.327.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tan(e+f\,x))^{3/2}\,\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)), x)
```

3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

3.328.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^3\arctan\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}} + \frac{d^3\operatorname{arctanh}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}$$

output
```
d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e
))^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b
^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(5/2)/f/(b*tan(f*x+e))
^(1/2)-2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

3.328.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\cos(e+fx)(d\sec(e+fx))^{7/2}\sin(e+fx)}{3f(b\tan(e+fx))^{5/2}} + \frac{\left(\arctan\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\tan(e+fx)}}{\sqrt[4]{\sec^2(e+fx)}}\right)\right)\cos^2(e+fx)(d\sec(e+fx))^{7/2}\tan^{\frac{5}{2}}(e+fx)}{f\sec^2(e+fx)^{3/4}(b\tan(e+fx))^{5/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*Sin[e + f*x])/(3*f*(b*Tan[e + f*x]
)^(5/2)) + ((ArcTan[Sqrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)] + ArcTanh[S
qrt[Tan[e + f*x]]/(Sec[e + f*x]^2)^(1/4)])*Cos[e + f*x]^2*(d*Sec[e + f*x])
^(7/2)*Tan[e + f*x]^(5/2))/(f*(Sec[e + f*x]^2)^(3/4)*(b*Tan[e + f*x])^(5/2
))
```

3.328.3 Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3042, 3088, 3042, 3096, 3042, 3044, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx}{b^2}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}dx}{b^2}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}dx}{b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\cos(e+fx)\sqrt{b\sin(e+fx)}}dx}{b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3044$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{b^2}{\sqrt{b\sin(e+fx)}\left(b^2-b^2\sin^2(e+fx)\right)}d(b\sin(e+fx))}{b^3f\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}\left(b^2-b^2\sin^2(e+fx)\right)}d(b\sin(e+fx))}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{b^2-b^4\sin^4(e+fx)}d\sqrt{b\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 756$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\int\frac{1}{b^2\sin^2(e+fx)+b}d\sqrt{b\sin(e+fx)}}{2b}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 216$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\int\frac{1}{b-b^2\sin^2(e+fx)}d\sqrt{b\sin(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{2d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\left(\frac{\arctan\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sin(e+fx)\right)}{2b^{3/2}}\right)}{bf\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

3.328. $\int\frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

output
```
(-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^3*(A
rcTan[Sqrt[b]*Sin[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sin[e + f*x]]/(2
*b^(3/2)))*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e +
f*x]])
```

3.328.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.328. $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

3.328.4 Maple [A] (verified)

Time = 17.89 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

method	result
default	$-\frac{\csc(fx+e) d^3 \sqrt{d\sec(fx+e)}\left(-3\arctan\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cot(fx+e)+\csc(fx+e))\right)\cos(fx+e)+3\,\mathrm{arctanh}\left(\sqrt{\frac{\sin(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cdots\right.\right.}{\cdots}$

input
```
int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*csc(f*x+e)*d^3*(d*sec(f*x+e))^(1/2)*(-3*arctan((sin(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+3*arctanh((sin(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x+e)+csc(f*x+e)))*cos(f*x+e)+2*(sin(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)+3*arctan((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(
f*x+e)+csc(f*x+e)))-3*arctanh((sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(cot(f*x
+e)+csc(f*x+e))))/(sin(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*tan(f*x+e))^(1/2)
/b^2
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(142) = 284$.

Time = 0.47 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.94

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f
*x + e) - 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*arctan(1/4*(cos(f*x
+ e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x
+ e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sq
rt(d/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) -
 d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*log((d*cos(f*x + e)^4 -
 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x +
 e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt
(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 7
2*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x +
 e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f), 1/24*(16*d^3*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e) + 6*(b*d^3*cos(f*x + e)^
2 - b*d^3)*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(
f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b
*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)
^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b
*d^3)*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x
 + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*
cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x ...
```

3.328.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.328.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

3.328.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(b\tan(e+f\,x))^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2), x)
```

3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$

3.329.1 Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} + \frac{2d^2\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

output
```
-2/3*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^
(1/2)/b^2/f/(b*tan(f*x+e))^(1/2)-2/3*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f
*x+e))^(3/2)
```

3.329.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.92 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}\left(-\cot^2(e+fx)+\frac{\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)}{\sqrt[4]{\sec^2(e+fx)}}\right)\sqrt{b\tan}}{3b^3f}$$

input
```
Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(2*d^2*Sqrt[d*Sec[e + f*x]]*(-Cot[e + f*x]^2 + Hypergeometric2F1[1/4, 3/4,
 5/4, -Tan[e + f*x]^2]/(Sec[e + f*x]^2)^(1/4))*Sqrt[b*Tan[e + f*x]])/(3*b^
3*f)
```

3.329.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3088, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3088}$$

$$\frac{d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3120$$

$$\frac{2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b
^2*f*Sqrt[b*Tan[e + f*x]])
```

3.329.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3088
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Simp[a^2*((m - 2)/(b^2*(n + 1)))   Int[(a*Sec[e + f
*x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
LtQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m,
2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.329. $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.329.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

method	result
default	$-\frac{d^2\sqrt{d\sec(fx+e)}\left(i(-\cos(fx+e)-1)\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(-i-\cot(fx+e)+\csc(fx+e))}\sqrt{i(\csc(fx+e)-\cot(fx+e)}\right.}{3f\,b^2\sqrt{b\tan(fx+e)}}$

input
```
int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*d^2*(d*sec(f*x+e))^(1/2)/b^2/(b*tan(f*x+e))^(1/2)*(I*(-cos(f*x+e)-1
)*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(I*(-I-cot(f*x+e)+csc(f*x+e)))^(1/2
)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)
))^(1/2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))*2^(1/2)
```

3.329.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)^2+\left(d^2\cos\left(fx+e\right)^2-d^2\right)\sqrt{-2i\,bd}\text{weierst}}{}$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x
+ e)^2 + (d^2*cos(f*x + e)^2 - d^2)*sqrt(-2*I*b*d)*weierstrassPInverse(4,
0, cos(f*x + e) + I*sin(f*x + e)) + (d^2*cos(f*x + e)^2 - d^2)*sqrt(2*I*b*
d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*
x + e)^2 - b^3*f)
```

3.329.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.329.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

3.329.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(b\tan\left(e+f\,x\right))^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2), x)
```

3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$

3.330.1 Optimal result

Integrand size = 25, antiderivative size = 34

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

output
```
-2/3*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

3.330.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

3.330.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3085$$

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

3.330.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

3.330.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\csc(fx+e)\sqrt{d\sec(fx+e)}\, d}{3f\, b^{2}\sqrt{b\tan(fx+e)}}$	36

input
```
int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*csc(f*x+e)*(d*sec(f*x+e))^(1/2)*d/b^2/(b*tan(f*x+e))^(1/2)
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\,d\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos\left(fx+e\right)}{3\left(b^3f\cos\left(fx+e\right)^2-b^3f\right)}$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/3*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/
(b^3*f*cos(f*x + e)^2 - b^3*f)
```

3.330.6 Sympy [A] (verification not implemented)

Time = 44.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \begin{cases} -\frac{2(d\sec(e+fx))^{\frac{3}{2}}\tan(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}} & \text{for } f\neq 0 \\ \frac{x(d\sec(e))^{\frac{3}{2}}}{(b\tan(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Piecewise((-2*(d*sec(e + f*x))**(3/2)*tan(e + f*x)/(3*f*(b*tan(e + f*x))**
(5/2)), Ne(f, 0)), (x*(d*sec(e))**(3/2)/(b*tan(e))**(5/2), True))
```

3.330.7 Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

3.330.8 Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

3.330.9 Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\,d\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{3\,b^2\,f\,\sin(e+f\,x)\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
-(2*d*(d/cos(e + f*x))^(1/2))/(3*b^2*f*sin(e + f*x)*((b*sin(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$

3.331.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

output
```
4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)
/b^2/f/(b*tan(f*x+e))^(1/2)-2/3*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(3
/2)
```

3.331.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2\left(\csc^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*d^2*(Csc[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, -Tan[e + f*x]
^2]*(Sec[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x]
)^(3/2))
```

3.331.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3042, 3089, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int \frac{1}{\sqrt{b\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3b^2\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

input
```
Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

output
```
(-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
 - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqr
t[b*Tan[e + f*x]])
```

3.331.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.331. $\int\frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.331.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.52

method	result
default	$-\frac{\sqrt{d\sec(fx+e)}\,\left(2i\sqrt{-i(i-\cot(fx+e)+\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{-i(i-\cot(\right.\right.}{}$

input
```
int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2*(2*I*cos(f*x+e)*(-I*(
I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(
cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/
2),1/2*2^(1/2))+2*I*(-I*(I-cot(f*x+e)+csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-c
sc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((-I*(I-co
t(f*x+e)+csc(f*x+e)))^(1/2),1/2*2^(1/2))+2^(1/2)*cot(f*x+e))*2^(1/2)
```

3.331.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2-\sqrt{-2i\,bd}\left(\cos(fx+e)^2-1\right)\mathrm{weierstrass}\right.}{}$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 - sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f*x +
 e) + I*sin(f*x + e)) - sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInv
erse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x + e)^2 - b^3*f)
```

3.331.6 Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{(b\tan{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Integral(sqrt(d*sec(e + f*x))/(b*tan(e + f*x))**(5/2), x)
```

3.331.7 Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{(b\tan{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

3.331.8 Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{(b\tan{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx=\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

input
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2),x)
```

output
```
int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2), x)
```

3.332 $\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx$

3.332.1 Optimal result

Integrand size = 25, antiderivative size = 69

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{3/2}} - \frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

output
```
-8/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e)
)^(1/2)/(b*tan(f*x+e))^(3/2)
```

3.332.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2(3+\csc^2(e+fx))\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

input
```
Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
(-2*(3 + Csc[e + f*x]^2)*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x
]])
```

3.332.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3089, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(b\tan(e+fx))^{5/2}\sqrt{d\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3089}$$
$$-\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$
$$\Big\downarrow \text{3085}$$
$$-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
-2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
 f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

3.332.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

3.332.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\cot(fx+e)-\frac{8\sec(fx+e)\csc(fx+e)}{3}}{f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2}$	52

input
```
int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2*(3*cot(f*x+e)-4*sec(f*
x+e)*csc(f*x+e))
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{3\left(b^3df\cos\left(fx+e\right)^2-b^3df\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/3*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(d/cos(f*x + e))/(b^3*d*f*cos(f*x + e)^2 - b^3*d*f)
```

3.332.6 Sympy [A] (verification not implemented)

Time = 56.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \begin{cases} -\frac{8\tan^3(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} - \frac{2\tan(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} & \text{for } f \ne \\ \frac{x}{(b\tan(e))^{\frac{5}{2}}\sqrt{d\sec(e)}} & \text{otherw} \end{cases}$$

input
```
integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e +
f*x))) - 2*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f*x)))
, Ne(f, 0)), (x/((b*tan(e))**(5/2)*sqrt(d*sec(e))), True))
```

3.332.7 Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

3.332.8 Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}\,(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

3.332.9 Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(b\tan(e+fx))^{5/2}}\,dx = \frac{\left(\frac{13\sin(e+fx)}{3}-\sin(3e+3fx)\right)\sqrt{\frac{d}{\cos(e+fx)}}}{b^2\,d\,f\,\left(\cos(2e+2fx)-1\right)\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2)),x)
```

output
```
(((13*sin(e + f*x))/3 - sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(b^2*d*f
*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2
))
```

3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

3.333.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{8\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}} - \frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}$$

output
```
8/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)
/b^2/d^2/f/(b*tan(f*x+e))^(1/2)-4/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+
e))^(3/2)-2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2)
```

3.333.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-3+\cos(2(e+fx)))\csc(e+fx) - 8\,\text{Hypergeometric2F1}\left(\frac{1}{4},\right.}{3b^2df\sqrt{d\sec(e+fx)}\sqrt{b}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
((-3 + Cos[2*(e + f*x)])*Csc[e + f*x] - 8*Hypergeometric2F1[1/4, 3/4, 5/4,
 -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x])/(3*b^2*d*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

3.333.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3089, 3042, 3092, 3042, 3096, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}dx}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(\frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}dx}{3d^2}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3096}$$

$$-\frac{2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{2\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{b\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3121

$$-\frac{2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2\left(\frac{2\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3120

$$-\frac{2\left(\frac{4\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}\right)}{b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
-2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (2*((4*Elliptic
F[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*d^2*f
*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e + f*x])^
(3/2))))/b^2
```

3.333. $\int\frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

3.333.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

rule 3096
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n))   Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /;
FreeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.333. $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

3.333.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

method	result
default	$\frac{\left(-4i\sqrt{i(-i+\cot(fx+e)-\csc(fx+e))}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e)+i)}\,\sqrt{-i(\cot(fx+e)-\csc(fx+e))}\,F\left(\sqrt{i(-i+\cot(fx+e)-\csc(fx\right.\right.}{}$

input
```
int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2/d*(-4*I*(I*(-I+cot(f*x
+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e
)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^
(1/2))-4*I*sec(f*x+e)*(I*(-I+cot(f*x+e)-csc(f*x+e)))^(1/2)*(-I*(cot(f*x+e)
-csc(f*x+e)+I))^(1/2)*(-I*(cot(f*x+e)-csc(f*x+e)))^(1/2)*EllipticF((I*(-I+
cot(f*x+e)-csc(f*x+e)))^(1/2),1/2*2^(1/2))+cos(f*x+e)*cot(f*x+e)*2^(1/2)-2
*csc(f*x+e)*2^(1/2))*2^(1/2)
```

3.333.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^2-1\right)\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2\sqrt{2i\,bd}\left(\cos\left(f\right.\right.\right.}{3\,(b^3}$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/3*(2*sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(
f*x + e) + I*sin(f*x + e)) + 2*sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstr
assPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + (cos(f*x + e)^4 - 2*cos
(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b^3*
d^2*f*cos(f*x + e)^2 - b^3*d^2*f)
```

3.333.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.333.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.333.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx=\int \frac{1}{(b\tan\left(e+f\,x\right))^{5/2}\,\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

3.334.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}} - \frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}$$

output
```
-16/15*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(5/2)-64/15*(b*tan(f*x+e)
)^(1/2)/b^3/d^2/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e))^(5/2)/(b*tan
(f*x+e))^(3/2)
```

3.334.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \frac{(-151+108\cos(2(e+fx))+3\cos(4(e+fx)))\csc(e+fx)\sqrt{}}{60b^2d^3f\sqrt{b\tan(e+fx)}}$$

input
```
Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

output
```
((-151 + 108*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)])*Csc[e + f*x]*Sqrt[d*Se
c[e + f*x]])/(60*b^2*d^3*f*Sqrt[b*Tan[e + f*x]])
```

3.334.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3089, 3042, 3092, 3042, 3085}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(b\tan(e+fx))^{5/2}(d\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3089}$$

$$-\frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}dx}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3092}$$

$$-\frac{8\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\left(\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}dx}{5d^2}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3085}$$

$$-\frac{8\left(\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}\right)}{3b^2}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

input
```
Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

3.334. $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

output
```
-2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (8*((2*Sqrt[b*T
an[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e + f*x]])/(5
*b*d^2*f*Sqrt[d*Sec[e + f*x]])))/(3*b^2)
```

3.334.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3085
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

rule 3089
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Simp[(m + n + 1)/(b^2*(n + 1))   Int[(a*Sec[e + f*x])^m*(b*Ta
n[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && I
ntegersQ[2*m, 2*n]
```

rule 3092
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Simp[(m + n + 1)/(a^2*m)   Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1
] && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

3.334.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2\sec(fx+e)\csc(fx+e)\left(3\left(\cos^4(fx+e)\right)+24\left(\cos^2(fx+e)\right)-32\right)}{15f\sqrt{d\sec(fx+e)}\,\sqrt{b\tan(fx+e)}\,b^2d^2}$	66

input
```
int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/f*sec(f*x+e)*csc(f*x+e)*(3*cos(f*x+e)^4+24*cos(f*x+e)^2-32)/(d*sec(f*
x+e))^(1/2)/(b*tan(f*x+e))^(1/2)/b^2/d^2
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\left(3\cos\left(fx+e\right)^{5}+24\cos\left(fx+e\right)^{3}-32\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{15\left(b^{3}d^{3}f\cos\left(fx+e\right)^{2}-b^{3}d^{3}f\right)}$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
-2/15*(3*cos(f*x + e)^5 + 24*cos(f*x + e)^3 - 32*cos(f*x + e))*sqrt(b*sin(
f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b^3*d^3*f*cos(f*x + e)^2 - b^
3*d^3*f)
```

3.334.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.334.7 Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.334.8 Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

3.334.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,(105\,\sin(3\,e+3\,f\,x)-410\,\sin(e+f\,x)+3\,\sin(5\,e+5\,f\,x))}{60\,b^2\,d^3\,f\,\left(\cos(2\,e+2\,f\,x)-1\right)\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

input
```
int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2)),x)
```

output
```
-((d/cos(e + f*x))^(1/2)*(105*sin(3*e + 3*f*x) - 410*sin(e + f*x) + 3*sin(
5*e + 5*f*x)))/(60*b^2*d^3*f*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/
(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

3.335.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)(b\sec(e+fx)$$

$$3df$$

output
```
2/3*(cos(f*x+e)^2)^(17/12)*hypergeom([3/4, 17/12],[7/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

3.335.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx=\frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt[4]{-\tan^2(e+}}{4f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*d*Hypergeometric2F1[1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/
3)*(-Tan[e + f*x]^2)^(1/4))/(4*f*Sqrt[d*Tan[e + f*x]])
```

3.335.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \; 3097$$

$$\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{12},\frac{7}{4},\sin^2(e+fx)\right)}{3df}$$

input
```
Int[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.335.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.335.4 Maple [F]

$$\int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}dx$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

3.335.5 Fricas [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sec(f*x + e), x)
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.335.7 Maxima [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

3.335.8 Giac [F]

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+fx)}\left(\frac{b}{\cos(e+fx)}\right)^{4/3}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3), x)
```

3.336 $\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$

3.336.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}}{3df}$$

output
```
2/3*(cos(f*x+e)^2)^(11/12)*hypergeom([3/4, 11/12],[7/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

3.336.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{4},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(3*d*Hypergeometric2F1[1/6, 1/4, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/
3)*(-Tan[e + f*x]^2)^(1/4))/(f*Sqrt[d*Tan[e + f*x]])
```

3.336.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{11}{12},\frac{7}{4},\sin^2(e+fx)\right)}{3df}$$

input
```
Int[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

3.336.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.336.4 Maple [F]

$$\int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}dx$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

3.336.5 Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.336.6 Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*sec(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

3.336.7 Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.336.8 Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{1}{3}}\sqrt{d\tan(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan(e+f\,x)}\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3), x)
```

3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

3.337.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

output
```
2/3*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(1/3)
```

3.337.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = -\frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{4},\frac{5}{6},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(-3*d*Hypergeometric2F1[-1/6, 1/4, 5/6, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^
(1/4))/(f*(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])
```

3.337.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3097}$$
$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

3.337.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.337.4 Maple [F]

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

3.337.5 Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sec(f*x + e)), x)
```

3.337.6 Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(1/3), x)
```

3.337.7 Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

3.337.8 Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\tan(e+f\,x)}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3), x)
```

3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$

3.338.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

output
```
2/3*(cos(f*x+e)^2)^(1/12)*hypergeom([1/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(4/3)
```

3.338.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = -\frac{3d\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{4f(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(-3*d*Hypergeometric2F1[-2/3, 1/4, 1/3, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^
(1/4))/(4*f*(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])
```

3.338.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}dx$$
$$\Big\downarrow \text{ 3097}$$
$$\frac{2\sqrt[12]{\cos^2(e+fx)}(d\tan(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{12},\frac{3}{4},\frac{7}{4},\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

input
```
Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

3.338.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.338.4 Maple [F]

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

3.338.5 Fricas [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*sec(f*x + e)^2),
 x)
```

3.338.6 Sympy [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan\left(e+fx\right)}}{\left(b\sec\left(e+fx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(4/3), x)
```

3.338.7 Maxima [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

3.338.8 Giac [F]

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(fx+e)}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{d\tan(e+f\,x)}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3), x)
```

3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

3.339.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)(b\sec(e+fx$$

$$5df$$

output
```
2/5*(cos(f*x+e)^2)^(23/12)*hypergeom([5/4, 23/12],[9/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

3.339.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{2}{3},\frac{5}{3},\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt{d\tan(e}}{4f\sqrt[4]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(3*d*Hypergeometric2F1[-1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4
/3)*Sqrt[d*Tan[e + f*x]])/(4*f*(-Tan[e + f*x]^2)^(1/4))
```

3.339.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{23}{12},\frac{9}{4},\sin^2(e+fx)\right)}{5df}$$

input
```
Int[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

3.339.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.339.4 Maple [F]

$$\int (b\sec(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

3.339.5 Fricas [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}} (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sec(f*x + e)*tan(
f*x + e), x)
```

3.339.6 Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.339.7 Maxima [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

3.339.8 Giac [F]

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{4}{3}}(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3), x)
```

3.340 $\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$

3.340.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx)$$

output
```
2/5*(cos(f*x+e)^2)^(17/12)*hypergeom([5/4, 17/12],[9/4],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

3.340.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \frac{3d\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\sqrt{d\tan(e+fx)}}{f\sqrt[4]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(3*d*Hypergeometric2F1[-1/4, 1/6, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1
/3)*Sqrt[d*Tan[e + f*x]])/(f*(-Tan[e + f*x]^2)^(1/4))
```

3.340.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{2\cos^2(e+fx)^{17/12}\sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{17}{12},\frac{9}{4},\sin^2(e+fx)\right)}{5df}$$

input
```
Int[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

3.340.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.340.4 Maple [F]

$$\int (b\sec (fx+e))^{\frac{1}{3}} (d\tan (fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

3.340.5 Fricas [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int (b\sec (fx+e))^{\frac{1}{3}} (d\tan (fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

3.340.6 Sympy [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx = \int \sqrt[3]{b\sec (e+fx)}(d\tan (e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*sec(e + f*x))**(1/3)*(d*tan(e + f*x))**(3/2), x)
```

3.340.7 Maxima [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\int (b\sec(fx+e))^{\frac{1}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

3.340.8 Giac [F]

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\int (b\sec(fx+e))^{\frac{1}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sec(e+fx)}(d\tan(e+fx))^{3/2}\,dx=\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3), x)
```

3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

3.341.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/}}{5df\sqrt[3]{b\sec(e+fx)}}$$

output
```
2/5*(cos(f*x+e)^2)^(13/12)*hypergeom([13/12, 5/4],[9/4],sin(f*x+e)^2)*(d*t
an(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(1/3)
```

3.341.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{f\sqrt[3]{b\sec(e+fx)}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(3*Cot[e + f*x]^3*Hypergeometric2F1[-1/4, -1/6, 5/6, Sec[e + f*x]^2]*(d*Ta
n[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(f*(b*Sec[e + f*x])^(1/3))
```

3.341.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\text{Hypergeometric2F1}\left(\frac{13}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x
]^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

3.341.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.341.4 Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

3.341.5 Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sec
(f*x + e)), x)
```

3.341.6 Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{\sqrt[3]{b\sec(e+fx)}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(1/3),x)
```

output
```
Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(1/3), x)
```

3.341.7 Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

3.341.8 Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3), x)
```

3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$

3.342.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

output
```
2/5*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 5/4],[9/4],sin(f*x+e)^2)*(d*tan
(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(4/3)
```

3.342.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{3\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{4},\frac{1}{3},\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{4f(b\sec(e+fx))^{4/3}}$$

input
```
Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(3*Cot[e + f*x]^3*Hypergeometric2F1[-2/3, -1/4, 1/3, Sec[e + f*x]^2]*(d*Ta
n[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(4*f*(b*Sec[e + f*x])^(4/3))
```

3.342.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}dx$$
$$\Big\downarrow \; 3097$$
$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{12},\frac{5}{4},\frac{9}{4},\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

input
```
Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

output
```
(2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

3.342.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.342.4 Maple [F]

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

output
```
int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

3.342.5 Fricas [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*s
ec(f*x + e)^2), x)
```

3.342.6 Sympy [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{(b\sec(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(4/3),x)
```

output
```
Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(4/3), x)
```

3.342.7 Maxima [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

3.342.8 Giac [F]

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{4/3}}\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3),x)
```

output
```
int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3), x)
```

3.343 $\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$

3.343.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx}{7df}$$

output
```
3/7*(cos(f*x+e)^2)^(17/12)*hypergeom([7/6, 17/12],[13/6],sin(f*x+e)^2)*(b*
sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

3.343.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{4},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}}{f\sqrt[6]{-\tan^2(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[-1/6, 1/4, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]
]*(d*Tan[e + f*x])^(1/3))/(f*(-Tan[e + f*x]^2)^(1/6))
```

3.343.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}dx$$
$$\Big\downarrow \text{3097}$$
$$\frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{17}{12},\frac{13}{6},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*
x]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

3.343.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.343.4 Maple [F]

$$\int \sqrt{b\sec(fx+e)}\,(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

3.343.5 Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

3.343.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.343.7 Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

3.343.8 Giac [F]

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int \sqrt{b\sec(fx+e)}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\,dx = \int (d\tan(e+f\,x))^{4/3}\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2), x)
```

3.344 $\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$

3.344.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{11/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}}{4df}$$

output
```
3/4*(cos(f*x+e)^2)^(11/12)*hypergeom([2/3, 11/12],[5/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

3.344.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\sqrt[3]{-\tan^2(e+fx)}}{f(d\tan(e+fx))^{2/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 1/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(1/3))/(f*(d*Tan[e + f*x])^(2/3))
```

3.344.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{11}{12},\frac{5}{3},\sin^2(e+fx)\right)}{4df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

3.344.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.344.4 Maple [F]

$$\int \sqrt{b \sec (fx+e)}\, (d \tan (fx+e))^{\frac{1}{3}}\, dx$$

input
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

3.344.5 Fricas [F]

$$\int \sqrt{b \sec(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx = \int \sqrt{b \sec (fx+e)} (d \tan (fx+e))^{\frac{1}{3}}\, dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.344.6 Sympy [F]

$$\int \sqrt{b \sec(e+fx)} \sqrt[3]{d \tan(e+fx)}\, dx = \int \sqrt{b \sec (e+fx)} \sqrt[3]{d \tan (e+fx)}\, dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

3.344.7 Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.344.8 Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(d\tan\left(fx+e\right))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2), x)
```

3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

3.345.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}}{2df}$$

output
```
3/2*(cos(f*x+e)^2)^(7/12)*hypergeom([1/3, 7/12],[4/3],sin(f*x+e)^2)*(b*sec
(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

3.345.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{2}{3},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{2/3}}{f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 2/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(2/3))/(f*(d*Tan[e + f*x])^(4/3))
```

3.345.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}dx$$
$$\Big\downarrow \text{3097}$$
$$\frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}(d\tan(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{12},\frac{4}{3},\sin^2(e+fx)\right)}{2df}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^
2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

3.345.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

3.345.4 Maple [F]

$$\int \frac{\sqrt{b \sec \left(f x+e\right)}}{\left(d \tan \left(f x+e\right)\right)^{\frac{1}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

3.345.5 Fricas [F]

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx=\int \frac{\sqrt{b \sec \left(f x+e\right)}}{\left(d \tan \left(f x+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

3.345.6 Sympy [F]

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx=\int \frac{\sqrt{b \sec \left(e+fx\right)}}{\sqrt[3]{d \tan \left(e+fx\right)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

3.345.7 Maxima [F]

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx = \int \frac{\sqrt{b \sec(fx+e)}}{(d \tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.345.8 Giac [F]

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx = \int \frac{\sqrt{b \sec(fx+e)}}{(d \tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b \sec(e+fx)}}{\sqrt[3]{d \tan(e+fx)}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{(d \tan(e+f\,x))^{1/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

3.346.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\sqrt[12]{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

output
```
-3*(cos(f*x+e)^2)^(1/12)*hypergeom([-1/6, 1/12],[5/6],sin(f*x+e)^2)*(b*sec
(f*x+e))^(1/2)/d/f/(d*tan(f*x+e))^(1/3)
```

3.346.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{6},\frac{5}{4},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{7/6}}{f(d\tan(e+fx))^{7/3}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/4, 7/6, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(-Tan[e + f*x]^2)^(7/6))/(f*(d*Tan[e + f*x])^(7/3))
```

3.346.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{3\sqrt[12]{\cos^2(e+fx)}\sqrt{b\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

3.346.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.346.4 Maple [F]

$$\int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

3.346.5 Fricas [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
 x)
```

3.346.6 Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(4/3), x)
```

3.346.7 Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.346.8 Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

3.346.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\left(d\tan\left(e+f\,x\right)\right)^{4/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

3.347.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{23/12}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)(b\sec(e+f\ldots}{7df}$$

output
```
3/7*(cos(f*x+e)^2)^(23/12)*hypergeom([7/6, 23/12],[13/6],sin(f*x+e)^2)*(b*
sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

3.347.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e\ldots}}{3f\sqrt[6]{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[-1/6, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3
/2)*(d*Tan[e + f*x])^(1/3))/(3*f*(-Tan[e + f*x]^2)^(1/6))
```

3.347.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{23}{12},\frac{13}{6},\sin^2(e+fx)\right)}{7df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*
x]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

3.347.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.347.4 Maple [F]

$$\int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{4}{3}}\, dx$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

3.347.5 Fricas [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\, dx = \int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{4}{3}}\, dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sec(f*x + e)*tan(
f*x + e), x)
```

3.347.6 Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\, dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

output
```
Timed out
```

3.347.7 Maxima [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

3.347.8 Giac [F]

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \int (d\tan(e+f\,x))^{4/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2),x)
```

output
```
int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2), x)
```

3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

3.348.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)(b\sec(e+fx)}{4df}$$

output
```
3/4*(cos(f*x+e)^2)^(17/12)*hypergeom([2/3, 17/12],[5/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

3.348.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{-\tan^2(e+}}{3f(d\tan(e+fx))^{2/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[1/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(1/3))/(3*f*(d*Tan[e + f*x])^(2/3))
```

3.348.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{17}{12},\frac{5}{3},\sin^2(e+fx)\right)}{4df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

3.348.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.348.4 Maple [F]

$$\int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

3.348.5 Fricas [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sec(f*x + e), x)
```

3.348.6 Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(e+fx))^{\frac{3}{2}} \sqrt[3]{d\tan(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)*(d*tan(e + f*x))**(1/3), x)
```

3.348.7 Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

3.348.8 Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \int (d\tan(e+f\,x))^{1/3}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2),x)
```

output
```
int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2), x)
```

3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

3.349.1 Optimal result

Integrand size = 25, antiderivative size = 64

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(}{2df}$$

output
```
3/2*(cos(f*x+e)^2)^(13/12)*hypergeom([1/3, 13/12],[4/3],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

3.349.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{4},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)}{3f(d\tan(e+fx))^{4/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(2*d*Hypergeometric2F1[2/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(2/3))/(3*f*(d*Tan[e + f*x])^(4/3))
```

3.349.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}dx$$

$$\Big\downarrow \; 3097$$

$$\frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{13}{12},\frac{4}{3},\sin^2(e+fx)\right)}{2df}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

output
```
(3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

3.349.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.349.4 Maple [F]

$$\int \frac{(b\sec (fx+e))^{\frac{3}{2}}}{(d\tan (fx+e))^{\frac{1}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

3.349.5 Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec (fx+e))^{\frac{3}{2}}}{(d\tan (fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d*tan
(f*x + e)), x)
```

3.349.6 Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec (e+fx))^{\frac{3}{2}}}{\sqrt[3]{d\tan (e+fx)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(1/3), x)
```

3.349.7 Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.349.8 Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{(d\tan(e+f\,x))^{1/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
```

output
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

3.350.1 Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

output
```
-3*(cos(f*x+e)^2)^(7/12)*hypergeom([-1/6, 7/12],[5/6],sin(f*x+e)^2)*(b*sec
(f*x+e))^(3/2)/d/f/(d*tan(f*x+e))^(1/3)
```

3.350.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{2d\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{6},\frac{7}{4},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)}{3f(d\tan(e+fx))^{7/3}}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(2*d*Hypergeometric2F1[3/4, 7/6, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(-Tan[e + f*x]^2)^(7/6))/(3*f*(d*Tan[e + f*x])^(7/3))
```

3.350.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}dx$$

$$\Big\downarrow \; 3097$$

$$-\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{7}{12},\frac{5}{6},\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

output
```
(-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

3.350.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.350.4 Maple [F]

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}dx$$

input
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

output
```
int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

3.350.5 Fricas [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d^2*t
an(f*x + e)^2), x)
```

3.350.6 Sympy [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{(d\tan(e+fx))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(4/3), x)
```

3.350.7 Maxima [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.350.8 Giac [F]

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{(d\tan(e+f\,x))^{4/3}}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
```

output
```
int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$

3.351.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m}{fm} - \frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)} + \frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}$$

output
```
(b*sec(f*x+e))^m/f/m-2*(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)+(b*sec(f*x+e))^(4+
m)/b^4/f/(4+m)
```

3.351.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{(b\sec(e+fx))^m\left(8+6m+m^2-2m(4+m)\sec^2(e+fx)+m(2+m)\sec^4(e+fx)\right)}{fm(2+m)(4+m)}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

output
```
((b*Sec[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Sec[e + f*x]^2 + m*(2 + m
)*Sec[e + f*x]^4))/(f*m*(2 + m)*(4 + m))
```

3.351.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^5(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int (b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)^2 d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left((b\sec(e+fx))^{m-1}-\frac{2(b\sec(e+fx))^{m+1}}{b^2}+\frac{(b\sec(e+fx))^{m+3}}{b^4}\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{m+4}}{b^5(m+4)}-\frac{2(b\sec(e+fx))^{m+2}}{b^3(m+2)}+\frac{(b\sec(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

output
```
(b*((b*Sec[e + f*x])^m/(b*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^3*(2 + m))
+ (b*Sec[e + f*x])^(4 + m)/(b^5*(4 + m))))/f
```

3.351.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.351.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.10 (sec) , antiderivative size = 6067, normalized size of antiderivative = 90.55

method	result	size
risch	Expression too large to display	6067

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx = \frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-2\left(m^2+4\,m\right)\cos\left(fx+e\right)^2+m^2+2\,m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{\left(fm^3+6\,fm^2+8\,fm\right)\cos\left(fx+e\right)^4}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="fricas")
```

output
```
((m^2 + 6*m + 8)*cos(f*x + e)^4 - 2*(m^2 + 4*m)*cos(f*x + e)^2 + m^2 + 2*m
)*(b/cos(f*x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4)
```

3.351.6 Sympy [F]

$$\int (b\sec(e+fx))^m\tan^5(e+fx)\,dx = \begin{cases} x(b\sec(e))^m\tan^5(e) & \text{for } f = \\ \frac{\int \frac{\tan^5(e+fx)}{\sec^4(e+fx)}\,dx}{b^4} & \text{for } m = \\ \frac{\int \frac{\tan^5(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m = \\ \frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f} & \text{for } m = \\ \frac{m^2(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{2m(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{4m(b\sec(e+fx))^m\tan^2(e+fx)}{fm^3+6fm^2+8fm}+\frac{8(b\sec(e+fx))^m}{fm^3+6fm^2+8fm} & \text{otherw} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**5,x)
```

output
```
Piecewise((x*(b*sec(e))**m*tan(e)**5, Eq(f, 0)), (Integral(tan(e + f*x)**5
/sec(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(tan(e + f*x)**5/sec(e + f
*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x
)**4/(4*f) - tan(e + f*x)**2/(2*f), Eq(m, 0)), (m**2*(b*sec(e + f*x))**m*t
an(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 2*m*(b*sec(e + f*x))**m*tan(e
 + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 4*m*(b*sec(e + f*x))**m*tan(e + f
*x)**2/(f*m**3 + 6*f*m**2 + 8*f*m) + 8*(b*sec(e + f*x))**m/(f*m**3 + 6*f*m
**2 + 8*f*m), True))
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \frac{\frac{b^m\cos(fx+e)^{-m}}{m} - \frac{2\,b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2} + \frac{b^m\cos(fx+e)^{-m}}{(m+4)\cos(fx+e)^4}}{f}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="maxima")
```

output
```
(b^m*cos(f*x + e)^(-m)/m - 2*b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2
) + b^m*cos(f*x + e)^(-m)/((m + 4)*cos(f*x + e)^4))/f
```

3.351.8 Giac [F]

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^5\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^5, x)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.97

$$\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$$
$$= \frac{(\cos(4e+4fx) - \sin(4e+4fx)\ 1\mathrm{i})\left(\frac{b}{\cos(e+fx)}\right)^m \left(\frac{2\cos(4e+4fx)\,(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})}{fm} + \frac{(\cos(4e+4fx)}{}\right.}{16\left(\frac{\cos(2e+2fx)}{2}+\frac{1}{2}\right)^2}$$

input
```
int(tan(e + f*x)^5*(b/cos(e + f*x))^m,x)
```

output
```
((cos(4*e + 4*f*x) - sin(4*e + 4*f*x)*1i)*(b/cos(e + f*x))^m*((2*cos(4*e +
 4*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i))/(f*m) + ((cos(4*e + 4*f*
x) + sin(4*e + 4*f*x)*1i)*(4*m + 6*m^2 + 48))/(f*m*(6*m + m^2 + 8)) - (2*c
os(2*e + 2*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(8*m + 4*m^2 - 32
))/(f*m*(6*m + m^2 + 8))))/(16*(cos(2*e + 2*f*x)/2 + 1/2)^2)
```

3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$

3.352.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\sec(e+fx))^m}{fm} + \frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}$$

output
```
-(b*sec(f*x+e))^m/f/m+(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)
```

3.352.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\sec(e+fx))^m\,(2+m-m\sec^2(e+fx))}{fm(2+m)}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Sec[e + f*x])^m*(2 + m - m*Sec[e + f*x]^2))/(f*m*(2 + m)))
```

3.352.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int -(b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int (b\sec(e+fx))^{m-1}\left(1-\sec^2(e+fx)\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left((b\sec(e+fx))^{m-1}-\frac{(b\sec(e+fx))^{m+1}}{b^2}\right)d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{m+2}}{b^3(m+2)}-\frac{(b\sec(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(b*(-((b*Sec[e + f*x])^m/(b*m)) + (b*Sec[e + f*x])^(2 + m)/(b^3*(2 + m))))
/f
```

3.352.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.352.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.06 (sec) , antiderivative size = 2423, normalized size of antiderivative = 56.35

method	result	size
risch	Expression too large to display	2423

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/(2+m)/f/(exp(2*I*(f*x+e))+1)^2/m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))+1)^
(-m)*2^m*b^m*(m*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*P
i*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*
(f*x+e)))*Pi*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*Pi
*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I
*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*ex
p(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e)
)/(exp(2*I*(f*x+e))+1))^2*Pi*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*
(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*m)
*exp(-1/2*I*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp(1/2*
I*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)*Pi*m)*exp(4*I*
f*x)*exp(4*I*e)+2*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3
*Pi*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(
I*(f*x+e)))*Pi*m)*exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*
Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*Pi*csgn(I/(exp(2*I*(f*x+e))+1))*m)*
exp(1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2*Pi*m)*exp(-1/2*I*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)*Pi*
m)*exp(-1/2*I*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3*Pi*m)*exp...
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
-((m + 2)*cos(f*x + e)^2 - m)*(b/cos(f*x + e))^m/((f*m^2 + 2*f*m)*cos(f*x
+ e)^2)
```

3.352.6 Sympy [F]

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \begin{cases} x(b\sec(e))^m \tan^3(e) & \text{for } f=0 \\ \frac{\int \frac{\tan^3(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m=-2 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f} + \frac{\tan^2(e+fx)}{2f} & \text{for } m=0 \\ \frac{m(b\sec(e+fx))^m \tan^2(e+fx)}{fm^2+2fm} - \frac{2(b\sec(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Piecewise((x*(b*sec(e))**m*tan(e)**3, Eq(f, 0)), (Integral(tan(e + f*x)**3
/sec(e + f*x)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) +
tan(e + f*x)**2/(2*f), Eq(m, 0)), (m*(b*sec(e + f*x))**m*tan(e + f*x)**2/(
f*m**2 + 2*f*m) - 2*(b*sec(e + f*x))**m/(f*m**2 + 2*f*m), True))
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = -\frac{\frac{b^m\cos(fx+e)^{-m}}{m} - \frac{b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2}}{f}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
-(b^m*cos(f*x + e)^(-m)/m - b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2)
)/f
```

3.352.8 Giac [F]

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^3, x)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$$
$$= -\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-m+2\,\cos\left(4\,e+4\,f\,x\right)+m\,\cos\left(4\,e+4\,f\,x\right)+6\right)}{f\,m\,\left(m+2\right)\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

input
```
int(tan(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

output
```
-((b/cos(e + f*x))^m*(8*cos(2*e + 2*f*x) - m + 2*cos(4*e + 4*f*x) + m*cos(
4*e + 4*f*x) + 6))/(f*m*(m + 2)*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3
))
```

3.353 $\int (b\sec(e+fx))^m\tan(e+fx)\,dx$

3.353.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx=\frac{(b\sec(e+fx))^m}{fm}$$

output
```
(b*sec(f*x+e))^m/f/m
```

3.353.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx=\frac{(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(b*Sec[e + f*x])^m/(f*m)
```

3.353.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3086, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int (b\sec(e+fx))^{m-1}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{17}$$

$$\frac{(b\sec(e+fx))^m}{fm}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(b*Sec[e + f*x])^m/(f*m)
```

3.353.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.353.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{(b\sec(fx+e))^m}{fm}$
default	$\frac{(b\sec(fx+e))^m}{fm}$
risch	$(e^{i(fx+e)})^m (e^{2i(fx+e)}+1)^{-m} 2^m b^m e^{\frac{i\pi m\left(-\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2 \operatorname{csgn}\left(ie^{i(fx+e)}\right)+\operatorname{csgn}\left(\frac{ie^{i(fx+}}{e^{2i(fx+e}}\right.}{}}$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(b*sec(f*x+e))^m/f/m
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(fx+e)}\right)^m}{fm}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
(b/cos(f*x + e))^m/(f*m)
```

3.353.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int (b\sec(e+fx))^m \tan(e+fx)\,dx = \begin{cases} x\tan(e) & \text{for } f=0 \wedge m=0 \\ x(b\sec(e))^m \tan(e) & \text{for } f=0 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} & \text{for } m=0 \\ \frac{(b\sec(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e),x)
```

output
```
Piecewise((x*tan(e), Eq(f, 0) & Eq(m, 0)), (x*(b*sec(e))**m*tan(e), Eq(f,
0)), (log(tan(e + f*x)**2 + 1)/(2*f), Eq(m, 0)), ((b*sec(e + f*x))**m/(f*m
), True))
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx = \frac{b^m\cos(fx+e)^{-m}}{fm}$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
b^m*cos(f*x + e)^(-m)/(f*m)
```

3.353.8 Giac [F]

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx = \int (b\sec(fx+e))^m\tan(fx+e)\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e), x)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^m\tan(e+fx)\,dx = \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m}{f\,m}$$

input
```
int(tan(e + f*x)*(b/cos(e + f*x))^m,x)
```

output
```
(b/cos(e + f*x))^m/(f*m)
```

3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$

3.354.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
-hypergeom([1, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

3.354.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

3.354.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{b\int -\frac{(b\sec(e+fx))^{m-1}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{1-\sec^2(e+fx)}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

3.354.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.354.4 Maple [F]

$$\int \cot (fx + e) (b \sec (fx + e))^m dx$$

input
```
int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

3.354.5 Fricas [F]

$$\int \cot(e + fx)(b \sec(e + fx))^m dx = \int (b \sec (fx + e))^m \cot (fx + e) dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e), x)
```

3.354.6 Sympy [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(e+fx)})^m\cot{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x), x)
```

3.354.7 Maxima [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m\cot{(fx+e)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

3.354.8 Giac [F]

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m\cot{(fx+e)}\,dx$$

input
```
integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(cot(e + f*x)*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)*(b/cos(e + f*x))^m, x)
```

3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$

3.355.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
hypergeom([2, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

3.355.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

output
```
(Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/
(f*m)
```

3.355.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3086, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^2}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

output
```
(Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/
(f*m)
```

3.355.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.355.4 Maple [F]

$$\int (\cot^3(fx+e))\,(b\sec(fx+e))^m\,dx$$

input
```
int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

3.355.5 Fricas [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

3.355.6 Sympy [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(e+fx))^m\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**3, x)
```

3.355.7 Maxima [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

3.355.8 Giac [F]

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^3\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^3*(b/cos(e + f*x))^m, x)
```

3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$

3.356.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

output
```
-hypergeom([3, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

3.356.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

3.356.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{b\int -\frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{m-1}}{(1-\sec^2(e+fx))^3}d\sec(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(3,\frac{m}{2},\frac{m+2}{2},\sec^2(e+fx)\right)}{fm}$$

input
```
Int[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m
)/(f*m))
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.356.4 Maple [F]

$$\int \left(\cot^{5}\left(f x +e \right)\right) \left(b \sec \left(f x +e \right)\right)^{m}d x$$

input
```
int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

3.356.5 Fricas [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int (b \sec\left(fx+e\right))^{m} \cot\left(fx+e\right)^{5}\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

3.356.6 Sympy [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(e+fx)})^m\cot^5{(e+fx)}\,dx$$

input
```
integrate(cot(f*x+e)**5*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**5, x)
```

3.356.7 Maxima [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m\cot{(fx+e)}^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

3.356.8 Giac [F]

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec{(fx+e)})^m\cot{(fx+e)}^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^5\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^5*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^5*(b/cos(e + f*x))^m, x)
```

3.357 $\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx$

3.357.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{5+m}{2}}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e+fx)}{5f}$$

output
```
1/5*(cos(f*x+e)^2)^(5/2+1/2*m)*hypergeom([5/2, 5/2+1/2*m],[7/2],sin(f*x+e)
^2)*(b*sec(f*x+e))^m*tan(f*x+e)^5/f
```

3.357.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[-3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*S
ec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

3.357.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
 + f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

3.357.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.357.4 Maple [F]

$$\int (b\sec (fx+e))^{m} \left(\tan^{4}(fx+e)\right) dx$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)
```

3.357.5 Fricas [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec (fx+e))^{m} \tan (fx+e)^{4}\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

3.357.6 Sympy [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec (e+fx))^{m} \tan^{4}(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((b*sec(e + f*x))**m*tan(e + f*x)**4, x)
```

3.357.7 Maxima [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

3.357.8 Giac [F]

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

3.357.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^4*(b/cos(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.358 $\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx$

3.358.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

output
```
1/3*(cos(f*x+e)^2)^(3/2+1/2*m)*hypergeom([3/2, 3/2+1/2*m],[5/2],sin(f*x+e)
^2)*(b*sec(f*x+e))^m*tan(f*x+e)^3/f
```

3.358.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\tan(e+fx)}{fm\sqrt{-\tan^2(e+fx)}}$$

input
```
Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^
m*Tan[e + f*x])/(f*m*Sqrt[-Tan[e + f*x]^2])
```

3.358.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^2(b\sec(e+fx))^m dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
 + f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

3.358.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.358.4 Maple [F]

$$\int (b\sec (fx+e))^{m} \left(\tan^{2}(fx+e)\right) dx$$

input
```
int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

3.358.5 Fricas [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\, dx = \int (b\sec (fx+e))^{m} \tan (fx+e)^{2}\, dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

3.358.6 Sympy [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\, dx = \int (b\sec (e+fx))^{m} \tan^{2}(e+fx)\, dx$$

input
```
integrate((b*sec(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((b*sec(e + f*x))**m*tan(e + f*x)**2, x)
```

3.358.7 Maxima [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

3.358.8 Giac [F]

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \int (b\sec(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$

3.359.1 Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\sin^2(e+fx)\right)(b\sec(e+fx))}{f}$$

output
```
-(cos(f*x+e)^2)^(-1/2+1/2*m)*cot(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2]
,sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

3.359.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[3/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*
Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

3.359.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\sin^2(e+fx)\right)}{f}$$

input
```
Int[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

output
```
-(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1
+ m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

3.359.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.359.4 Maple [F]

$$\int \left(\cot^{2}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

3.359.5 Fricas [F]

$$\int \cot^{2}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

3.359.6 Sympy [F]

$$\int \cot^{2}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(e+fx\right)\right)^{m}\cot^{2}\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**2, x)
```

3.359.7 Maxima [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

3.359.8 Giac [F]

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$

3.360.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\sin^2(e+fx)\right)\left(b\sec(e+f\ldots\right.}{3f}$$

output
```
-1/3*(cos(f*x+e)^2)^(-3/2+1/2*m)*cot(f*x+e)^3*hypergeom([-3/2, -3/2+1/2*m]
,[-1/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

3.360.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[5/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Se
c[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m)
```

3.360.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\sin^2(e+fx)\right)}{3f}$$

input
```
Int[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

output
```
-1/3*((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2,
 (-3 + m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

3.360.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

3.360.4 Maple [F]

$$\int \left(\cot^{4}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

3.360.5 Fricas [F]

$$\int \cot^{4}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

3.360.6 Sympy [F]

$$\int \cot^{4}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**4, x)
```

3.360.7 Maxima [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

3.360.8 Giac [F]

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^4*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$

3.361.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}(-5+m),-\frac{3}{2},\sin^2(e+fx)\right)(b\sec(e+f$$
$$5f$$

output
```
-1/5*(cos(f*x+e)^2)^(-5/2+1/2*m)*cot(f*x+e)^5*hypergeom([-5/2, -5/2+1/2*m]
,[-3/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

3.361.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^m\sqrt{-\tan^2(e+fx)}}{fm}$$

input
```
Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[7/2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*
Sec[e + f*x])^m*Sqrt[-Tan[e + f*x]^2])/(f*m))
```

3.361.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^m}{\tan(e+fx)^6}dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},-\frac{3}{2},\sin^2(e+fx)\right)}{5f}$$

input
```
Int[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

output
```
-1/5*((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2,
 (-5 + m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

3.361.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

3.361.4 Maple [F]

$$\int \left(\cot^{6}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

3.361.5 Fricas [F]

$$\int \cot^{6}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(fx+e\right)\right)^{m}\cot\left(fx+e\right)^{6}\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

3.361.6 Sympy [F]

$$\int \cot^{6}(e+fx)(b\sec(e+fx))^{m}\,dx=\int \left(b\sec\left(e+fx\right)\right)^{m}\cot^{6}\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**6*(b*sec(f*x+e))**m,x)
```

output
```
Integral((b*sec(e + f*x))**m*cot(e + f*x)**6, x)
```

3.361.7 Maxima [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

3.361.8 Giac [F]

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int (b\sec(fx+e))^m\cot(fx+e)^6\,dx$$

input
```
integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = \int \cot(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^6*(b/cos(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^6*(b/cos(e + f*x))^m, x)
```

3.362 $\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx$

3.362.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1+m+n),\frac{3+n}{2},\sin^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(}{bf(1+n)}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3
/2+1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

3.362.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{b\,\text{Hypergeometric2F1}\left(\frac{m}{2},\frac{1-n}{2},\frac{2+m}{2},\sec^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)}{fm}$$

input
```
Integrate[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
(b*Hypergeometric2F1[m/2, (1 - n)/2, (2 + m)/2, Sec[e + f*x]^2]*(a*Sec[e +
 f*x])^m*(b*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2))/(f*m)
```

3.362.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3097}$$

$$\frac{(a\sec(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(m+n+1),\frac{n+3}{2},\sin^2(e+\right.}{bf(n+1)}$$

input
```
Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)
/2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 + n))
```

3.362.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.362.4 Maple [F]

$$\int (a\sec(fx+e))^m (b\tan(fx+e))^n\,dx$$

input
```
int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

3.362.5 Fricas [F]

$$\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx = \int (a\sec(fx+e))^m (b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.362.6 Sympy [F]

$$\int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx = \int (a\sec(e+fx))^m (b\tan(e+fx))^n\,dx$$

input
```
integrate((a*sec(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*sec(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.362.7 Maxima [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.362.8 Giac [F]

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (a\sec(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\int (b\tan(e+f\,x))^n\left(\frac{a}{\cos(e+f\,x)}\right)^m\,dx$$

input
```
int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m,x)
```

output
```
int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m, x)
```

3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$

3.363.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)}+\frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)+2*(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)+(d*tan(b
*x+a))^(5+n)/b/d^5/(5+n)
```

3.363.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$$
$$=\frac{d(d\tan(a+bx))^{-1+n}\left((8+6n+n^2+2(3+n)\cos(2(a+bx))+\cos(4(a+bx)))\sec^4(a+bx)\tan^2(a+bx\right.}{b(1+n)(3+n)(5+n)}$$

input
```
Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

output
```
(d*(d*Tan[a + b*x])^(-1 + n)*((8 + 6*n + n^2 + 2*(3 + n)*Cos[2*(a + b*x)]
+ Cos[4*(a + b*x)])*Sec[a + b*x]^4*Tan[a + b*x]^2 + 8*(-Tan[a + b*x]^2)^((
1 - n)/2)))/(b*(1 + n)*(3 + n)*(5 + n))
```

3.363.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^6(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^n\left(\tan^2(a+bx)+1\right)^2 d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((d\tan(a+bx))^n+\frac{2(d\tan(a+bx))^{n+2}}{d^2}+\frac{(d\tan(a+bx))^{n+4}}{d^4}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d\tan(a+bx))^{n+5}}{d^5(n+5)}+\frac{2(d\tan(a+bx))^{n+3}}{d^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{d(n+1)}}{b}$$

input
```
Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

output
```
((d*Tan[a + b*x])^(1 + n)/(d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(d^3*
(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(d^5*(5 + n)))/b
```

3.363.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.363.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\frac{\tan\left(bx+a\right) \mathrm{e}^{n \ln\left(d \tan\left(bx+a\right)\right)}}{b\left(1+n\right)}+\frac{\left(\tan^{5}\left(bx+a\right)\right) \mathrm{e}^{n \ln\left(d \tan\left(bx+a\right)\right)}}{b\left(5+n\right)}+\frac{2\left(\tan^{3}\left(bx+a\right)\right) \mathrm{e}^{n \ln\left(d \tan\left(bx+a\right)\right)}}{b\left(3+n\right)}$$

input
```
int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x)
```

output
```
1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(5+n)*tan(b*x+a)^5*exp(n*
ln(d*tan(b*x+a)))+2/b/(3+n)*tan(b*x+a)^3*exp(n*ln(d*tan(b*x+a)))
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(8\,\cos\left(bx+a\right)^{4}+4\left(n+1\right)\cos\left(bx+a\right)^{2}+n^{2}+4\,n+3\right)\left(\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}\right)^{n}\sin\left(bx+a\right)}{\left(bn^{3}+9\,bn^{2}+23\,bn+15\,b\right)\cos\left(bx+a\right)^{5}}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(8*cos(b*x + a)^4 + 4*(n + 1)*cos(b*x + a)^2 + n^2 + 4*n + 3)*(d*sin(b*x +
 a)/cos(b*x + a))^n*sin(b*x + a)/((b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cos(b*
x + a)^5)
```

3.363.6 Sympy [F]

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec^6(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**6*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**6, x)
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^5}{n+5} + \frac{2\,d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3} + \frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d^n*tan(b*x + a)^n*tan(b*x + a)^5/(n + 5) + 2*d^n*tan(b*x + a)^n*tan(b*x
+ a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

3.363.8 Giac [F(-2)]

Exception generated.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+%%%{1,[0,1,0,4,0]%
%%} / %%%
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)^6}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^6,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^6, x)
```

3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$

3.364.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx=\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)+(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)
```

3.364.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$$
$$=\frac{d(d\tan(a+bx))^{-1+n}\left((2+n+\cos(2(a+bx)))\sec^2(a+bx)\tan^2(a+bx)+2(-\tan^2(a+bx))^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

input
```
Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(d*(d*Tan[a + b*x])^(-1 + n)*((2 + n + Cos[2*(a + b*x)])*Sec[a + b*x]^2*Ta
n[a + b*x]^2 + 2*(-Tan[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

3.364.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^4(d\tan(a+bx))^ndx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (d\tan(a+bx))^n\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left((d\tan(a+bx))^n+\frac{(d\tan(a+bx))^{n+2}}{d^2}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(d\tan(a+bx))^{n+3}}{d^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{d(n+1)}}{b}$$

input
```
Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
((d*Tan[a + b*x])^(1 + n)/(d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(d^3*(3 +
 n)))/b
```

3.364.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.364.4 Maple [A] (verified)

Time = 56.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{\tan(bx+a)\mathrm{e}^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{(\tan^3(bx+a))\mathrm{e}^{n\ln(d\tan(bx+a))}}{b(3+n)}$	58
default	$\frac{\tan(bx+a)\mathrm{e}^{n\ln(d\tan(bx+a))}}{b(1+n)}+\frac{(\tan^3(bx+a))\mathrm{e}^{n\ln(d\tan(bx+a))}}{b(3+n)}$	58
risch	Expression too large to display	5283

input
```
int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
1/b/(1+n)*tan(b*x+a)*exp(n*ln(d*tan(b*x+a)))+1/b/(3+n)*tan(b*x+a)^3*exp(n*
ln(d*tan(b*x+a)))
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(2\,\cos\left(bx+a\right)^2+n+1\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{\left(bn^2+4\,bn+3\,b\right)\cos\left(bx+a\right)^3}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(2*cos(b*x + a)^2 + n + 1)*(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((
b*n^2 + 4*b*n + 3*b)*cos(b*x + a)^3)
```

3.364.6 Sympy [F]

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(a+bx\right)\right)^n\sec^4\left(a+bx\right)\,dx$$

input
```
integrate(sec(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**4, x)
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d^n*tan(b*x + a)^n*tan(b*x + a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(
n + 1)))/b
```

3.364.8 Giac [F(-2)]

Exception generated.

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} / %%%{1,[0,0,3,0,1
]%%%} Err
```

3.364.9 Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx = -\frac{2\left(-\frac{d\sin(2a+2bx)}{2\sin(a+bx)^2-2}\right)^n(9\sin(2a+2bx)+6\sin(4a+4bx)+\sin(6a+6bx)+4n\sin(2a+2bx)+2}{b\,(n^2+4n+3)\,\left(30\sin(a+bx)^2+12\sin(2a+2bx)^2+2\sin(3a+3bx)^2-32\right)}$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^4,x)
```

output
```
-(2*(-(d*sin(2*a + 2*b*x))/(2*sin(a + b*x)^2 - 2))^n*(9*sin(2*a + 2*b*x) +
 6*sin(4*a + 4*b*x) + sin(6*a + 6*b*x) + 4*n*sin(2*a + 2*b*x) + 2*n*sin(4*
a + 4*b*x)))/(b*(4*n + n^2 + 3)*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x
)^2 + 30*sin(a + b*x)^2 - 32))
```

3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$

3.365.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.365.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Tan[a + b*x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

3.365.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3087, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(a+bx)^2(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int (d\tan(a+bx))^n d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

3.365.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.365.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
default	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
risch	Expression too large to display	1752

input
```
int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

output
```
(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin(bx+a)}{(bn+b)\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b*n + b)*cos(b*x + a))
```

3.365.6 Sympy [F]

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n\sec^2(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**2, x)
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{(d\tan(bx+a))^{n+1}}{bd(n+1)}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
(d*tan(b*x + a))^(n + 1)/(b*d*(n + 1))
```

3.365.8 Giac [F(-2)]

Exception generated.

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument
Value
```

3.365.9 Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\sin(2a+2bx)\left(\frac{d\sin(2a+2bx)}{2\cos(a+bx)^2}\right)^n}{2b\cos(a+bx)^2\,(n+1)}$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^2,x)
```

output
```
(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(2*cos(a + b*x)^2))^n)/(2*b*cos(a
+ b*x)^2*(n + 1))
```

3.366 $\int (d\tan(a+bx))^n\,dx$

3.366.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

3.366.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int (d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

3.366.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3957, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3957}$$

$$\frac{d\int \frac{(d\tan(a+bx))^n}{\tan^2(a+bx)d^2+d^2}d(d\tan(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

3.366.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

3.366.4 Maple [F]

$$\int (d\tan (bx+a))^n\,dx$$

input
```
int((d*tan(b*x+a))^n,x)
```

output
```
int((d*tan(b*x+a))^n,x)
```

3.366.5 Fricas [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan (bx+a))^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n, x)
```

3.366.6 Sympy [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan (a+bx))^n\,dx$$

input
```
integrate((d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n, x)
```

3.366.7 Maxima [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n, x)
```

3.366.8 Giac [F]

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\,dx$$

input
```
integrate((d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n, x)
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int (d\tan(a+bx))^n\,dx = \int (d\tan(a+b\,x))^n\,dx$$

input
```
int((d*tan(a + b*x))^n,x)
```

output
```
int((d*tan(a + b*x))^n, x)
```

3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$

3.367.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

3.367.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

3.367.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3087, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^2}dx$$

$$\Big\downarrow \ 3087$$

$$\frac{\int \frac{(d\tan(a+bx))^n}{(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \ 278$$

$$\frac{(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

3.367.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.367.4 Maple [F]

$$\int \left(\cos^2\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^n dx$$

input
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

3.367.5 Fricas [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(bx+a\right)\right)^n\cos\left(bx+a\right)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

3.367.6 Sympy [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \left(d\tan\left(a+bx\right)\right)^n\cos^2\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x)**2, x)
```

3.367.7 Maxima [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

3.367.8 Giac [F]

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx = \int \cos(a+b\,x)^2\,(d\tan(a+b\,x))^n\,dx$$

input
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^2*(d*tan(a + b*x))^n, x)
```

3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$

3.368.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b
/d/(1+n)
```

3.368.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*x]*
(d*Tan[a + b*x])^n)/(b*(1 + n))
```

3.368.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3087, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^4}dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \frac{(d\tan(a+bx))^n}{(\tan^2(a+bx)+1)^3}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},-\tan^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

output
```
(Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*
x])^(1 + n))/(b*d*(1 + n))
```

3.368.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

3.368.4 Maple [F]

$$\int \left(\cos^{4}\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^{n}dx$$

input
```
int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

3.368.5 Fricas [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int \left(d\tan\left(bx+a\right)\right)^{n}\cos\left(bx+a\right)^{4}\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

3.368.6 Sympy [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx=\int \left(d\tan\left(a+bx\right)\right)^{n}\cos^{4}\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x)**4, x)
```

3.368.7 Maxima [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

3.368.8 Giac [F]

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx = \int \cos(a+b\,x)^4\,(d\tan(a+b\,x))^n\,dx$$

input
```
int(cos(a + b*x)^4*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^4*(d*tan(a + b*x))^n, x)
```

3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$

3.369.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{6+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(3+1/2*n)*hypergeom([3+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)^5*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.369.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \frac{d\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{7}{2},\sec^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{5b}$$

input
```
Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Hypergeometric2F1[5/2, (1 - n)/2, 7/2, Sec[a + b*x]^2]*Sec[a + b*x]^5*(
d*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(5*b)
```

3.369.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^5(d\tan(a+bx))^ndx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+6}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

3.369.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.369.4 Maple [F]

$$\int \left(\sec^5(bx+a)\right)\left(d\tan(bx+a)\right)^n dx$$

input
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

3.369.5 Fricas [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

3.369.6 Sympy [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**5, x)
```

3.369.7 Maxima [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

3.369.8 Giac [F]

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)^5\,dx$$

input
```
integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

3.369.9 Mupad [F(-1)]

Timed out.

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)^5}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^5,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^5, x)
```

3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$

3.370.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{4+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)^3*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.370.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{d\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{5}{2},\sec^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{3b}$$

input
```
Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(d*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[a + b*x]^2]*Sec[a + b*x]^3*(
d*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(3*b)
```

3.370.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)^3(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+4}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

3.370.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.370.4 Maple [F]

$$\int \left(\sec^3 (bx+a)\right) (d\tan (bx+a))^n \, dx$$

input
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

3.370.5 Fricas [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n \, dx = \int (d\tan (bx+a))^n \sec (bx+a)^3 \, dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

3.370.6 Sympy [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n \, dx = \int (d\tan (a+bx))^n \sec^3 (a+bx) \, dx$$

input
```
integrate(sec(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x)**3, x)
```

3.370.7 Maxima [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

3.370.8 Giac [F]

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

3.370.9 Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)^3}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x)^3,x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x)^3, x)
```

3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$

3.371.1 Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{2+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
(cos(b*x+a)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*
x+a)^2)*sec(b*x+a)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.371.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\sec^2(a+bx)\right)(d\tan(a+bx))^n\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{b}$$

input
```
Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(Csc[a + b*x]*Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Sec[a + b*x]^2]*(d*Ta
n[a + b*x])^n*(-Tan[a + b*x]^2)^((1 - n)/2))/b
```

3.371.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(a+bx)(d\tan(a+bx))^n dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{n+2}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

3.371.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.371.4 Maple [F]

$$\int \sec(bx+a)\,(d\tan(bx+a))^n\,dx$$

input
```
int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

3.371.5 Fricas [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*sec(b*x + a), x)
```

3.371.6 Sympy [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(a+bx))^n \sec(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*sec(a + b*x), x)
```

3.371.7 Maxima [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

3.371.8 Giac [F]

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \int \frac{(d\tan(a+b\,x))^n}{\cos(a+b\,x)}\,dx$$

input
```
int((d*tan(a + b*x))^n/cos(a + b*x),x)
```

output
```
int((d*tan(a + b*x))^n/cos(a + b*x), x)
```

3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$

3.372.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

output
```
cos(b*x+a)*(cos(b*x+a)^2)^(1/2*n)*hypergeom([1/2*n, 1/2+1/2*n],[3/2+1/2*n]
,sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.372.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.02 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.28

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx =$$

$$-\frac{2\left(\text{AppellF1}\left(\frac{1+n}{2},n,\right.\right.}{b(1+n)\left(-\text{AppellF1}\left(\frac{1+n}{2},n,1,\frac{3+n}{2},\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+\frac{\left(-\left(\left(\text{AppellF1}\left(\frac{3+n}{2},n,2,\frac{5+n}{2},\tan\right.\right.\right.\right.}{}\right.}$$

input
```
Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(-2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2])*Cos[(a + b*x)/2]*Cos[a + b*x]*Sin[(a + b*x)/2]*(d*Tan[a
+ b*x])^n)/(b*(1 + n)*(-AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)
/2]^2, -Tan[(a + b*x)/2]^2] + ((-((AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + n)/2, n, 3, (5 +
n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 +
 n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*n*AppellF1[
(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*
(-1 + Cos[a + b*x])) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sec[(a + b*x)/2]^2)
/(3 + n)))
```

3.372.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)}dx$$

$$\Big\downarrow \; 3097$$

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

output
```
(Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3
+ n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

3.372.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.372.4 Maple [F]

$$\int \cos(bx+a)\,(d\tan(bx+a))^n\,dx$$

input
```
int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

3.372.5 Fricas [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a), x)
```

3.372.6 Sympy [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(a+bx)})^n\cos{(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))**n,x)
```

output
```
Integral((d*tan(a + b*x))**n*cos(a + b*x), x)
```

3.372.7 Maxima [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\cos{(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

3.372.8 Giac [F]

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan{(bx+a)})^n\cos{(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \int \cos\left(a+b\,x\right)\,\left(d\tan(a+b\,x)\right)^n\,dx$$

input
```
int(cos(a + b*x)*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)*(d*tan(a + b*x))^n, x)
```

3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$

3.373.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$$
$$=\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-2+n),\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(d\tan(a+bx))}{bd(1+n)}$$

output
```
cos(b*x+a)^3*(cos(b*x+a)^2)^(-1+1/2*n)*hypergeom([-1+1/2*n, 1/2+1/2*n],[3/
2+1/2*n],sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

3.373.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 4.85 (sec) , antiderivative size = 1313, normalized size of antiderivative = 16.83

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(4*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2] - 6*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + 12*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*AppellF1[(1 + n)/2, n, 4, (3 + n)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos[a + b*x]^
3*Sin[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(1 + n
)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a
+ b*x]) - 2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + 36*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/
2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n,
 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 6*n*AppellF1[(3
+ n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12
*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 18*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 6*n*AppellF1[(1 + n
)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*
x)/2]^2 + 8*(3 + n)*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/...
```

3.373.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\tan(a+bx))^n}{\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{ 3097}$$

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{n-2}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

output
```
(Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2
, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1
+ n))
```

3.373.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

3.373.4 Maple [F]

$$\int \left(\cos^3(bx+a)\right)(d\tan(bx+a))^n\,dx$$

input
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

output
```
int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

3.373.5 Fricas [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

3.373.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

output
```
Timed out
```

3.373.7 Maxima [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

3.373.8 Giac [F]

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \int (d\tan(bx+a))^n\cos(bx+a)^3\,dx$$

input
```
integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx=\int \cos\left(a+b\,x\right)^3\,{\left(d\tan(a+b\,x)\right)}^n\,dx$$

input
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^n,x)
```

output
```
int(cos(a + b*x)^3*(d*tan(a + b*x))^n, x)
```

3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$

3.374.1 Optimal result

Integrand size = 19, antiderivative size = 40

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

output
```
-(b*csc(f*x+e))^m*hypergeom([2, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

3.374.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,2-\frac{m}{2},3-\frac{m}{2},\sin^2(e+fx)\right)\sin^4(e+fx)}{f(-4+m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, 2 - m/2, 3 - m/2, Sin[e + f*x]^
2]*Sin[e + f*x]^4)/(f*(-4 + m)))
```

3.374.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 25, 3086, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m}{\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3}dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int \frac{(b\csc(e+fx))^{m-1}}{(1-\csc^2(e+fx))^2}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2]
)/(f*m))
```

3.374.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.374.4 Maple [F]

$$\int \left(b \csc \left(f x +e \right)\right)^{m} \left(\tan^{3}\left(f x +e \right)\right)d x$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

3.374.5 Fricas [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int \left(b \csc \left(f x + e\right)\right)^{m} \tan \left(f x + e\right)^{3}\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

3.374.6 Sympy [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**3, x)
```

3.374.7 Maxima [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

3.374.8 Giac [F]

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^3\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx = \int \tan(e+f\,x)^3 \left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^3*(b/sin(e + f*x))^m, x)
```

3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$

3.375.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\csc^2(e+fx)\right)}{fm}$$

output
```
(b*csc(f*x+e))^m*hypergeom([1, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

3.375.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = -\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1,1-\frac{m}{2},2-\frac{m}{2},\sin^2(e+fx)\right)\sin^2(e+fx)}{f(-2+m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

output
```
-(((b*Csc[e + f*x])^m*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, Sin[e + f*x]^
2]*Sin[e + f*x]^2)/(f*(-2 + m)))
```

3.375.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 25, 3086, 25, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m}{\tan\left(\frac{1}{2}(2e-\pi)+fx\right)}dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int -\frac{(b\csc(e+fx))^{m-1}}{1-\csc^2(e+fx)}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{(b\csc(e+fx))^{m-1}}{1-\csc^2(e+fx)}d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{278}$$

$$\frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\csc^2(e+fx)\right)}{fm}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/
(f*m)
```

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.375.4 Maple [F]

$$\int (b\csc (fx+e))^{m} \tan (fx+e)\, dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

3.375.5 Fricas [F]

$$\int (b\csc(e+fx))^m\tan(e+fx)\,dx=\int (b\csc (fx+e))^{m} \tan (fx+e)\ dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e), x)
```

3.375.6 Sympy [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc{(e+fx)})^m \tan{(e+fx)}\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e),x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x), x)
```

3.375.7 Maxima [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc{(fx+e)})^m \tan{(fx+e)}\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

3.375.8 Giac [F]

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int (b\csc{(fx+e)})^m \tan{(fx+e)}\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx = \int \tan(e+f\,x)\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int(tan(e + f*x)*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)*(b/sin(e + f*x))^m, x)
```

3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$

3.376.1 Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx=-\frac{(b\csc(e+fx))^m}{fm}$$

output
```
-(b*csc(f*x+e))^m/f/m
```

3.376.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx=-\frac{(b\csc(e+fx))^m}{fm}$$

input
```
Integrate[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*Csc[e + f*x])^m/(f*m))
```

3.376.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 25, 3086, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{b\int (b\csc(e+fx))^{m-1}d\csc(e+fx)}{f}$$

$$\Big\downarrow \; 17$$

$$-\frac{(b\csc(e+fx))^m}{fm}$$

input
```
Int[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*Csc[e + f*x])^m/(f*m))
```

3.376.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.376.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{(b\csc(fx+e))^m}{fm}$
default	$-\frac{(b\csc(fx+e))^m}{fm}$
risch	$-\frac{\left(e^{i(fx+e)}\right)^m \left(e^{2i(fx+e)}-1\right)^{-m} 2^m b^m e^{\frac{i\pi m\left(\mathrm{csgn}\left(\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^3+\mathrm{csgn}\left(\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)^2 \mathrm{csgn}\left(\frac{ib\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}\right)-\mathrm{csgn}\left(\frac{ib\,e}{e^{2i(}\right.\right.}{}}}{}$

input
```
int(cot(f*x+e)*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
-(b*csc(f*x+e))^m/f/m
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-(b/sin(f*x + e))^m/(f*m)
```

3.376.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} x\cot(e) & \text{for } f=0 \wedge m=0 \\ x(b\csc(e))^m\cot(e) & \text{for } f=0 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f} + \frac{\log(\tan(e+fx))}{f} & \text{for } m=0 \\ -\frac{(b\csc(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))**m,x)
```

output
```
Piecewise((x*cot(e), Eq(f, 0) & Eq(m, 0)), (x*(b*csc(e))**m*cot(e), Eq(f,
0)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f, Eq(m, 0)), (-
(b*csc(e + f*x))**m/(f*m), True))
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{b^m\sin(fx+e)^{-m}}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
-b^m*sin(f*x + e)^(-m)/(f*m)
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

input
```
integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
-(b/sin(f*x + e))^m/(f*m)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \cot(e+fx)(b\csc(e+fx))^m\,dx = \begin{cases} -\frac{\ln\left(\frac{b}{\sin(e+f\,x)}\right)}{f} & \text{if } m=0 \\ -\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{f\,m} & \text{if } m\neq 0 \end{cases}$$

input
```
int(cot(e + f*x)*(b/sin(e + f*x))^m,x)
```

output
```
piecewise(m == 0, -log(b/sin(e + f*x))/f, m ~= 0, -(b/sin(e + f*x))^m/(f*m
))
```

3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$

3.377.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}$$

output
```
(b*csc(f*x+e))^m/f/m-(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)
```

3.377.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx=\frac{(b\csc(e+fx))^m\,(2+m-m\csc^2(e+fx))}{fm(2+m)}$$

input
```
Integrate[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

output
```
((b*Csc[e + f*x])^m*(2 + m - m*Csc[e + f*x]^2))/(f*m*(2 + m))
```

3.377.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^3\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int -(b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int (b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left((b\csc(e+fx))^{m-1}-\frac{(b\csc(e+fx))^{m+1}}{b^2}\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(b\csc(e+fx))^{m+2}}{b^3(m+2)}-\frac{(b\csc(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*(-((b*Csc[e + f*x])^m/(b*m)) + (b*Csc[e + f*x])^(2 + m)/(b^3*(2 + m))
))/f)
```

3.377.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.377.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.16 (sec) , antiderivative size = 3514, normalized size of antiderivative = 81.72

method	result	size
risch	Expression too large to display	3514

input
```
int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
1/(2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*b^m*exp(I*(f*x+e))^m*(exp(2*I*(f*x+e))-
1)^(-m)*2^m*(m*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*
m)*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*csgn(I*b/(ex
p(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+
e))-1)*exp(I*(f*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp
(I*(f*x+e)))^2*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(-1/2*I*P
i*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I/(exp(2*I*(f*x+e))-1))*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*m)*exp(-1/2*I*Pi*csgn(I/(exp(
2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*
(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m
)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*
(f*x+e)))*m)*exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*m
)*exp(-1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*Pi*csgn(I*b/(exp(
2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(4*I*e)*exp(4*I*f*x)
+2*exp(1/2*I*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*Pi*m)*exp(1/2*I
*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*Pi*csgn(I*b/(exp(2*I*(f*x+e
))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I
*(f*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e...
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\left((m+2)\cos\left(fx+e\right)^2-2\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{fm^2-\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2+2\,fm}$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-((m + 2)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^m/(f*m^2 - (f*m^2 + 2*f*m)*
cos(f*x + e)^2 + 2*f*m)
```

3.377.6 Sympy [F]

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$
$$= \begin{cases} x(b\csc(e))^m\cot^3(e) & \text{for } f=0 \\ \frac{\int \frac{\cot^3(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for } m=-2 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} - \frac{\log(\tan(e+fx))}{f} - \frac{1}{2f\tan^2(e+fx)} & \text{for } m=0 \\ -\frac{m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^2+2fm} + \frac{2(b\csc(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(f*x+e)**3*(b*csc(f*x+e))**m,x)
```

output
```
Piecewise((x*(b*csc(e))**m*cot(e)**3, Eq(f, 0)), (Integral(cot(e + f*x)**3
/csc(e + f*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) - l
og(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2), Eq(m, 0)), (-m*(b*csc(e + f*
x))**m*cot(e + f*x)**2/(f*m**2 + 2*f*m) + 2*(b*csc(e + f*x))**m/(f*m**2 +
2*f*m), True))
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \frac{\frac{b^m\sin(fx+e)^{-m}}{m} - \frac{b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2}}{f}$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
(b^m*sin(f*x + e)^(-m)/m - b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2))
/f
```

3.377.8 Giac [F]

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^3, x)
```

3.377.9 Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$$
$$= \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m \left(m+4\sin\left(2\,e+2\,f\,x\right)^2+m\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)-16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,\left(m+2\right)}$$

input
```
int(cot(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

output
```
((b/sin(e + f*x))^m*(m + 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
1) - 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m
 + 2))
```

3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

3.378.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)} - \frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}$$

output
```
-(b*csc(f*x+e))^m/f/m+2*(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)-(b*csc(f*x+e))^(4
+m)/b^4/f/(4+m)
```

3.378.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m\left(8+6m+m^2-2m(4+m)\csc^2(e+fx)+m(2+m)\csc^4(e+fx)\right)}{fm(2+m)(4+m)}$$

input
```
Integrate[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

output
```
-(((b*Csc[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Csc[e + f*x]^2 + m*(2 +
 m)*Csc[e + f*x]^4))/(f*m*(2 + m)*(4 + m)))
```

3.378.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 25, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^5\left(-\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(b\sec\left(\frac{1}{2}(2e-\pi)+fx\right)\right)^m\tan\left(\frac{1}{2}(2e-\pi)+fx\right)^5dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{b\int (b\csc(e+fx))^{m-1}\left(1-\csc^2(e+fx)\right)^2d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left((b\csc(e+fx))^{m-1}-\frac{2(b\csc(e+fx))^{m+1}}{b^2}+\frac{(b\csc(e+fx))^{m+3}}{b^4}\right)d\csc(e+fx)}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(b\csc(e+fx))^{m+4}}{b^5(m+4)}-\frac{2(b\csc(e+fx))^{m+2}}{b^3(m+2)}+\frac{(b\csc(e+fx))^m}{bm}\right)}{f}$$

input
```
Int[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

output
```
-((b*((b*Csc[e + f*x])^m/(b*m) - (2*(b*Csc[e + f*x])^(2 + m))/(b^3*(2 + m)
) + (b*Csc[e + f*x])^(4 + m)/(b^5*(4 + m))))/f)
```

3.378.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

3.378.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.14 (sec) , antiderivative size = 8846, normalized size of antiderivative = 128.20

method	result	size
risch	Expression too large to display	8846

input
```
int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.378.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.67

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx =$$
$$-\frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-4\,(m+4)\cos\left(fx+e\right)^2+8\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^4+fm^3+6\,fm^2-2\,(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^2+8\,fm}$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
-((m^2 + 6*m + 8)*cos(f*x + e)^4 - 4*(m + 4)*cos(f*x + e)^2 + 8)*(b/sin(f*
x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 + 6*f*m^2 - 2*
(f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

3.378.6 Sympy [F]

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$$
$$=\begin{cases} x(b\csc(e))^m\cot^5(e) & \text{for } f \\ \frac{\int\frac{\cot^5(e+fx)}{\csc^4(e+fx)}\,dx}{b^4} & \text{for } m \\ \frac{\int\frac{\cot^5(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for } m \\ -\frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\log(\tan(e+fx))}{f}+\frac{1}{2f\tan^2(e+fx)}-\frac{1}{4f\tan^4(e+fx)} & \text{for } m \\ -\frac{m^2(b\csc(e+fx))^m\cot^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{2m(b\csc(e+fx))^m\cot^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{4m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^3+6fm^2+8fm}-\frac{8(b\csc(e+fx))^m}{fm^3+6fm^2+8fm} & \text{other} \end{cases}$$

input
```
integrate(cot(f*x+e)**5*(b*csc(f*x+e))**m,x)
```

output
```
Piecewise((x*(b*csc(e))**m*cot(e)**5, Eq(f, 0)), (Integral(cot(e + f*x)**5
/csc(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(cot(e + f*x)**5/csc(e + f
*x)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e
+ f*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4), Eq(m, 0)),
(-m**2*(b*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 2
*m*(b*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 4*m*(
b*csc(e + f*x))**m*cot(e + f*x)**2/(f*m**3 + 6*f*m**2 + 8*f*m) - 8*(b*csc(
e + f*x))**m/(f*m**3 + 6*f*m**2 + 8*f*m), True))
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\frac{b^m\sin(fx+e)^{-m}}{m} - \frac{2\,b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2} + \frac{b^m\sin(fx+e)^{-m}}{(m+4)\sin(fx+e)^4}}{f}$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
-(b^m*sin(f*x + e)^(-m)/m - 2*b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^
2) + b^m*sin(f*x + e)^(-m)/((m + 4)*sin(f*x + e)^4))/f
```

3.378.8 Giac [F]

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^5, x)
```

3.378.9 Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.22

$$\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx =$$

$$-\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m\left(2\sin\left(2\,e+2\,f\,x\right)^2+\sin\left(4\,e+4\,f\,x\right)\,1\mathrm{i}-1\right)\left(\frac{2\left(2\sin(2\,e+2\,f\,x)^2-1\right)\left(-2\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\ldots\right.}{f\,m}\right.}{1\ldots}$$

input
```
int(cot(e + f*x)^5*(b/sin(e + f*x))^m,x)
```

output
```
-((b/sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*((2*
(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1
))/(f*m) - ((sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(4*m + 6*m^2
+ 48))/(f*m*(6*m + m^2 + 8)) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)
*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(8*m + 4*m^2 - 32))/(f*m*(6*m + m^2 + 8)))
)/(16*sin(e + f*x)^4)
```

3.378. $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$

3.379.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(-3+m),-\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e+f}{3f}$$

output
```
1/3*(b*csc(f*x+e))^m*hypergeom([-3/2, -3/2+1/2*m],[-1/2],cos(f*x+e)^2)*(si
n(f*x+e)^2)^(-3/2+1/2*m)*tan(f*x+e)^3/f
```

3.379.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 240 vs. 2(63) = 126.

Time = 8.73 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.81

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = -\frac{\cos(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3}{2},\cos^2(e+fx)\right)\sin(e+fx)\sin^2(e+fx)^{\frac{1}{2}(-}}{f} + \frac{4(b\csc(e+fx))^m\left(\frac{m\,\text{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\tan\left(\frac{1}{2}(e+fx)\right)}{-1+m}-\frac{1}{2}\tan(e+\right.}{f} + \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan(e}{f(1-m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
-((Cos[e + f*x]*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
Cos[e + f*x]^2]*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 + m)/2))/f) + (4*(b*Csc
[e + f*x])^m*((m*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/2 - m/2, -Tan[(e +
f*x)/2]^2]*Tan[(e + f*x)/2])/((-1 + m)*(Sec[(e + f*x)/2]^2)^m) - Tan[e + f
*x]/2))/f + ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1 - m/2, 1/2 - m/2, 3/2
 - m/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(f*(1 - m)*(Sec[e + f*x]^2)^(m/2))
```

3.379.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \ 3097$$

$$\frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},-\frac{1}{2},\cos^2(e+fx)\right)}{3f}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]
^2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

3.379.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.379.4 Maple [F]

$$\int (b\csc (fx+e))^m \left(\tan^4 (fx+e)\right) dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

3.379.5 Fricas [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\, dx = \int (b\csc (fx+e))^m \tan (fx+e)^4\, dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

3.379.6 Sympy [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**4, x)
```

3.379.7 Maxima [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

3.379.8 Giac [F]

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^4\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

3.379.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4 \left(\frac{b}{\sin(e+f\,x)}\right)^m dx$$

input
```
int(tan(e + f*x)^4*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$

3.380.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(-1+m),\frac{1}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+fx)}{f}$$

output
```
(b*csc(f*x+e))^m*hypergeom([-1/2, -1/2+1/2*m],[1/2],cos(f*x+e)^2)*(sin(f*x
+e)^2)^(-1/2+1/2*m)*tan(f*x+e)/f
```

3.380.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(1-\frac{m}{2},\frac{3}{2}-\frac{m}{2},\frac{5}{2}-\frac{m}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan^3(e+f}{f(3-m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 3/2 - m/2, 5/2 - m/2, -Tan[
e + f*x]^2]*Tan[e + f*x]^3)/(f*(3 - m)*(Sec[e + f*x]^2)^(m/2))
```

3.380.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m}{\tan\left(e+fx-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3097}$$

$$\frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{1}{2},\cos^2(e+fx)\right)}{f}$$

input
```
Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

3.380.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.380.4 Maple [F]

$$\int (b\csc(fx+e))^m \left(\tan^2(fx+e)\right) dx$$

input
```
int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

3.380.5 Fricas [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

3.380.6 Sympy [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$$

input
```
integrate((b*csc(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((b*csc(e + f*x))**m*tan(e + f*x)**2, x)
```

3.380.7 Maxima [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

3.380.8 Giac [F]

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int (b\csc(fx+e))^m \tan(fx+e)^2\,dx$$

input
```
integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2 \left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int(tan(e + f*x)^2*(b/sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$

3.381.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^3(e+fx)(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+m}{2},\frac{5}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

output
```
-1/3*cot(f*x+e)^3*(b*csc(f*x+e))^m*hypergeom([3/2, 3/2+1/2*m],[5/2],cos(f*
x+e)^2)*(sin(f*x+e)^2)^(3/2+1/2*m)/f
```

3.381.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(63) = 126.

Time = 1.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.95

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = -\frac{(b\csc(e+fx))^m\left(-4(1+m)\text{Hypergeometric2F1}\left(1-m,\frac{1}{2}-\frac{m}{2},\frac{3}{2}-\frac{m}{2},-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(-1+\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

output
```
-1/2*((b*Csc[e + f*x])^m*(-4*(1 + m)*Hypergeometric2F1[1 - m, 1/2 - m/2, 3
/2 - m/2, -Tan[(e + f*x)/2]^2] + (-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometri
c2F1[-1/2 - m/2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2] + (1 + m)*Hypergeomet
ric2F1[1/2 - m/2, -m, 3/2 - m/2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(
f*(-1 + m^2)*(Sec[(e + f*x)/2]^2)^m)
```

3.381.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^2\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+3}{2},\frac{5}{2},\cos^2(e+fx)\right)}{3f}$$

input
```
Int[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

output
```
-1/3*(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2,
5/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/f
```

3.381.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.381.4 Maple [F]

$$\int \left(\cot^2\left(fx+e\right)\right)\left(b\csc\left(fx+e\right)\right)^m dx$$

input
```
int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

3.381.5 Fricas [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int \left(b\csc\left(fx+e\right)\right)^m \cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

3.381.6 Sympy [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(e+fx))^m \cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(b*csc(f*x+e))**m,x)
```

output
```
Integral((b*csc(e + f*x))**m*cot(e + f*x)**2, x)
```

3.381.7 Maxima [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

3.381.8 Giac [F]

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx = \int \cot(e+f\,x)^2\left(\frac{b}{\sin(e+f\,x)}\right)^m dx$$

input
```
int(cot(e + f*x)^2*(b/sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$

3.382.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^5(e+fx)(b\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{5}{2},\frac{5+m}{2},\frac{7}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

output
```
-1/5*cot(f*x+e)^5*(b*csc(f*x+e))^m*hypergeom([5/2, 5/2+1/2*m],[7/2],cos(f*
x+e)^2)*(sin(f*x+e)^2)^(5/2+1/2*m)/f
```

3.382.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot(e+fx)(b\csc(e+fx))^m \left(\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3}{2},\cos^2(e+fx)\right)-2\operatorname{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{f}$$

input
```
Integrate[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

output
```
-((Cot[e + f*x]*(b*Csc[e + f*x])^m*(Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
 Cos[e + f*x]^2] - 2*Hypergeometric2F1[1/2, (3 + m)/2, 3/2, Cos[e + f*x]^2
] + Hypergeometric2F1[1/2, (5 + m)/2, 3/2, Cos[e + f*x]^2])*(Sin[e + f*x]^
2)^((1 + m)/2))/f)
```

3.382.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(e+fx-\frac{\pi}{2}\right)^4\left(b\sec\left(e+fx-\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3097}$$

$$-\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+5}{2},\frac{7}{2},\cos^2(e+fx)\right)}{5f}$$

input
```
Int[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

output
```
-1/5*(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2,
7/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/f
```

3.382.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

3.382.4 Maple [F]

$$\int \left(\cot^4(fx+e)\right)\left(b\csc(fx+e)\right)^m dx$$

input
```
int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

3.382.5 Fricas [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

3.382.6 Sympy [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(e+fx))^m \cot^4(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**4*(b*csc(f*x+e))**m,x)
```

output
```
Integral((b*csc(e + f*x))**m*cot(e + f*x)**4, x)
```

3.382.7 Maxima [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

3.382.8 Giac [F]

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int (b\csc(fx+e))^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = \int \cot(e+f\,x)^4\left(\frac{b}{\sin(e+f\,x)}\right)^m\,dx$$

input
```
int(cot(e + f*x)^4*(b/sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$

3.383.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+fx\right.}{df(5-2m)}$$

output
```
2*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^m*hypergeom([5/4, 5/4-1/2*m],[9/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(5/2)/d/f/(5-2*m)
```

3.383.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),1-\frac{m}{2},\frac{1}{4}(9-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}(}{df(-5+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(5 - 2*m)/4, 1 - m/2, (9 - 2*m)/4
, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(-5 + 2*m)*(Sec[e + f*x]^2
)^(m/2))
```

3.383.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\tan(e+fx))^{3/2}(b\csc(e+fx))^m dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\tan(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+3}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}-m}}{\cos^{\frac{3}{2}}(e+fx)}dx}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+3}\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{3}{2}-m}}{\cos(e+fx)^{3/2}}dx}{bd}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{4}(5-2m),\frac{1}{4}(9-2m),\sin^2(e+fx)\right)}{df(5-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

output
```
(2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2
*m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m)
)
```

3.383.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.383.4 Maple [F]

$$\int (b\csc(fx+e))^m (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

3.383.5 Fricas [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(fx+e))^{\frac{3}{2}}(b\csc(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m*d*tan(f*x + e), x)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.383.7 Maxima [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(fx+e))^{\frac{3}{2}}(b\csc(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

3.383.8 Giac [F]

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \int (d\tan{(fx+e)})^{\frac{3}{2}}\,(b\csc{(fx+e)})^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m(d\tan(e+fx))^{3/2}\,dx = \int (d\tan(e+f\,x))^{3/2}\,\left(\frac{b}{\sin{(e+f\,x)}}\right)^m\,dx$$

input
```
int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m,x)
```

output
```
int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m, x)
```

3.384 $\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx$

3.384.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right)(d\tan(e+\cdots}{df(3-2m)}$$

output
```
2*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^m*hypergeom([3/4, 3/4-1/2*m],[7/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(3/2)/d/f/(3-2*m)
```

3.384.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m),1-\frac{m}{2},\frac{1}{4}(7-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m\cdots}}{df(-3+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(3 - 2*m)/4, 1 - m/2, (7 - 2*m)/4
, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(-3 + 2*m)*(Sec[e + f*x]^2
)^(m/2))
```

3.384.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\tan(e+fx)}(b\csc(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{d\tan(e+fx)}(b\csc(e+fx))^m dx$$

$$\Big\downarrow \ 3098$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \ 3042$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \left(\frac{\sin(e+fx)}{b}\right)^{-m} \sqrt{d\tan(e+fx)}dx$$

$$\Big\downarrow \ 3082$$

$$\frac{\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+2}\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}dx}{bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}}(b\csc(e+fx))^{m+2}\int\frac{\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}-m}}{\sqrt{\cos(e+fx)}}dx}{bd}$$

$$\Big\downarrow \ 3057$$

$$\frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}(3-2m),\frac{1}{4}(7-2m),\sin^2(e+fx)\right)}{df(3-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2
*m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m)
)
```

3.384.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.384.4 Maple [F]

$$\int (b\csc(fx+e))^m \sqrt{d\tan(fx+e)}dx$$

input
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

3.384.5 Fricas [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

3.384.6 Sympy [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \int (b\csc\left(e+fx\right))^m\sqrt{d\tan\left(e+fx\right)}\,dx$$

input
```
integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**m*sqrt(d*tan(e + f*x)), x)
```

3.384.7 Maxima [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

3.384.8 Giac [F]

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(fx+e\right)}(b\csc\left(fx+e\right))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

3.384.9 Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx=\int \sqrt{d\tan\left(e+f\,x\right)}\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

input
```
int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m,x)
```

output
```
int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m, x)
```

3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$

3.385.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \frac{2\sqrt[4]{\cos^2(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)\sqrt{d\tan(e}}{df(1-2m)}$$

output
```
2*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^m*hypergeom([1/4, 1/4-1/2*m],[5/4-1/
2*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(1/2)/d/f/(1-2*m)
```

3.385.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = -\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),1-\frac{m}{2},\frac{1}{4}(5-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m}}{df(-1+2m)}$$

input
```
Integrate[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(1 - 2*m)/4, 1 - m/2, (5 - 2*m)/4
, -Tan[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(-1 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

3.385.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^{m+1} \int \sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{1}{2}}dx}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^{m+1} \int \sqrt{\cos(e+fx)}\left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{1}{2}}dx}{bd}$$

$$\Big\downarrow \text{3057}$$

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4}(1-2m),\frac{1}{4}(5-2m),\sin^2(e+fx)\right)}{df(1-2m)}$$

input
```
Int[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

output
```
(2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2
*m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

3.385.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.385.4 Maple [F]

$$\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}dx$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

output
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

3.385.5 Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d*tan(f*x + e)), x)
```

3.385.6 Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$$

input
```
integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(1/2),x)
```

output
```
Integral((b*csc(e + f*x))**m/sqrt(d*tan(e + f*x)), x)
```

3.385.7 Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

3.385.8 Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

3.385.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx=\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{\sqrt{d\tan(e+f\,x)}}\,dx$$

input
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2),x)
```

output
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2), x)
```

3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$

3.386.1 Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-1-2m),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

output
```
-2*(b*csc(f*x+e))^m*hypergeom([-1/4, -1/4-1/2*m],[3/4-1/2*m],sin(f*x+e)^2)
/d/f/(1+2*m)/(cos(f*x+e)^2)^(1/4)/(d*tan(f*x+e))^(1/2)
```

3.386.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = -\frac{2(b\csc(e+fx))^m \,\text{Hypergeometric2F1}\left(\frac{1}{4}(-1-2m),1-\frac{m}{2},\frac{1}{4}(3-2m),-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}}{df(1+2m)\sqrt{d\tan(e+fx)}}$$

input
```
Integrate[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(-1 - 2*m)/4, 1 - m/2, (3 - 2*m)/
4, -Tan[e + f*x]^2])/(d*f*(1 + 2*m)*(Sec[e + f*x]^2)^(m/2)*Sqrt[d*Tan[e +
f*x]])
```

3.386.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{b}\right)^m (b\csc(e+fx))^m \int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^m \int \cos^{\frac{3}{2}}(e+fx) \left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{3}{2}}dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(\frac{\sin(e+fx)}{b}\right)^{m+\frac{1}{2}} (b\csc(e+fx))^m \int \cos(e+fx)^{3/2} \left(\frac{\sin(e+fx)}{b}\right)^{-m-\frac{3}{2}}dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$\Big\downarrow \text{3057}$$

$$-\frac{2(b\csc(e+fx))^m \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4}(-2m-1),\frac{1}{4}(3-2m),\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

input
```
Int[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

output
```
(-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4,
Sin[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]
])
```

3.386.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.386.4 Maple [F]

$$\int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

output
```
int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

3.386.5 Fricas [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d^2*tan(f*x + e)^2), x)
```

3.386.6 Sympy [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(3/2),x)
```

output
```
Integral((b*csc(e + f*x))**m/(d*tan(e + f*x))**(3/2), x)
```

3.386.7 Maxima [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

3.386.8 Giac [F]

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{(d\tan(e+f\,x))^{3/2}}\,dx$$

input
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2),x)
```

output
```
int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2), x)
```

3.387 $\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx$

3.387.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^m \operatorname{Hypergeometric2F1}\left(\frac{1+n}{2},\frac{1}{2}(1-m+n),\frac{1}{2}(3-m+n),\sin^2(e+fx)\right)(b$$
$$\overline{\qquad bf(1-m+n)\qquad}$$

output
```
(cos(f*x+e)^2)^(1/2+1/2*n)*(a*csc(f*x+e))^m*hypergeom([1/2+1/2*n, 1/2-1/2*
m+1/2*n],[3/2-1/2*m+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

3.387.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.00 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx =$$

$$-\frac{a(}{f(-1+m-n)\left((-3+m-n)\operatorname{AppellF1}\left(\frac{1}{2}(1-m+n),n,1-m,\frac{1}{2}(3-m+n),\tan^2\left(\frac{1}{2}(e+fx)\right),-\right.\right.}$$

input
```
Integrate[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
-((a*(-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m + n)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(a*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f*x
])^n)/(f*(-1 + m - n)*((-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 -
 m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m)*AppellF1
[(3 - m + n)/2, n, 2 - m, (5 - m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + n*AppellF1[(3 - m + n)/2, 1 + n, 1 - m, (5 - m + n)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

3.387.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3098, 3042, 3082, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3098}$$

$$\left(\frac{\sin(e+fx)}{a}\right)^m(a\csc(e+fx))^m\int \left(\frac{\sin(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\sin(e+fx)}{a}\right)^m(a\csc(e+fx))^m\int \left(\frac{\sin(e+fx)}{a}\right)^{-m}(b\tan(e+fx))^ndx$$

$$\Big\downarrow \text{3082}$$

$$\frac{\cos^{n+1}(e+fx)(a\csc(e+fx))^{m+1}(b\tan(e+fx))^{n+1}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}\int \cos^{-n}(e+fx)\left(\frac{\sin(e+fx)}{a}\right)^{n-m}dx}{ab}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{n+1}(e+fx)(a\csc(e+fx))^{m+1}(b\tan(e+fx))^{n+1}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}\int \cos(e+fx)^{-n}\left(\frac{\sin(e+fx)}{a}\right)^{n-m}dx}{ab}$$

$\big\downarrow$ 3057

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\csc(e+fx))^m(b\tan(e+fx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1),\frac{1}{2}(-m+n+3),\text{si}\right.}{bf(-m+n+1)}$$

input
```
Int[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

output
```
((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)
/2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(1 - m + n))
```

3.387.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3082
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1)))   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x
], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

rule 3098
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m]   Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.387.4 Maple [F]

$$\int (a\csc(fx+e))^m (b\tan(fx+e))^n \, dx$$

input
```
int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

output
```
int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

3.387.5 Fricas [F]

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\csc(fx+e))^m (b\tan(fx+e))^n \, dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.387.6 Sympy [F]

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n \, dx = \int (a\csc(e+fx))^m (b\tan(e+fx))^n \, dx$$

input
```
integrate((a*csc(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

output
```
Integral((a*csc(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

3.387.7 Maxima [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\csc(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.387.8 Giac [F]

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (a\csc(fx+e))^m(b\tan(fx+e))^n\,dx$$

input
```
integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int (a\csc(e+fx))^m(b\tan(e+fx))^n\,dx = \int (b\tan(e+f\,x))^n\left(\frac{a}{\sin(e+f\,x)}\right)^m dx$$

input
```
int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m,x)
```

output
```
int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```